Mathematical Methods Units 1 & 2

Chapter 1 — Linear equations

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Contents

Section	Page
1A Solving linear equations	2
1B Forming linear equations	6
1C Solving simultaneous equations	10
1D Forming simultaneous equations	13
1E Linear inequalities	18
1F Transposition of formulas	22

Theory

A **linear equation** in one unknown contains that unknown to the first power only (no x^2, no x in a way that cannot be simplified to first degree). To **solve** an equation we find the value of the unknown that makes it true, by doing the **same operation to both sides** until the unknown is the subject.

Key idea. Whatever you do to one side of an equation you must do to the other, so the two sides stay equal. Aim to gather the unknown on one side and the numbers on the other.

Common types and the first move for each:

- **One- and two-step:** undo the operations in reverse order (undo $+$ / $-$ first, then $\times$ / $\div$).
- **Brackets: expand** first, then collect like terms.
- **Unknown on both sides:** collect the unknown terms on one side (usually the side that keeps the coefficient positive) and the numbers on the other.
- **Fractions:** multiply **every term** by the lowest common denominator (LCD) to clear the fractions, then solve.
- **Unknown in a denominator:** multiply both sides by that denominator; the equation then reduces to a linear one (assume the denominator is not zero).

You can always **check** a solution by substituting it back into the original equation.

Worked examples

Example 1 — scaffolded

Solve $3x+5=20$.

Working	Comment
$3x+5=20$	Aim to make x the subject.
$3x=20-5=15$	Subtract 5 from both sides.
$x=\dfrac{15}{3}=5$	Divide both sides by 3.
Check: $3(5)+5=20$. ✓	Substitute back to confirm.

Example 2 — scaffolded

Solve $2(x-4)=3x+1$.

Working	Comment
$2x-8=3x+1$	Expand the bracket first.
$-8-1=3x-2x$	Collect x-terms on one side, numbers on the other.
$-9=x$	Simplify.
$\therefore x=-9$	Check: $2(-9-4)=-26$ and $3(-9)+1=-26$. ✓

Example 3 — unscaffolded

Solve $\dfrac{x}{3}+\dfrac{x}{4}=7$.

Multiply every term by the LCD, 12:

$$12\times\frac{x}{3}+12\times\frac{x}{4}=12\times7\Rightarrow 4x+3x=84.$$

$7x = 84$, so $\therefore x = 12$.

Example 4 — unscaffolded

Solve $\frac{5}{x-2} = 3$.

Multiply both sides by $(x-2)$: $5 = 3(x-2) = 3x - 6$.

Then $3x = 11$, so $\therefore x = \frac{11}{3}$.

Practice questions

Scaffolded

Q1. Solve $4x - 7 = 13$.

Q2. Solve $5 - 2x = 11$.

Q3. Solve $3(x+2) = 24$.

Q4. Solve $7x - 4 = 3x + 32$.

Q5. Solve $\frac{x}{5} + 2 = 6$.

Q6. Solve $\frac{2x-1}{3} = 5$.

Unscaffolded

Solve each of the following for x.

Q7. $6x + 11 = 53$

Q8. $8 - 3x = 23$

Q9. $4(2x - 3) = 20$

Q10. $3(x+5) - 2(x-1) = 20$

Q11. $5x - 8 = 2x + 31$

Q12. $4 - x = 3x + 12$

Q13. $9 - 2x = 5x - 5$

Q14. $\frac{x}{2} + \frac{x}{3} = 10$

Q15. $\frac{x+1}{4} = \frac{x-2}{3}$

Q16. $\frac{2x}{5} - 3 = 1$

Q17. $\frac{6}{x+1} = 3$

Q18. $\dfrac{3}{2x-1}=\dfrac{4}{x+3}$

Answers

Q1. $x = 5$. **Q2.** $x = -3$. **Q3.** $x = 6$. **Q4.** $x = 9$. **Q5.** $x = 20$. **Q6.** $x = 8$. **Q7.** $x = 7$. **Q8.** $x = -5$. **Q9.** $x = 4$. **Q10.** $x = 3$.
Q11. $x = 13$. **Q12.** $x = -2$. **Q13.** $x = 2$. **Q14.** $x = 12$. **Q15.** $x = 11$. **Q16.** $x = 10$. **Q17.** $x = 1$. **Q18.** $x = \frac{13}{5}$.

Fully-worked solutions

Q1. $4x - 7 = 13 \Rightarrow 4x = 20 \Rightarrow x = 5$.

Q2. $5 - 2x = 11 \Rightarrow -2x = 6 \Rightarrow x = -3$.

Q3. $3(x+2) = 24 \Rightarrow x + 2 = 8 \Rightarrow x = 6$.

Q4. $7x - 4 = 3x + 32 \Rightarrow 4x = 36 \Rightarrow x = 9$.

Q5. $\frac{x}{5} + 2 = 6 \Rightarrow \frac{x}{5} = 4 \Rightarrow x = 20$.

Q6. $\frac{2x-1}{3} = 5 \Rightarrow 2x - 1 = 15 \Rightarrow 2x = 16 \Rightarrow x = 8$.

Q7. $6x + 11 = 53 \Rightarrow 6x = 42 \Rightarrow x = 7$.

Q8. $8 - 3x = 23 \Rightarrow -3x = 15 \Rightarrow x = -5$.

Q9. $4(2x-3) = 20 \Rightarrow 2x - 3 = 5 \Rightarrow 2x = 8 \Rightarrow x = 4$.

Q10. $3(x+5) - 2(x-1) = 20 \Rightarrow 3x + 15 - 2x + 2 = 20 \Rightarrow x + 17 = 20 \Rightarrow x = 3$.

Q11. $5x - 8 = 2x + 31 \Rightarrow 3x = 39 \Rightarrow x = 13$.

Q12. $4 - x = 3x + 12 \Rightarrow 4 - 12 = 3x + x \Rightarrow -8 = 4x \Rightarrow x = -2$.

Q13. $9 - 2x = 5x - 5 \Rightarrow 9 + 5 = 5x + 2x \Rightarrow 14 = 7x \Rightarrow x = 2$.

Q14. Multiply by the LCD 6: $3x + 2x = 60 \Rightarrow 5x = 60 \Rightarrow x = 12$.

Q15. Cross-multiply: $3(x+1) = 4(x-2) \Rightarrow 3x + 3 = 4x - 8 \Rightarrow 11 = x$, so $x = 11$.

Q16. $\frac{2x}{5} - 3 = 1 \Rightarrow \frac{2x}{5} = 4 \Rightarrow 2x = 20 \Rightarrow x = 10$.

Q17. $\frac{6}{x+1} = 3 \Rightarrow 6 = 3(x+1) \Rightarrow 6 = 3x + 3 \Rightarrow 3x = 3 \Rightarrow x = 1$.

