General Mathematics Units 1 & 2

Practice Examinations

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Contents

Section	Page
Exam 1 — Multiple-choice (Easy)	2
Exam 1 Formula Sheet	13
Exam 1 — Answers and solutions	15
Exam 2 — Multiple-choice (Medium)	17
Exam 2 Formula Sheet	29
Exam 2 — Answers and solutions	31
Exam 3 — Multiple-choice (Hard)	34
Exam 3 Formula Sheet	46
Exam 3 — Answers and solutions	48
Exam 4 — Short-answer (Easy)	51
Exam 4 Formula Sheet	56
Exam 4 — Answers and solutions	58
Exam 5 — Short-answer (Medium)	61
Exam 5 Formula Sheet	66
Exam 5 — Answers and solutions	68
Exam 6 — Short-answer (Hard)	73
Exam 6 Formula Sheet	78
Exam 6 — Answers and solutions	80

Exam 1 — Multiple-choice (Easy)

Reading time: 15 minutes. Writing time: 1 hour 30 minutes. 40 multiple-choice questions, 40 marks. One bound reference and one approved technology (CAS calculator or software) are permitted; a scientific calculator may also be used. Choose the response that is ***correct*** *for the question. Unless otherwise indicated, diagrams are not drawn to scale.*

Section A — Multiple-choice

Question 1

A café records four variables for each customer: the customer's membership level (bronze, silver or gold), the number of coffees they buy, the temperature of their coffee (in °C), and their preferred payment method (cash, card or phone). How many of these four variables are categorical?

A. 1

B. 2

C. 3

D. 4

Question 2

The numbers of goals scored by a netball team in seven games were

$$12, 9, 15, 11, 8, 14, 10.$$

The median number of goals scored is

A. 11

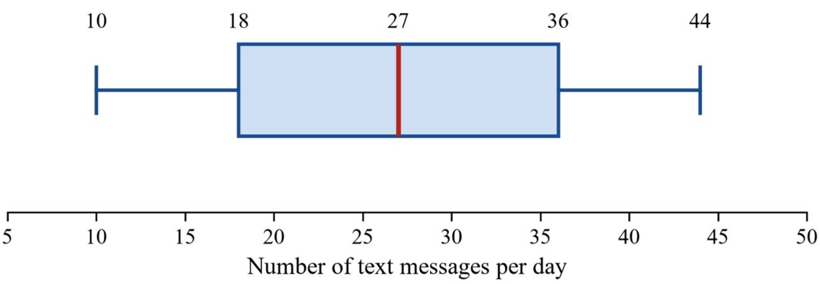
B. 10

C. 12

D. 14

Question 3

The boxplot below summarises the number of text messages sent per day by a student over 11 days.



The interquartile range (IQR) of this data set is

A. 34

B. 9

C. 18

D. 26

Question 4

For the data set

$$4, 7, 10, 13, 16,$$

the sample standard deviation is closest to

A. 4.24

B. 6.00

C. 22.50

D. 4.74

Question 5

A sequence is defined by the recurrence relation

$$t_0 = 4, t_{n+1} = 4t_n - 7.$$

The value of t_2 is

A. 9

B. 36

C. 29

D. 109

Question 6

The arithmetic sequence

$$7, 11, 15, 19, \ldots$$

has first term $a = 7$ and common difference $d = 4$. The 20th term of the sequence is

A. 83

B. 87

C. 80

D. 76

Question 7

A geometric sequence has first term 6 and common ratio 2. The 5th term of the sequence is

A. 48

B. 192

C. 60

D. 96

Question 8

Which one of the following recurrence relations generates the sequence

$$40, 34, 28, 22, \ldots ?$$

A. $t_0 = 40, t_{n+1} = t_n + 6$

B. $t_0 = 40, t_{n+1} = t_n - 6$

C. $t_0 = 40, t_{n+1} = 6t_n$

D. $t_0 = 40, t_{n+1} = -6t_n$

Question 9

Rohan invests $2500 in an account paying simple interest at 4% per annum. The interest earned after 3 years is