Q18. Cross-multiply: $3(x+3) = 4(2x-1) \Rightarrow 3x + 9 = 8x - 4 \Rightarrow 13 = 5x$, so $x = \frac{13}{5}$.

Chapter 1 Linear equations — 1B Forming linear equations

Theory

Many problems can be solved by **constructing a linear equation** — translating the words of a problem into an equation, then solving it. A reliable method has four steps.

1. **Define the unknown.** Choose a pronumeral for the quantity you are asked to find, and state what it represents **including units** (e.g. "let d be the distance in kilometres").
2. **Construct the equation.** Translate each phrase of the problem into algebra and write a linear equation.
3. **Solve** the equation.
4. **Interpret and check.** Answer the question in words with correct units, and check the answer satisfies the original problem.

Some phrases translate directly:

- "the sum of" → $+$; "the difference" → $-$; "the product" → $\times$; "is" / "gives" → $=$.
- "5 more than x" → $x+5$; "3 less than x" → $x-3$; "twice x" → $2x$.
- **Consecutive integers**: $n, n+1, n+2, \ldots$
- **Consecutive even (or odd) numbers**: $n, n+2, n+4, \ldots$

Worked examples

Example 1 — scaffolded

The sum of three consecutive integers is 72. Find the integers.

Working	Comment
Let the smallest integer be n. The three integers are n, $n+1$, $n+2$.	Define the unknown; write consecutive integers in terms of n.
$n+(n+1)+(n+2)=72$	Construct the equation: their sum is 72.
$3n+3=72 \Rightarrow 3n=69 \Rightarrow n=23$	Collect like terms and solve.
The integers are 23, 24 and 25.	Interpret. Check: $23+24+25=72$. ✓

Example 2 — scaffolded

A taxi charges a flagfall of \$5.00 plus \$1.80 per kilometre. A trip costs \$23.00. How far was the trip?

Working	Comment
Let d be the distance in kilometres.	Define the unknown with units.
$5+1.8d=23$	Cost = flagfall + rate × distance.
$1.8d=18 \Rightarrow d=10$	Subtract 5, then divide by 1.8.
The trip was 10 km.	Interpret. Check: $5+1.8\times 10=23$. ✓

Example 3 — unscaffolded

Sarah is 4 years older than her brother Tom. The sum of their ages is 26. How old is each?

Let Tom's age be t years. Then Sarah's age is $t+4$ years.

$$t+(t+4)=26 \Rightarrow 2t+4=26 \Rightarrow 2t=22 \Rightarrow t=11.$$

∴ Tom is 11 and Sarah is 15. (Check: $11+15=26$ and $15-11=4$. ✓)

Example 4 — unscaffolded

The perimeter of a rectangle is 54 cm. Its length is 5 cm more than its width. Find the dimensions.

Let the width be w cm. Then the length is $(w+5)$ cm.

$$2(w+(w+5))=54 \Rightarrow 2(2w+5)=54 \Rightarrow 4w+10=54 \Rightarrow 4w=44 \Rightarrow w=11.$$

∴ the width is 11 cm and the length is 16 cm. (Check: $2(11+16)=54$. ✓)

Practice questions

Scaffolded

Q1. The sum of two consecutive integers is 45. (a) Let the smaller integer be n and write an equation. (b) Solve it. (c) State the two integers.

Q2. A number increased by 17 gives 43. Let the number be x, write an equation, and find the number.

Q3. Three times a number, decreased by 8, is 25. Find the number.

Q4. A rectangle's length is twice its width and its perimeter is 48 cm. (a) Let the width be w cm and write an equation. (b) Solve for the dimensions.

Q5. Anna has \$5 more than twice the amount Ben has. Together they have \$50. (a) Let Ben have \$$b$ and write an equation. (b) Find how much each has.

Q6. A plumber charges an \$80 call-out fee plus \$65 per hour. A job costs \$340. How many hours did it take?

Unscaffolded

For each problem, define the unknown, construct a linear equation, solve it, and answer in words.

Q7. The sum of three consecutive integers is 90. Find the integers.

Q8. The sum of three consecutive even numbers is 84. Find the numbers.

Q9. A mother is three times as old as her daughter. In 12 years she will be twice as old as her daughter. Find their present ages.

Q10. Tom is 5 years older than Jerry. Five years ago, Tom was twice as old as Jerry. Find their present ages.

Q11. After a 20% discount, a jacket costs \$72. Find its original price.

Q12. Three friends share a restaurant bill equally. After a \$12 service charge is added to the bill, each friend pays \$28. What was the bill before the service charge?

Q13. An isosceles triangle has two equal sides and a base that is 4 cm shorter than each equal side. Its perimeter is 41 cm. Find the length of each side.

Q14. A rope 60 m long is cut into two pieces so that one piece is 8 m longer than the other. Find the length of each piece.

Q15. A mobile phone plan charges a fixed \$19 per month plus \$0.12 per minute of calls. One month's bill is \$37. How many minutes of calls were made?

Q16. A car hire costs \$45 plus \$0.25 per kilometre travelled. The total charge is \$145. How far was the car driven?

Q17. A sum of money is shared between two people in the ratio $3:5$. The larger share is \$120 more than the smaller share. Find the total amount of money.

Q18. The sum of a number and 6 equals four times the number, decreased by 9. Find the number.

Answers

Q1. 22 and 23. **Q2.** 26. **Q3.** 11. **Q4.** width 8 cm, length 16 cm. **Q5.** Ben \$15, Anna \$35. **Q6.** 4 hours. **Q7.** 29, 30, 31 . **Q8.** 26, 28, 30. **Q9.** daughter 12, mother 36. **Q10.** Jerry 10, Tom 15. **Q11.** \$90. **Q12.** \$72. **Q13.** equal sides 15 cm, base 11 cm. **Q14.** 26 m and 34 m. **Q15.** 150 minutes. **Q16.** 400 km. **Q17.** \$480. **Q18.** 5.

Fully-worked solutions

Q1. Let the smaller integer be n; the integers are n and $n+1$. $n+(n+1)=45 \Rightarrow 2n+1=45 \Rightarrow 2n=44 \Rightarrow n=22$. ∴ the integers are 22 and 23.

Q2. Let the number be x. $x+17=43 \Rightarrow x=26$. ∴ the number is 26.

Q3. Let the number be x. $3x-8=25 \Rightarrow 3x=33 \Rightarrow x=11$. ∴ the number is 11.

Q4. Let the width be w cm; the length is $2w$ cm. $2(w+2w)=48 \Rightarrow 6w=48 \Rightarrow w=8$. ∴ the width is 8 cm and the length is 16 cm.

Q5. Let Ben have \$$b$; Anna has \$$(2b+5)$. $b+(2b+5)=50 \Rightarrow 3b+5=50 \Rightarrow 3b=45 \Rightarrow b=15$. ∴ Ben has \$15 and Anna has \$35.

Q6. Let the job take h hours. $80+65h=340 \Rightarrow 65h=260 \Rightarrow h=4$. ∴ the job took 4 hours.

Q7. Let the smallest integer be n; the integers are n, $n+1$, $n+2$.
$n+(n+1)+(n+2)=90 \Rightarrow 3n+3=90 \Rightarrow 3n=87 \Rightarrow n=29$. ∴ the integers are 29, 30 and 31.

Q8. Let the smallest even number be n; the numbers are n, $n+2$, $n+4$.
$n+(n+2)+(n+4)=84 \Rightarrow 3n+6=84 \Rightarrow 3n=78 \Rightarrow n=26$. ∴ the numbers are 26, 28 and 30.

Q9. Let the daughter's age be d years; the mother's age is $3d$ years. In 12 years:
$3d+12=2(d+12) \Rightarrow 3d+12=2d+24 \Rightarrow d=12$. ∴ the daughter is 12 and the mother is 36. (Check: in 12 years, $48=2\times 24$. ✓)

Q10. Let Jerry's age be j years; Tom's age is $(j+5)$ years. Five years ago:
$(j+5)-5=2(j-5) \Rightarrow j=2j-10 \Rightarrow j=10$. ∴ Jerry is 10 and Tom is 15. (Check: five years ago $10=2\times 5$. ✓)

Q11. Let the original price be \$$x$. A 20% discount leaves 80% of the price: $0.8x=72 \Rightarrow x=90$. ∴ the original price was \$90.

Q12. Let the bill be \$$b$. $\dfrac{b+12}{3}=28 \Rightarrow b+12=84 \Rightarrow b=72$. ∴ the bill was \$72.

Q13. Let each equal side be s cm; the base is $(s-4)$ cm. $2s+(s-4)=41 \Rightarrow 3s-4=41 \Rightarrow 3s=45 \Rightarrow s=15$. ∴ the equal sides are 15 cm and the base is 11 cm.

Q14. Let the shorter piece be x m; the longer piece is $(x+8)$ m. $x+(x+8)=60 \Rightarrow 2x+8=60 \Rightarrow 2x=52 \Rightarrow x=26$. ∴ the pieces are 26 m and 34 m.

Q15. Let the call time be m minutes. $19+0.12m=37 \Rightarrow 0.12m=18 \Rightarrow m=150$. ∴ 150 minutes of calls were made.

Q16. Let the distance be k km. $45+0.25k=145 \Rightarrow 0.25k=100 \Rightarrow k=400$. ∴ the car was driven 400 km.

Q17. Let the shares be $3x$ and $5x$ dollars. $5x-3x=120 \Rightarrow 2x=120 \Rightarrow x=60$. The shares are \$180 and \$300, so the total is $8x=\$480$. ∴ the total amount is \$480.

Q18. Let the number be x. $x+6=4x-9 \Rightarrow 6+9=4x-x \Rightarrow 15=3x \Rightarrow x=5$. ∴ the number is 5. (Check: $5+6=11$ and $4\times 5-9=11$. ✓)

Chapter 1 Linear equations — 1C Solving simultaneous equations

Theory

Simultaneous equations are two or more equations that must hold at the same time. For two linear equations in x and y, a **solution** is a pair (x, y) that satisfies **both** equations.

Graphical meaning. Each linear equation is a straight line, so the solution is the **point of intersection** of the two lines. Two lines usually meet at one point (one solution), but parallel lines never meet (**no solution**) and two copies of the same line meet everywhere (**infinitely many solutions**).

Substitution method. Make one variable the subject of one equation, then substitute that expression into the other equation, leaving a single equation in one variable.

Elimination method. Add or subtract multiples of the two equations so that one variable is **eliminated**. If necessary, first multiply one or both equations so that the coefficients of the variable to be eliminated are equal in size.

Special cases. When solving, if you reach a **false** statement (such as $0=3$) the lines are parallel and there is **no solution**; if you reach an **always-true** statement (such as $0=0$) the equations describe the same line and there are **infinitely many solutions**.

Worked examples

Example 1 — scaffolded

Solve the simultaneous equations $y=2x-1$ and $3x+y=9$.

Working	Comment
The first equation already has y as the subject.	Choose the equation where a variable is isolated.
Substitute into the second: $3x+(2x-1)=9$.	Replace y with $2x-1$.
$5x-1=9 \Rightarrow 5x=10 \Rightarrow x=2$.	Solve the single equation for x.
$y=2(2)-1=3$.	Substitute back to find y.
Solution $(2,3)$. Check: $3(2)+3=9$. ✓	Always check in the *other* equation.

Example 2 — scaffolded

Solve the simultaneous equations $2x+3y=12$ and $2x-y=4$.

Working	Comment
The x-coefficients are equal. Subtract: $(2x+3y)-(2x-y)=12-4$.	Subtracting the equations eliminates x.
$4y=8 \Rightarrow y=2$.	Solve for y.
Substitute into $2x-y=4$: $2x-2=4 \Rightarrow 2x=6 \Rightarrow x=3$.	Back-substitute to find x.
Solution $(3,2)$. Check: $2(3)+3(2)=12$. ✓	Check in the first equation.

Example 3 — unscaffolded

Solve $3x+2y=16$ and $5x-4y=12$.

Multiply the first equation by 2: $6x+4y=32$.

Add this to the second equation: $(6x+4y)+(5x-4y)=32+12 \Rightarrow 11x=44 \Rightarrow x=4$.

Substitute into $3x+2y=16$: $12+2y=16 \Rightarrow 2y=4 \Rightarrow y=2$.

∴ the solution is $(4, 2)$.

Example 4 — unscaffolded

Solve $x + y = 1$ and $2x - y = -7$.

Adding the equations eliminates y: $(x + y) + (2x - y) = 1 + (-7) \Rightarrow 3x = -6 \Rightarrow x = -2$.

Substitute into $x + y = 1$: $-2 + y = 1 \Rightarrow y = 3$.

∴ the solution is $(-2, 3)$.

Practice questions

Scaffolded

Q1. Solve by substitution: $y = 2x$ and $x + y = 9$.

Q2. Solve by substitution: $y = x - 3$ and $2x + y = 12$.

Q3. Solve by elimination: $x + y = 7$ and $x - y = 1$.

Q4. Solve by elimination: $3x + y = 10$ and $x + y = 6$.

Q5. Solve by elimination: $2x + 3y = 8$ and $x - y = -1$ *(hint: multiply the second equation by* 3 *first)*.

Q6. Solve by substitution: $x = 2y + 1$ and $3x - y = 8$.

Unscaffolded

Solve each pair of simultaneous equations.