A. $300

B. $100

C. $312

D. $2800

Question 10

An amount of $8000 is invested at 5% per annum, compounding annually. The value of the investment after 2 years is

A. $8800

B. $9261

C. $8820

D. $8000

Question 11

A car bought for $24,000 depreciates by the reducing-balance method at 15% per annum. The value of the car after 2 years is

A. $20,400

B. $17,340

C. $16,800

D. $18,000

Question 12

A jacket is marked at $120 and is discounted by 25%. The sale price of the jacket is

A. $95

B. $150

C. $30

D. $90

Question 13

The cost \$C of hiring a carpet cleaner for h hours is given by

$$C = 30 + 15h.$$

The cost of hiring the cleaner for 5 hours is

A. \$75

B. \$45

C. \$150

D. \$105

Question 14

A phone plan charges a monthly cost, in dollars, given by

$$C = 20 + 0.10n,$$

where n is the number of calls made. In this model, the number 0.10 represents

A. the fixed monthly charge

B. the cost of each call

C. the number of calls included free

D. the total monthly cost

Question 15

A straight line passes through the points $(2, 5)$ and $(6, 17)$. The gradient of the line is

A. 3

B. 4

C. 12

D. $\frac{1}{3}$

Question 16

The solution to the simultaneous equations

$$y = 2x + 1 \text{ and } y = x + 4$$

is

A. $(1, 3)$

B. $(2, 5)$

C. $(3, 7)$

D. $(5, 9)$

Question 17

Consider the matrix

$$M = \begin{bmatrix} 3 & -1 & 5 \\ 2 & 4 & 0 \end{bmatrix}.$$

The order of M and the value of the element m_{12} are, respectively,

A. 3×2 and -1

B. 2×3 and 2

C. 2×3 and -1

D. 2×3 and 5

Question 18

Given

$$A = \begin{bmatrix} 6 & 3 \\ 2 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix},$$

the matrix $A + B$ is equal to

A. $\begin{bmatrix} 7 & 7 \\ 5 & 7 \end{bmatrix}$

B. $\begin{bmatrix} 5 & -1 \\ -1 & 3 \end{bmatrix}$

C. $\begin{bmatrix} 6 & 12 \\ 6 & 10 \end{bmatrix}$

D. $\begin{bmatrix} 7 & 5 \\ 7 & 7 \end{bmatrix}$

Question 19

Given

$$P = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}, Q = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix},$$

the product PQ is equal to

A. $\begin{bmatrix} 4 & 3 \\ 16 & 5 \end{bmatrix}$

B. $\begin{bmatrix} 8 & 7 \\ 28 & 17 \end{bmatrix}$

C. $\begin{bmatrix} 16 & 12 \\ 17 & 9 \end{bmatrix}$

D. $\begin{bmatrix} 16 & 17 \\ 12 & 9 \end{bmatrix}$

Question 20

Each week, customers switch between brand A and brand B according to the transition matrix below, in which the columns sum to 1.

$$\begin{bmatrix} 0.7 & 0.5 \\ 0.3 & 0.5 \end{bmatrix} \begin{array}{l} A \\ B \end{array}$$

This week 300 customers use brand A and 200 use brand B. According to this model, the number of customers using brand A next week will be