Q7. $y = x + 1$ and $3x + 2y = 12$

Q8. $y = 2x - 3$ and $x + y = 3$

Q9. $x + y = 5$ and $x - y = 9$

Q10. $2x + y = 7$ and $3x - y = 8$

Q11. $3x + 2y = 5$ and $x + 4y = 15$

Q12. $2x + 3y = 4$ and $3x + 2y = 1$

Q13. $2x + y = 3$ and $4x - y = 0$

Q14. $2x + 4y = 5$ and $2x - 4y = 1$

Q15. $x + 3y = -2$ and $2x - y = 3$

Q16. $x + 2y = 5$ and $2x + 4y = 3$

Q17. $3x - y = 2$ and $6x - 2y = 4$

Q18. $4x + 3y = 18$ and $2x - y = 4$

Answers

Q1. $(3,6)$. **Q2.** $(5,2)$. **Q3.** $(4,3)$. **Q4.** $(2,4)$. **Q5.** $(1,2)$. **Q6.** $(3,1)$. **Q7.** $(2,3)$. **Q8.** $(2,1)$. **Q9.** $(7,-2)$. **Q10.** $(3,1)$. **Q11.** $(-1,4)$. **Q12.** $(-1,2)$. **Q13.** $(1/2,2)$. **Q14.** $(3/2,1/2)$. **Q15.** $(1,-1)$. **Q16.** no solution (parallel lines). **Q17.** infinitely many solutions (the same line). **Q18.** $(3,2)$.

Fully-worked solutions

Q1. Substitute $y=2x$ into $x+y=9$: $x+2x=9 \Rightarrow 3x=9 \Rightarrow x=3$; then $y=2(3)=6$. $\therefore (3,6)$.

Q2. Substitute $y=x-3$ into $2x+y=12$: $2x+(x-3)=12 \Rightarrow 3x-3=12 \Rightarrow x=5$; then $y=5-3=2$. $\therefore (5,2)$.

Q3. Add the equations: $(x+y)+(x-y)=7+1 \Rightarrow 2x=8 \Rightarrow x=4$; then $4+y=7 \Rightarrow y=3$. $\therefore (4,3)$.

Q4. Subtract: $(3x+y)-(x+y)=10-6 \Rightarrow 2x=4 \Rightarrow x=2$; then $2+y=6 \Rightarrow y=4$. $\therefore (2,4)$.

Q5. Multiply the second equation by 3: $3x-3y=-3$. Add to $2x+3y=8$: $5x=5 \Rightarrow x=1$; then $1-y=-1 \Rightarrow y=2$. $\therefore (1,2)$.

Q6. Substitute $x=2y+1$ into $3x-y=8$: $3(2y+1)-y=8 \Rightarrow 6y+3-y=8 \Rightarrow 5y=5 \Rightarrow y=1$; then $x=2(1)+1=3$. $\therefore (3,1)$.

Q7. Substitute $y=x+1$ into $3x+2y=12$: $3x+2(x+1)=12 \Rightarrow 5x+2=12 \Rightarrow x=2$; then $y=3$. $\therefore (2,3)$.

Q8. Substitute $y=2x-3$ into $x+y=3$: $x+(2x-3)=3 \Rightarrow 3x=6 \Rightarrow x=2$; then $y=1$. $\therefore (2,1)$.

Q9. Add: $2x=14 \Rightarrow x=7$; then $7+y=5 \Rightarrow y=-2$. $\therefore (7,-2)$.

Q10. Add: $(2x+y)+(3x-y)=7+8 \Rightarrow 5x=15 \Rightarrow x=3$; then $2(3)+y=7 \Rightarrow y=1$. $\therefore (3,1)$.

Q11. Multiply the first by 2: $6x+4y=10$. Subtract the second ($x+4y=15$): $5x=-5 \Rightarrow x=-1$; then $-1+4y=15 \Rightarrow 4y=16 \Rightarrow y=4$. $\therefore (-1,4)$.

Q12. Multiply the first by 3 and the second by 2: $6x+9y=12$ and $6x+4y=2$. Subtract: $5y=10 \Rightarrow y=2$; then $2x+3(2)=4 \Rightarrow 2x=-2 \Rightarrow x=-1$. $\therefore (-1,2)$.

Q13. Add: $(2x+y)+(4x-y)=3+0 \Rightarrow 6x=3 \Rightarrow x=1/2$; then $4(1/2)-y=0 \Rightarrow y=2$. $\therefore (1/2,2)$.

Q14. Add: $4x=6 \Rightarrow x=3/2$. Subtract: $8y=4 \Rightarrow y=1/2$. $\therefore (3/2,1/2)$.

Q15. From the second equation, $y=2x-3$. Substitute into the first: $x+3(2x-3)=-2 \Rightarrow 7x-9=-2 \Rightarrow 7x=7 \Rightarrow x=1$; then $y=2(1)-3=-1$. $\therefore (1,-1)$.

Q16. Multiply the first equation by 2: $2x+4y=10$. But the second equation says $2x+4y=3$. These cannot both be true ($10 \neq 3$), so the lines are parallel and there is **no solution**.

Q17. Divide the second equation by 2: $3x-y=2$ — identical to the first equation. The two equations describe the **same line**, so there are **infinitely many solutions** (every point on $3x-y=2$).

Q18. From the second equation, $y=2x-4$. Substitute into the first: $4x+3(2x-4)=18 \Rightarrow 10x-12=18 \Rightarrow 10x=30 \Rightarrow x=3$; then $y=2(3)-4=2$. $\therefore (3,2)$.

Theory

Many problems describe **two unknown quantities** linked by **two conditions**. Each condition can be written as a linear equation, and the pair is then solved **simultaneously**.

Method.

1. **Define two variables**, including their units.
2. **Translate each condition** in the problem into a linear equation.
3. **Solve the two equations simultaneously**, by elimination (add or subtract to remove a variable) or by substitution.
4. **Interpret** the solution in context, state the units, and **check** it satisfies both conditions.

Choosing variables well and writing one equation per condition is the key step — once the two equations are set up, the algebra is the same simultaneous-equation technique from Section 1C.

Worked examples

Example 1 — scaffolded

At a café, 3 coffees and 2 muffins cost \$23, while 5 coffees and 2 muffins cost \$33. Find the cost of one coffee and one muffin.

Working	Comment
Let c = cost of a coffee (\$) and m = cost of a muffin (\$).	Define both variables, with units.
$3c+2m=23$ and $5c+2m=33$.	One equation per condition.
Subtract: $(5c+2m)-(3c+2m)=33-23 \Rightarrow 2c=10 \Rightarrow c=5$.	Eliminate m — its coefficients are equal.
$3(5)+2m=23 \Rightarrow 15+2m=23 \Rightarrow m=4$.	Substitute back to find m.
A coffee costs \$5 and a muffin costs \$4.	Interpret with units. Check: $5(5)+2(4)=33$ ✓.

Example 2 — scaffolded

Sara is 4 years older than her brother Tom. In 3 years, the sum of their ages will be 30. Find their present ages.