A. 290

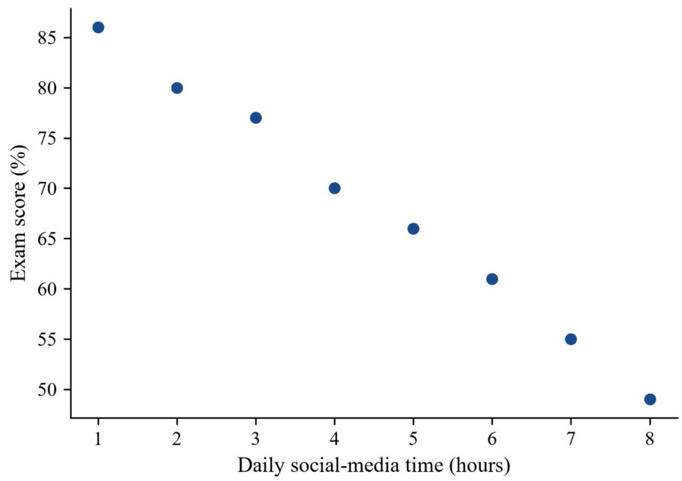
B. 310

C. 300

D. 250

Question 21

The scatterplot below shows the exam score and the daily time spent on social media for eight students.



The association between the two variables is best described as

A. strong positive

B. strong negative

C. weak positive

D. no association

Question 22

A researcher investigates whether the number of hours a student studies affects the student's test score. In this investigation, the response variable is

A. the number of hours of study

B. the student

C. the researcher

D. the test score

Question 23

A line of good fit is found for a set of data in which the explanatory variable x ranges from 10 to 50. Using the line to predict a value of y when $x = 70$ is an example of

A. extrapolation

B. interpolation

C. correlation

D. causation

Question 24

The line of good fit relating a plant's height y (in cm) to its age x (in weeks) is

$$y = 3 + 1.5x.$$

According to this line, on average the plant's height increases by

A. 3 cm each week

B. 1.5 cm in total

C. 1.5 cm each week

D. 4.5 cm each week

Question 25

A graph has five vertices whose degrees are

$$3, 4, 4, 4, 3.$$

The number of edges in the graph is

A. 9

B. 18

C. 10

D. 5

Question 26

A connected planar graph has 8 vertices and 13 edges. Using Euler's formula $v - e + f = 2$, the number of faces (including the infinite outer region) is

A. 5

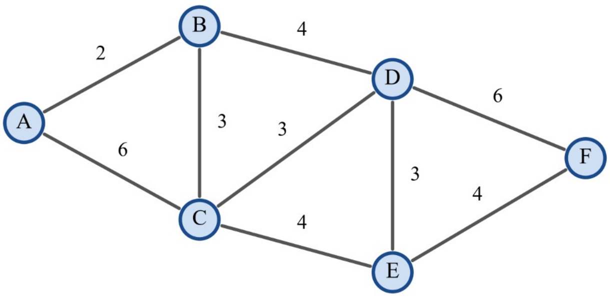
B. 6

C. 7

D. 23

Question 27

The network below shows the roads connecting six towns, A to F. The number on each edge is the length of that road, in kilometres.



The length of the shortest path from town A to town F, in kilometres, is

A. 13

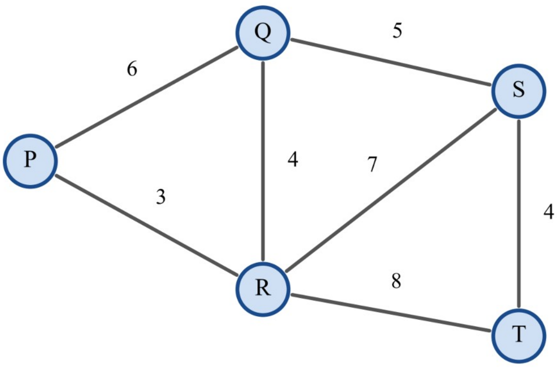
B. 12

C. 14

D. 15

Question 28

The network below shows the possible cable connections between five houses, P to T. The number on each edge is the cost, in thousands of dollars, of laying that cable.



The minimum cost of connecting all five houses (the total weight of the minimum spanning tree), in thousands of dollars, is

A. 18

B. 15

C. 17

D. 16

Question 29

The variable y varies directly as x, and $y = 20$ when $x = 4$. The constant of variation k is

A. 4

B. 80

C. 16

D. 5

Question 30

The variable y varies inversely as x, and $y = 10$ when $x = 3$. The value of y when $x = 6$ is

A. 20

B. 5

C. 15

D. 60

Question 31

The distance d travelled by a car varies directly with the time t for which it travels. The car travels 150 km in 2 hours. At the same rate, the distance it travels in 5 hours is

A. 300 km

B. 350 km

C. 375 km

D. 750 km

Question 32

The stopping distance d of a car varies directly with the square of its speed v, so that $d = k v^2$. When $v = 20$ the stopping distance is $d = 8$. The stopping distance when $v = 40$ is

A. 32

B. 16

C. 24

D. 64

Question 33

A sector of a circle has radius 6 cm and angle 60°. Using $A = \frac{\theta}{360} \times \pi r^2$, the area of the sector is closest to

A. 6.28 cm²

B. 37.70 cm²

C. 18.85 cm²

D. 3.14 cm²

Question 34

A cylinder has radius 5 cm and height 10 cm. Using $V = \pi r^2 h$, the volume of the cylinder is closest to

A. 785 cm^3

B. 250 cm^3

C. 1571 cm^3

D. 500 cm^3

Question 35

Two similar solids have their lengths in the ratio $1:3$. The ratio of their volumes is

A. $1:9$

B. $1:3$

C. $1:6$

D. $1:27$

Question 36

A sphere has radius 4 cm. Using $V = \frac{4}{3}\pi r^3$, the volume of the sphere is closest to

A. 201 cm^3

B. 268 cm^3

C. 804 cm^3

D. 67 cm^3

Question 37

From a point 20 m from the base of a tree on level ground, the angle of elevation to the top of the tree is 40°. Using $\tan 40° = \frac{\text{height}}{20}$, the height of the tree is closest to

A. 16.78 m

B. 12.86 m

C. 15.32 m

D. 23.84 m

Question 38

In triangle ABC, angle $A = 40°$, angle $B = 75°$ and side $a = 10$. Using the sine rule, the length of side b is closest to

A. 6.65

B. 9.66

C. 11.90

D. 15.03

Question 39

In triangle ABC, $a=7$, $b=9$ and the included angle $C=60°$. Using the cosine rule $c^2=a^2+b^2-2ab\cos C$, the length of side c is closest to

A. 8.06

B. 8.19

C. 11.40

D. 4.36

Question 40

A hiker walks 5 km due east and then 5 km due north. The bearing of the hiker's finishing point from her starting point is

A. 090°

B. 135°

C. 045°

D. 225°

End of Exam 1

Exam 1 Formula Sheet

This Formula Sheet is provided with every examination in this booklet.

Data analysis

mean	$\bar{x} = \frac{\sum x}{n}$
sample standard deviation	$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$
interquartile range	$\text{IQR} = Q_3 - Q_1$
lower fence of a boxplot	$Q_1 - 1.5 \times \text{IQR}$
upper fence of a boxplot	$Q_3 + 1.5 \times \text{IQR}$
line of good fit	$y = a + bx$

Sequences, recurrence and financial mathematics

first-order linear recurrence	$t_0 = a, t_{n+1} = R t_n + d$
arithmetic sequence	$t_n = a + (n-1)d$
geometric sequence	$t_n = a R^{n-1}$
simple interest	$I = \frac{P r n}{100}$
flat-rate / unit-cost depreciation	$V_n = P - nD$
compound interest	$A = P\left(1 + \frac{r}{100}\right)^n$
reducing-balance depreciation	$V_n = P\left(1 - \frac{r}{100}\right)^n$

Functions, graphs and variation

linear function	$y = a + bx$
slope of a line	$b = \frac{y_2 - y_1}{x_2 - x_1}$
direct variation	$y = kx$
inverse variation	$y = \frac{k}{x}$