Working	Comment
Let s = Sara's age and t = Tom's age (years).	Define both variables.
$s=t+4$.	Sara is 4 years older.
$(s+3)+(t+3)=30 \Rightarrow s+t=24$.	In 3 years each age increases by 3.
Substitute $s=t+4$: $(t+4)+t=24 \Rightarrow 2t=20 \Rightarrow t=10$, so $s=14$.	Substitution, then back-substitute.
Sara is 14 and Tom is 10.	Check: in 3 years $17+13=30$ ✓.

Example 3 — unscaffolded

A chemist mixes a 20% acid solution with a 50% acid solution to make 300 mL of a 40% acid solution. How much of each is used?

Let x = volume of the 20% solution and y = volume of the 50% solution (mL).

Total volume: $x+y=300$.

Acid content: $0.2x+0.5y=0.4\times 300=120$.

From the first equation $x = 300 - y$. Substituting:
$0.2(300 - y) + 0.5y = 120 \Rightarrow 60 + 0.3y = 120 \Rightarrow 0.3y = 60 \Rightarrow y = 200$, so $x = 100$.

∴ the chemist uses 100 mL of the 20% solution and 200 mL of the 50% solution.

Example 4 — unscaffolded

A two-digit number has digits that add to 11. When the digits are reversed, the new number is 27 more than the original. Find the number.

Let t = the tens digit and u = the units digit. The original number is $10t + u$.

Digit sum: $t + u = 11$.

Reversed number is 27 more: $(10u + t) - (10t + u) = 27 \Rightarrow 9u - 9t = 27 \Rightarrow u - t = 3$.

Adding $t + u = 11$ and $u - t = 3$: $2u = 14 \Rightarrow u = 7$, so $t = 4$.

∴ the number is 47. (Check: $4 + 7 = 11$; $74 - 47 = 27$ ✓.)

Practice questions

Scaffolded

Q1. A concert sold 250 tickets for a total of $3150. Adult tickets cost $15 and child tickets cost $9. (a) Letting a be the number of adult tickets and c the number of child tickets, write two equations. (b) Solve them. (c) State how many of each were sold.

Q2. Two pens and three notebooks cost $16, while four pens and one notebook cost $12. (a) Set up two equations for the cost of a pen p and a notebook n (in dollars). (b) Solve. (c) State each cost.

Q3. A mother is 3 times as old as her daughter. In 12 years, she will be twice as old as her daughter. (a) Define two variables. (b) Form two equations. (c) Find their present ages.

Q4. A rectangle's length is 5 cm more than its width, and its perimeter is 50 cm. Find its dimensions.

Q5. A cash box contains only $5 and $10 notes — 30 notes worth $260 in total. How many of each are there?

Q6. The sum of two numbers is 40 and their difference is 8. Find the two numbers.

Unscaffolded

For each problem, define suitable variables, form two equations, and solve.

Q7. Three apples and four bananas cost $7.20; five apples and two bananas cost $7.80. Find the cost of one apple and one banana.

Q8. Four pies and three sausage rolls cost $27.50; two pies and five sausage rolls cost $26.00. Find the cost of a pie and a sausage roll.

Q9. Ben is twice as old as his sister Amy. Five years ago, the sum of their ages was 20. Find their present ages.

Q10. The sum of Maria's age and her father's age is 50. In 5 years, her father will be twice as old as Maria. Find their present ages.

Q11. The digits of a two-digit number add to 9. When the digits are reversed, the number decreases by 45. Find the number.

Q12. The tens digit of a two-digit number is 3 more than the units digit, and the number equals 7 times the sum of its digits. Find the number.

Q13. A cinema sold 300 tickets for $4600. Adult tickets are $18 and child tickets are $10. How many of each were sold?

Q14. A shopkeeper mixes lollies worth $8/kg with lollies worth $12/kg to make 20 kg of a mixture worth $9.50/kg. How many kilograms of each are used?

Q15. A total of $10 000 is invested, part at 4% p.a. and part at 6% p.a. The total interest after one year is $520. How much is invested at each rate?

Q16. A rectangular garden is 6 m longer than it is wide and has a perimeter of 68 m. Find its dimensions and its area.

Q17. An isosceles triangle has two equal sides and a base. Each equal side is 3 cm longer than the base, and the perimeter is 36 cm. Find the length of each side.

Q18. Two numbers differ by 4. Three times the larger plus twice the smaller is 72. Find the two numbers.

Answers

Q1. 150 adult, 100 child. **Q2.** Pen \$2, notebook \$4. **Q3.** Daughter 12, mother 36. **Q4.** 15 cm by 10 cm. **Q5.** Eight \$5 notes and twenty-two \$10 notes. **Q6.** 24 and 16. **Q7.** Apple \$1.20, banana \$0.90. **Q8.** Pie \$4.25, sausage roll \$3.50. **Q9.** Amy 10, Ben 20. **Q10.** Maria 15, father 35. **Q11.** 72. **Q12.** 63. **Q13.** 200 adult, 100 child. **Q14.** 12.5 kg at \$8/kg and 7.5 kg at \$12/kg. **Q15.** \$4000 at 4% and \$6000 at 6%. **Q16.** 20 m by 14 m; area 280 m². **Q17.** Base 10 cm, each equal side 13 cm. **Q18.** 16 and 12.

Fully-worked solutions

Q1. Let a = adult tickets, c = child tickets. $a+c=250$ and $15a+9c=3150$. From the first, $c=250-a$; substituting: $15a+9(250-a)=3150 \Rightarrow 6a+2250=3150 \Rightarrow a=150$, so $c=100$. ∴ 150 adult and 100 child tickets.

Q2. Let p, n = cost of a pen and a notebook (\$). $2p+3n=16$ and $4p+n=12$. From the second, $n=12-4p$; substituting: $2p+3(12-4p)=16 \Rightarrow -10p+36=16 \Rightarrow p=2$, so $n=4$. ∴ a pen costs \$2 and a notebook \$4.

Q3. Let m, d = mother's and daughter's ages (years). $m=3d$ and $m+12=2(d+12)$. Substituting: $3d+12=2d+24 \Rightarrow d=12$, so $m=36$. ∴ the daughter is 12 and the mother is 36. (Check: in 12 years, $48=2\times 24$ ✓.)

Q4. Let L, w = length and width (cm). $L=w+5$ and $2(L+w)=50$, i.e. $L+w=25$. Substituting: $(w+5)+w=25 \Rightarrow 2w=20 \Rightarrow w=10$, so $L=15$. ∴ 15 cm by 10 cm.

Q5. Let f, t = number of \$5 and \$10 notes. $f+t=30$ and $5f+10t=260$. From the first, $f=30-t$; substituting: $5(30-t)+10t=260 \Rightarrow 150+5t=260 \Rightarrow t=22$, so $f=8$. ∴ eight \$5 notes and twenty-two \$10 notes.