Matrices

determinant of a 2×2 matrix: for $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $det\ A = ad - bc$

inverse of a 2×2 matrix: $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $ad - bc \neq 0$

transition (state) recurrence $S_{n+1} = T S_n$

Graphs and networks

Euler's formula (connected planar graph) $v - e + f = 2$

Measurement

Heron's rule: area of a triangle $=\sqrt{s(s-a)(s-b)(s-c)}$, where $s=\dfrac{a+b+c}{2}$

Pythagoras' theorem	$c^2=a^2+b^2$
circumference of a circle	$C=2\pi r$
area of a circle	$A=\pi r^2$
arc length	$\ell=\dfrac{\theta}{360}\times 2\pi r$
area of a sector	$A=\dfrac{\theta}{360}\times \pi r^2$
area of a triangle	$A=\dfrac{1}{2}bh$
volume of a prism	$V=A_{\text{cross}}\times h$
volume of a cylinder	$V=\pi r^2 h$
volume of a pyramid	$V=\dfrac{1}{3}A_{\text{base}}h$
volume of a sphere	$V=\dfrac{4}{3}\pi r^3$
surface area of a sphere	$A=4\pi r^2$
curved surface area of a cylinder	$A=2\pi rh$
similar-figure scale factors	length k, area k^2, volume k^3

Trigonometry

trigonometric ratios: $\sin\theta=\dfrac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta=\dfrac{\text{adjacent}}{\text{hypotenuse}}$, $\tan\theta=\dfrac{\text{opposite}}{\text{adjacent}}$

sine rule: $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

cosine rule: $a^2=b^2+c^2-2bc\cos A$

area of a triangle $A=\dfrac{1}{2}ab\sin C$

End of Formula Sheet

Exam 1 — Answers and solutions

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	B	A	C	D	C	A	D	B	A	C	B	D	D	B	A	C	C	A	D	B

Q	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	B	D	A	C	A	C	B	D	D	B	C	A	C	A	D	B	A	D	B	C

Q1. B. Membership level (ordinal) and payment method (nominal) are categorical; the number of coffees and the temperature are numerical, so 2 of the four variables are categorical.

Q2. A. In order the values are $8, 9, 10, 11, 12, 14, 15$; with $n=7$ the median is the 4th value, 11.

Q3. C. From the boxplot $Q_1=18$ and $Q_3=36$, so $\text{IQR}=Q_3-Q_1=36-18=18$.

Q4. D. The mean is 10, so $s=\sqrt{\dfrac{36+9+0+9+36}{5-1}}=\sqrt{22.5}\approx 4.74$ (dividing by $n-1$).

Q5. C. $t_1=4\times 4-7=9$ and $t_2=4\times 9-7=29$.

Q6. A. $t_{20}=a+19d=7+19\times 4=7+76=83$.

Q7. D. $t_5=aR^4=6\times 2^4=6\times 16=96$.

Q8. B. The sequence decreases by 6 each term, so it starts at $t_0=40$ and satisfies $t_{n+1}=t_n-6$.

Q9. A. $I=\dfrac{Prn}{100}=\dfrac{2500\times 4\times 3}{100}=\$300$.

Q10. C. $8000\times 1.05^2=8000\times 1.1025=\$8820$.

Q11. B. $V=24\,000\times(1-0.15)^2=24\,000\times 0.7225=\$17\,340$.

Q12. D. Sale price $=120\times(1-0.25)=120\times 0.75=\$90$.

Q13. D. $C=30+15\times 5=30+75=\$105$.

Q14. B. The coefficient of n is the rate of change of cost: each additional call adds \$0.10, so 0.10 is the cost of each call.

Q15. A. $b=\dfrac{17-5}{6-2}=\dfrac{12}{4}=3$.

Q16. C. Setting $2x+1=x+4$ gives $x=3$, and then $y=7$, so the solution is $(3,7)$.

Q17. C. M has 2 rows and 3 columns, so its order is 2×3; m_{12} is the entry in row 1, column 2, which is -1.