Q6. Let the numbers be x and y. $x+y=40$ and $x-y=8$. Adding: $2x=48 \Rightarrow x=24$, so $y=16$. ∴ 24 and 16.

Q7. Let a, b = cost of an apple and a banana (\$). $3a+4b=7.20$ and $5a+2b=7.80$. Multiply the second by 2: $10a+4b=15.60$. Subtracting the first: $7a=8.40 \Rightarrow a=1.20$; then $3(1.20)+4b=7.20 \Rightarrow 4b=3.60 \Rightarrow b=0.90$. ∴ an apple costs \$1.20 and a banana \$0.90.

Q8. Let p, r = cost of a pie and a sausage roll (\$). $4p+3r=27.50$ and $2p+5r=26.00$. Multiply the second by 2: $4p+10r=52.00$. Subtracting the first: $7r=24.50 \Rightarrow r=3.50$; then $4p+3(3.50)=27.50 \Rightarrow 4p=17.00 \Rightarrow p=4.25$. ∴ a pie costs \$4.25 and a sausage roll \$3.50.

Q9. Let A, B = Amy's and Ben's present ages (years). $B=2A$ and $(B-5)+(A-5)=20$, i.e. $A+B=30$. Substituting: $A+2A=30 \Rightarrow A=10$, so $B=20$. ∴ Amy is 10 and Ben is 20.

Q10. Let M, F = Maria's and her father's present ages (years). $M+F=50$ and $F+5=2(M+5)$, i.e. $F=2M+5$. Substituting: $M+2M+5=50 \Rightarrow 3M=45 \Rightarrow M=15$, so $F=35$. ∴ Maria is 15 and her father is 35.

Q11. Let t, u = tens and units digits. $t+u=9$ and the number decreases by 45 when reversed: $(10t+u)-(10u+t)=45 \Rightarrow 9t-9u=45 \Rightarrow t-u=5$. Adding $t+u=9$: $2t=14 \Rightarrow t=7$, so $u=2$. ∴ the number is 72. (Check: $72-27=45$ ✓.)

Q12. Let t, u = tens and units digits. $t=u+3$ and $10t+u=7(t+u)$, i.e. $10t+u=7t+7u \Rightarrow 3t=6u \Rightarrow t=2u$. Then $2u=u+3 \Rightarrow u=3$, so $t=6$. ∴ the number is 63. (Check: $63=7\times 9$ ✓.)

Q13. Let a, c = adult and child tickets. $a+c=300$ and $18a+10c=4600$. From the first, $c=300-a$; substituting: $18a+10(300-a)=4600 \Rightarrow 8a+3000=4600 \Rightarrow a=200$, so $c=100$. ∴ 200 adult and 100 child tickets.

Q14. Let x, y = kilograms of the \$8/kg and \$12/kg lollies. $x+y=20$ and $8x+12y=9.50\times 20=190$. From the first, $x=20-y$; substituting: $8(20-y)+12y=190 \Rightarrow 160+4y=190 \Rightarrow y=7.5$, so $x=12.5$. ∴ 12.5 kg at \$8/kg and 7.5 kg at \$12/kg.

Q15. Let x, y = dollars invested at 4% and 6%. $x + y = 10\,000$ and $0.04x + 0.06y = 520$. From the first, $x = 10\,000 - y$; substituting: $0.04(10\,000 - y) + 0.06y = 520 \Rightarrow 400 + 0.02y = 520 \Rightarrow y = 6000$, so $x = 4000$. ∴ \$4000 at 4% and \$6000 at 6%.

Q16. Let L, w = length and width (m). $L = w + 6$ and $2(L + w) = 68$, i.e. $L + w = 34$. Substituting: $(w + 6) + w = 34 \Rightarrow 2w = 28 \Rightarrow w = 14$, so $L = 20$. Area = $L \times w = 20 \times 14 = 280$. ∴ 20 m by 14 m, with area 280 m².

Q17. Let b = base and s = each equal side (cm). $s = b + 3$ and $b + 2s = 36$. Substituting: $b + 2(b + 3) = 36 \Rightarrow 3b + 6 = 36 \Rightarrow b = 10$, so $s = 13$. ∴ the base is 10 cm and each equal side is 13 cm.

Q18. Let x = the larger and y = the smaller number. $x - y = 4$ and $3x + 2y = 72$. From the first, $x = y + 4$; substituting: $3(y + 4) + 2y = 72 \Rightarrow 5y + 12 = 72 \Rightarrow y = 12$, so $x = 16$. ∴ the numbers are 16 and 12.

Theory

An **inequality** uses one of the symbols < (less than), ≤ (less than or equal to), > (greater than) or ≥ (greater than or equal to). **Solving** an inequality means finding *all* values of the pronumeral that make it true — so the solution is usually a whole set of numbers, not a single value.

Solve an inequality just like an equation, with one crucial exception. You may:

- add or subtract the same number on both sides; and
- multiply or divide both sides by the same **positive** number,

and the inequality still holds. **But when you multiply or divide both sides by a *negative* number, you must reverse (flip) the inequality sign.** For example, from $-x<3$, dividing by -1 gives $x>-3$.

Writing the solution. Two standard forms are used:

- **Set-builder notation** (Australian colon form): $\{x:x>2\}$, read "the set of all x such that $x>2$".
- **Interval notation**: $(2,\infty)$. A square bracket $[\,]$ includes the endpoint (for ≤ or ≥); a round bracket $(\,)$ excludes it (for < or >). Infinity is always open: $(-\infty,4]$ means $x\leq 4$.

A solution may also be shown on a **number line**, using an open circle ○ for a strict inequality (< or >) and a closed circle • for an inclusive one (≤ or ≥).

Worked examples

Example 1 — scaffolded

Solve $3x-5\leq 7$ and write the solution in set-builder and interval notation.

Working	Comment
$3x-5\leq 7$	Solve as for an equation.
$3x\leq 12$	Add 5 to both sides.
$x\leq 4$	Divide both sides by 3 (positive — the sign is unchanged).
$\{x:x\leq 4\}$, or $(-\infty,4]$.	State the solution set; ≤ gives a closed bracket.

Example 2 — scaffolded

Solve $4-2x<10$.

Working	Comment
$4-2x<10$	
$-2x<6$	Subtract 4 from both sides.
$x>-3$	Divide both sides by -2; **reverse** the sign (dividing by a negative).
$\{x:x>-3\}$, or $(-3,\infty)$.	State the solution set.

Example 3 — unscaffolded

Solve $5(x-2)\geq 2x+7$.

$$5(x-2)\geq 2x+7\Rightarrow 5x-10\geq 2x+7\Rightarrow 3x\geq 17\Rightarrow x\geq 17/3.$$

$\therefore \{x : x \geq 17/3\}$, or $[17/3, \infty)$.

Example 4 — unscaffolded

Solve $\frac{2x}{3} - 1 > \frac{x}{2}$.