Q18. A. $A+B=\begin{bmatrix}6+1 & 3+4\\ 2+3 & 5+2\end{bmatrix}=\begin{bmatrix}7 & 7\\ 5 & 7\end{bmatrix}$.

Q19. D. $PQ=\begin{bmatrix}2(2)+3(4) & 2(1)+3(5)\\ 4(2)+1(4) & 4(1)+1(5)\end{bmatrix}=\begin{bmatrix}16 & 17\\ 12 & 9\end{bmatrix}$.

Q20. B. Next week brand A has $0.7\times 300+0.5\times 200=210+100=310$ customers.

Q21. B. The points fall steadily from upper-left to lower-right in a tight, near-straight-line pattern, indicating a strong negative (linear) association.

Q22. D. The test score is the outcome being explained, so it is the response variable; the number of hours of study is the explanatory variable.

Q23. A. The value $x=70$ lies outside the range of the data (10 to 50), so predicting there is extrapolation.

Q24. C. The slope 1.5 is the change in y for each 1-unit increase in x: the height rises 1.5 cm for each extra week.

Q25. A. The sum of the degrees is $3+4+4+4+3=18$; the number of edges is half of this, $18/2=9$.

Q26. C. Euler's formula gives $f=2-v+e=2-8+13=7$.

Q27. B. The shortest route is $A-B-D-F$ with length $2+4+6=12$ km.

Q28. D. The minimum spanning tree uses edges $PR(3)$, $QR(4)$, $ST(4)$ and $QS(5)$, giving a total of $3+4+4+5=16$ thousand dollars.

Q29. D. For direct variation $y=kx$, so $k=\frac{y}{x}=\frac{20}{4}=5$.

Q30. B. For inverse variation $k=xy=3\times 10=30$, so when $x=6$, $y=\frac{30}{6}=5$.

Q31. C. The rate is $\frac{150}{2}=75$ km/h, so in 5 hours the distance is $75\times 5=375$ km.

Q32. A. Since $d\propto v^2$, doubling v multiplies d by $2^2=4$, so $d=8\times 4=32$.

Q33. C. $A=\frac{60}{360}\times\pi\times 6^2=\frac{1}{6}\times 36\pi=6\pi\approx 18.85$ cm².

Q34. A. $V=\pi r^2 h=\pi\times 5^2\times 10=250\pi\approx 785$ cm³.

Q35. D. Volumes scale by the cube of the length ratio, so $1^3:3^3=1:27$.

Q36. B. $V=\frac{4}{3}\pi\times 4^3=\frac{256}{3}\pi\approx 268$ cm³.

Q37. A. Height $=20\tan 40°=20\times 0.8391\approx 16.78$ m.

Q38. D. $b=\frac{a\sin B}{\sin A}=\frac{10\sin 75°}{\sin 40°}\approx 15.03$.

Q39. B. $c^2=7^2+9^2-2\times 7\times 9\times\cos 60°=130-63=67$, so $c=\sqrt{67}\approx 8.19$.

Q40. C. Equal distances due east and due north place the finishing point exactly north-east of the start, a bearing of 045°.

Exam 2 — Multiple-choice (Medium)

Reading time: 15 minutes. Writing time: 1 hour 30 minutes. 40 multiple-choice questions, 40 marks. One bound reference and one approved technology (CAS calculator or software) are permitted; a scientific calculator may also be used. Choose the response that is ***correct*** *for the question. Unless otherwise indicated, diagrams are not drawn to scale.*

Section A — Multiple-choice

Question 1

For each customer at a café, a barista records four variables: the drink ordered (for example, latte or tea), the number of drinks bought, whether the customer dined in (yes or no), and the amount spent, in dollars. How many of these four variables are numerical?