Multiply every term by the lowest common denominator, 6:

$$6 \times 2x/3 - 6 \times 1 > 6 \times x/2 \Rightarrow 4x - 6 > 3x \Rightarrow x > 6.$$

$\therefore \{x : x > 6\}$, or $(6, \infty)$.

Practice questions

Scaffolded

Q1. Solve $x + 7 > 12$.

Q2. Solve $2x - 3 \leq 9$.

Q3. Solve $5 - x \geq 1$.

Q4. Solve $-3x < 15$.

Q5. Solve $2(x + 4) \leq 3x + 5$.

Q6. Solve $\frac{x}{4} + 1 > 2$.

Unscaffolded

Solve each inequality, giving your answer in set-builder notation (and in interval notation where indicated).

Q7. $4x + 3 < 19$

Q8. $7 - 2x \geq 1$

Q9. $3(x - 2) \leq 12$

Q10. $5x + 2 > 2x - 7$

Q11. $2 - 4x \leq 14$

Q12. $6 - x > 3x - 2$

Q13. $-2(x - 3) < 8$

Q14. $\frac{x}{2} - 3 \geq 1$ *(give both set-builder and interval notation)*

Q15. $\frac{2x - 1}{3} < 5$

Q16. $\frac{x}{2} + \frac{x}{3} \leq 5$

Q17. $4 - 3x \geq x + 12$ *(give both set-builder and interval notation)*

Q18. $\frac{3 - x}{2} > \frac{x + 1}{4}$

Answers

Q1. $\{x: x>5\}$. **Q2.** $\{x: x\le 6\}$. **Q3.** $\{x: x\le 4\}$. **Q4.** $\{x: x>-5\}$. **Q5.** $\{x: x\ge 3\}$. **Q6.** $\{x: x>4\}$. **Q7.** $\{x: x<4\}$. **Q8.** $\{x: x\le 3\}$. **Q9.** $\{x: x\le 6\}$. **Q10.** $\{x: x>-3\}$. **Q11.** $\{x: x\ge -3\}$. **Q12.** $\{x: x<2\}$. **Q13.** $\{x: x>-1\}$. **Q14.** $\{x: x\ge 8\}$, or $[8,\infty)$. **Q15.** $\{x: x<8\}$. **Q16.** $\{x: x\le 6\}$. **Q17.** $\{x: x\le -2\}$, or $(-\infty,-2]$. **Q18.** $\{x: x<5/3\}$.

Fully-worked solutions

Q1. $x+7>12 \Rightarrow x>5$. $\therefore \{x: x>5\}$.

Q2. $2x-3\le 9 \Rightarrow 2x\le 12 \Rightarrow x\le 6$. $\therefore \{x: x\le 6\}$.

Q3. $5-x\ge 1 \Rightarrow -x\ge -4 \Rightarrow x\le 4$ (divide by -1, reverse the sign). $\therefore \{x: x\le 4\}$.

Q4. $-3x<15 \Rightarrow x>-5$ (divide by -3, reverse the sign). $\therefore \{x: x>-5\}$.

Q5. $2(x+4)\le 3x+5 \Rightarrow 2x+8\le 3x+5 \Rightarrow 3\le x$. $\therefore \{x: x\ge 3\}$.

Q6. $\frac{x}{4}+1>2 \Rightarrow \frac{x}{4}>1 \Rightarrow x>4$. $\therefore \{x: x>4\}$.

Q7. $4x+3<19 \Rightarrow 4x<16 \Rightarrow x<4$. $\therefore \{x: x<4\}$.

Q8. $7-2x\ge 1 \Rightarrow -2x\ge -6 \Rightarrow x\le 3$ (divide by -2, reverse). $\therefore \{x: x\le 3\}$.

Q9. $3(x-2)\le 12 \Rightarrow x-2\le 4 \Rightarrow x\le 6$. $\therefore \{x: x\le 6\}$.

Q10. $5x+2>2x-7 \Rightarrow 3x>-9 \Rightarrow x>-3$. $\therefore \{x: x>-3\}$.

Q11. $2-4x\le 14 \Rightarrow -4x\le 12 \Rightarrow x\ge -3$ (divide by -4, reverse). $\therefore \{x: x\ge -3\}$.

Q12. $6-x>3x-2 \Rightarrow 8>4x \Rightarrow x<2$. $\therefore \{x: x<2\}$.

Q13. $-2(x-3)<8 \Rightarrow -2x+6<8 \Rightarrow -2x<2 \Rightarrow x>-1$ (divide by -2, reverse). $\therefore \{x: x>-1\}$.

Q14. $\frac{x}{2}-3\ge 1 \Rightarrow \frac{x}{2}\ge 4 \Rightarrow x\ge 8$. $\therefore \{x: x\ge 8\}$, or $[8,\infty)$.

Q15. $\frac{2x-1}{3}<5 \Rightarrow 2x-1<15 \Rightarrow 2x<16 \Rightarrow x<8$. $\therefore \{x: x<8\}$.

Q16. $\frac{x}{2}+\frac{x}{3}\le 5$. Multiply by 6: $3x+2x\le 30 \Rightarrow 5x\le 30 \Rightarrow x\le 6$. $\therefore \{x: x\le 6\}$.

Q17. $4-3x\ge x+12 \Rightarrow -4x\ge 8 \Rightarrow x\le -2$ (divide by -4, reverse). $\therefore \{x: x\le -2\}$, or $(-\infty,-2]$.

Q18. $\frac{3-x}{2}>\frac{x+1}{4}$. Multiply by 4: $2(3-x)>x+1 \Rightarrow 6-2x>x+1 \Rightarrow 5>3x \Rightarrow x<5/3$. $\therefore \{x: x<5/3\}$.

Theory

A **formula** is an equation that relates two or more quantities — for example $v = u + at$ relates final velocity v to initial velocity u, acceleration a and time t. The variable written by itself, usually on the left, is called the **subject** of the formula (v is the subject above).

Using a formula (substitution). To use a formula, substitute the known values for the pronumerals and evaluate the remaining variable.

Transposing a formula (also called *changing the subject* or *rearranging*) means using inverse operations to make a *different* variable the subject. As with any equation, whatever operation you apply to one side you must apply to the other, so the equation stays balanced.

Squares and square roots.

- If the required subject is **squared**, take the square root of both sides. In a measurement context (length, radius, time) take the **positive** root, e.g. $A = \pi r^2 \Rightarrow r = \sqrt{\frac{A}{\pi}}$.
- If the required subject is **under a square root**, square both sides to remove the root.

Subject appearing more than once. If the required variable appears in more than one term, collect all terms containing it on one side, then **factorise** it out and divide, e.g. $ax + b = cx + d \Rightarrow x(a - c) = d - b \Rightarrow x = \frac{d - b}{a - c}$.

Worked examples

Example 1 — scaffolded

The perimeter of a rectangle is $P = 2(l + w)$. (a) Find P when $l = 8$ and $w = 5$. (b) Make w the subject of the formula.

Working	Comment
(a) $P = 2(8 + 5) = 2 \times 13 = 26$.	Substitute $l = 8$, $w = 5$ and evaluate.
(b) $P = 2(l + w)$	Start from the formula.
$\frac{P}{2} = l + w$	Divide both sides by 2 (inverse of × 2).
$w = \frac{P}{2} - l$	Subtract l from both sides.

Example 2 — scaffolded

Make a the subject of $v = u + at$.

Working	Comment
$v = u + at$	Start from the formula.
$v - u = at$	Subtract u from both sides.
$a = \frac{v - u}{t}$	Divide both sides by t.

Example 3 — unscaffolded

The area of a circle is $A = \pi r^2$. Make r the subject.

$$A=\pi r^2 \Rightarrow r^2=\frac{A}{\pi} \Rightarrow r=\sqrt{\frac{A}{\pi}}.$$

$\therefore r=\sqrt{\frac{A}{\pi}}$ (the positive root, since $r>0$).

Example 4 — unscaffolded

Make x the subject of $ax+b=cx+d$.

The pronumeral x appears in two terms, so collect them on one side and factorise:

$$ax+b=cx+d \Rightarrow ax-cx=d-b \Rightarrow x(a-c)=d-b.$$

$\therefore x=\frac{d-b}{a-c}$.

Practice questions

Scaffolded

Q1. For $v=u+at$, find v when $u=5$, $a=2$ and $t=3$.

Q2. Make t the subject of $v=u+at$.

Q3. For $A=1/2\,bh$: (a) find A when $b=10$ and $h=6$; (b) make h the subject.

Q4. Make r the subject of $C=2\pi r$.

Q5. Make t the subject of $s=\frac{d}{t}$.

Q6. Make C the subject of $F=9/5\,C+32$.

Unscaffolded

Make the variable shown the subject of each formula (or evaluate where asked).

Q7. $A=1/2\,bh$ — find A when $b=12$ and $h=7$.

Q8. $I=\frac{PRT}{100}$ — find I when $P=2000$, $R=5$ and $T=3$.

Q9. Make b the subject of $A=1/2\,bh$.

Q10. Make u the subject of $v=u+at$.

Q11. Make P the subject of $I=\frac{PRT}{100}$.

Q12. Make r the subject of $V=\pi r^2 h$.

Q13. Make v the subject of $E=1/2\,mv^2$.

Q14. Make ℓ the subject of $T=2\pi\sqrt{\frac{\ell}{g}}$.

Q15. Make g the subject of $T=2\pi\sqrt{\frac{\ell}{g}}$.

Q16. Make x the subject of $px = qx + r$.

Q17. Make x the subject of $a(x+b) = x + c$.

Q18. Make R the subject of $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$.

Answers

Q1. $v=11$. **Q2.** $t=\frac{v-u}{a}$. **Q3.** (a) $A=30$; (b) $h=\frac{2A}{b}$. **Q4.** $r=\frac{C}{2\pi}$. **Q5.** $t=\frac{d}{s}$. **Q6.** $C=\frac{5(F-32)}{9}$. **Q7.** $A=42$. **Q8.** $I=300$. **Q9.** $b=\frac{2A}{h}$. **Q10.** $u=v-at$. **Q11.** $P=\frac{100I}{RT}$. **Q12.** $r=\sqrt{\frac{V}{\pi h}}$. **Q13.** $v=\sqrt{\frac{2E}{m}}$. **Q14.** $\ell=\frac{gT^2}{4\pi^2}$. **Q15.** $g=\frac{4\pi^2\ell}{T^2}$. **Q16.** $x=\frac{r}{p-q}$. **Q17.** $x=\frac{c-ab}{a-1}$. **Q18.** $R=\frac{R_1R_2}{R_1+R_2}$.

Fully-worked solutions

Q1. $v=u+at=5+2\times3=5+6=11$.

Q2. $v=u+at\Rightarrow v-u=at\Rightarrow t=\frac{v-u}{a}$.

Q3. (a) $A=1/2(10)(6)=30$. (b) $A=1/2bh\Rightarrow 2A=bh\Rightarrow h=\frac{2A}{b}$.

Q4. $C=2\pi r\Rightarrow r=\frac{C}{2\pi}$.

Q5. $s=\frac{d}{t}\Rightarrow st=d\Rightarrow t=\frac{d}{s}$.

Q6. $F=9/5C+32\Rightarrow F-32=9/5C\Rightarrow C=\frac{5(F-32)}{9}$.

Q7. $A=1/2(12)(7)=42$.

Q8. $I=\frac{PRT}{100}=\frac{2000\times5\times3}{100}=\frac{30\,000}{100}=300$.

Q9. $A=1/2bh\Rightarrow 2A=bh\Rightarrow b=\frac{2A}{h}$.

Q10. $v=u+at\Rightarrow u=v-at$.

Q11. $I=\frac{PRT}{100}\Rightarrow 100I=PRT\Rightarrow P=\frac{100I}{RT}$.

Q12. $V=\pi r^2h\Rightarrow r^2=\frac{V}{\pi h}\Rightarrow r=\sqrt{\frac{V}{\pi h}}$ (positive root, since $r>0$).

Q13. $E=1/2mv^2\Rightarrow 2E=mv^2\Rightarrow v^2=\frac{2E}{m}\Rightarrow v=\sqrt{\frac{2E}{m}}$ (positive root).

Q14. $T=2\pi\sqrt{\frac{\ell}{g}}\Rightarrow\frac{T}{2\pi}=\sqrt{\frac{\ell}{g}}\Rightarrow\left(\frac{T}{2\pi}\right)^2=\frac{\ell}{g}\Rightarrow\frac{T^2}{4\pi^2}=\frac{\ell}{g}\Rightarrow\ell=\frac{gT^2}{4\pi^2}$.

Q15. From $\frac{T^2}{4\pi^2}=\frac{\ell}{g}$ (as in Q14), multiply both sides by g and by $\frac{4\pi^2}{T^2}$: $g=\frac{4\pi^2\ell}{T^2}$.

Q16. $px=qx+r\Rightarrow px-qx=r\Rightarrow x(p-q)=r\Rightarrow x=\frac{r}{p-q}$.

Q17. $a(x+b)=x+c \Rightarrow ax+ab=x+c \Rightarrow ax-x=c-ab \Rightarrow x(a-1)=c-ab \Rightarrow x=\dfrac{c-ab}{a-1}$.

Q18. $\dfrac{1}{R}=\dfrac{1}{R_1}+\dfrac{1}{R_2}=\dfrac{R_2+R_1}{R_1 R_2}$. Taking the reciprocal of both sides: $R=\dfrac{R_1 R_2}{R_1+R_2}$.