A. 1

B. 2

C. 3

D. 4

Question 2

The numbers of goals scored by a netball team in each of five games were

$$8, 5, 11, 6, 10.$$

The sample standard deviation of these scores is closest to

A. 2.28

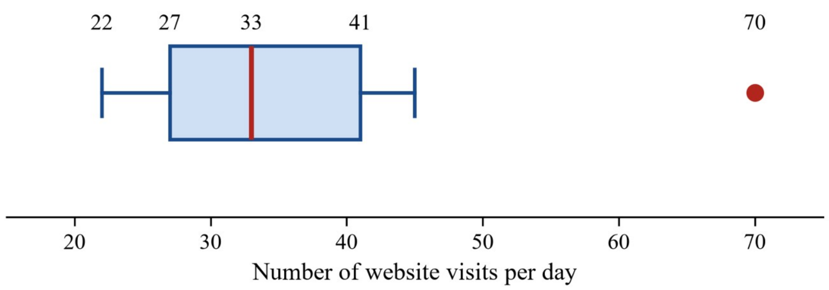
B. 6.50

C. 5.20

D. 2.55

Question 3

The boxplot below summarises the number of visits to a website on each of 11 days.



The interquartile range (IQR) of this data set is

A. 14

B. 8

C. 21

D. 48

Question 4

For a large set of house sale prices, the mean price is \$720,000 and the median price is \$610,000. The shape of this distribution is best described as

A. approximately symmetric

B. negatively skewed

C. positively skewed

D. bimodal

Question 5

A sequence is generated by the recurrence relation

$$t_0 = 6, t_{n+1} = 2t_n + 5.$$

The value of t_2 is

A. 39

B. 17

C. 34

D. 83

Question 6

The rows of seats in a theatre form an arithmetic sequence. Row 1 has 14 seats, and each row after that has 3 more seats than the row in front of it. The number of seats in row 10 is

A. 44

B. 38

C. 41

D. 30

Question 7

A ball is dropped and, on each bounce, it rises to half the height of the previous bounce. The heights of successive bounces form a geometric sequence. If the first bounce reaches a height of 80 cm, the height of the fourth bounce is

A. 20 cm

B. 5 cm

C. 40 cm

D. 10 cm

Question 8

The first four terms of a sequence are

$$3, 8, 13, 18.$$

A recurrence relation that generates this sequence is

A. $t_0 = 3, t_{n+1} = 5t_n$

B. $t_0 = 3, t_{n+1} = t_n + 5$

C. $t_0 = 3, t_{n+1} = t_n + 8$

D. $t_0 = 5, t_{n+1} = t_n + 3$

Question 9

In a sale, a jacket is discounted by 30% and now sells for $91. The original price of the jacket, before the discount, was

A. $118.30

B. $121.00

C. $127.00

D. $130.00

Question 10

A machine is purchased for $24,000. It depreciates by a flat rate of 15% of its purchase price each year. The value of the machine after 4 years is

A. $9600

B. $12,528.15

C. $14,400

D. $12,000

Question 11

An amount of $8000 is invested at 6% per annum, compounding annually. The value of the investment after 3 years is

A. $9440.00

B. $8988.80

C. $9528.13

D. $10,099.82

Question 12

A supermarket sells the same laundry detergent in two sizes: Brand A offers 750 g for $6.00, and Brand B offers 1.2 kg for $9.00. Comparing the price per kilogram, the better value is

A. Brand A, by $0.50 per kg

B. Brand B, by $0.50 per kg

C. Brand B, by $1.50 per kg

D. neither — they are equal value

Question 13

The straight line passing through the points $(2,5)$ and $(6,17)$ has a slope of

A. 3

B. 4

C. $\frac{1}{3}$

D. -3

Question 14

A plumber charges a $60 call-out fee plus $45 for each hour worked. The cost, C dollars, of a job lasting h hours is modelled by $C=60+45h$. The cost of a job that lasts 3.5 hours is

A. $157.50

B. $202.50

C. $217.50

D. $180.00

Question 15

At a market stall, 3 coffees and 2 muffins cost $25, while 1 coffee and 4 muffins cost $20. The cost of one coffee is

A. $3.50

B. $4.00

C. $5.00

D. $6.00

Question 16

Under a certain tax scale, no tax is payable on the first $18,000 of annual income, and tax of 20 cents is payable for each $1 of income above $18,000. The tax payable on an annual income of $25,000 is

A. $5000

B. $1400

C. $3600

D. $1360

Question 17

Consider the matrix

$$A = \begin{bmatrix} 4 & 7 \\ 2 & 0 \\ 5 & -3 \end{bmatrix}.$$

The order of A and the value of the element a_{31} are, respectively,

A. 3×2 and 5

B. 2×3 and 5

C. 3×2 and -3

D. 3×2 and 2

Question 18

Given

$$P = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}, Q = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix},$$

the product PQ is equal to

A. $\begin{bmatrix} 6 & 10 \\ 13 & 25 \end{bmatrix}$

B. $\begin{bmatrix} 6 & 0 \\ 2 & 20 \end{bmatrix}$

C. $\begin{bmatrix} 11 & 8 \\ 25 & 20 \end{bmatrix}$

D. $\begin{bmatrix} 11 & 25 \\ 8 & 20 \end{bmatrix}$

Question 19

The matrix

$$\begin{bmatrix} k & 3 \\ 4 & 6 \end{bmatrix}$$

does not have an inverse (is singular) when k is equal to

A. 8

B. -2

C. 2

D. 0

Question 20

Each year, subscribers move between two streaming services, service A and service B, according to the transition matrix

$$T = \begin{bmatrix} 0.85 & 0.20 \\ 0.15 & 0.80 \end{bmatrix},$$

in which the first row and column correspond to service A and the second to service B, and the columns sum to 1. This year service A has 600 subscribers and service B has 400 subscribers. According to this model, the number of subscribers to service A next year will be

A. 510

B. 590

C. 600

D. 680

Question 21

A scatterplot of two numerical variables shows points that lie close to a straight line falling from the upper left to the lower right. The association between the two variables is best described as

A. strong, positive and linear

B. weak, negative and linear

C. strong, negative and linear

D. strong, negative and non-linear

Question 22

A survey of country towns finds that towns with more public libraries tend to have higher rates of coffee consumption. Which one of the following statements is the most appropriate conclusion?

A. The association may be explained by a common cause, such as town population, so causation cannot be concluded.

B. Building more libraries in a town would cause its coffee consumption to increase.

C. Drinking more coffee causes more libraries to be built.

D. There is no association between the two variables.

Question 23

For a set of documents, a line of good fit relating the printing time y (in seconds) to the number of pages x is

$$y = 4 + 0.5x.$$

The slope of this line indicates that, on average,

A. printing time increases by 4 seconds for each additional page

B. printing time is 0.5 seconds when there are no pages

C. printing time decreases by 0.5 seconds for each additional page

D. printing time increases by 0.5 seconds for each additional page

Question 24

Using the line of good fit $y = 4 + 0.5x$, where y is the printing time in seconds and x is the number of pages, the predicted printing time for a 60-page document is

A. 30 seconds

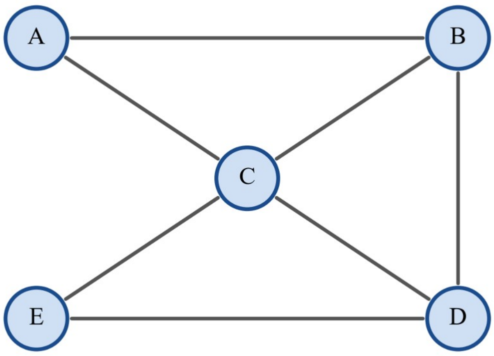
B. 34 seconds

C. 54 seconds

D. 64 seconds

Question 25

The graph below shows the direct flight connections between five airports, A to E.



The degree of vertex C is

A. 4

B. 3

C. 5

D. 2

Question 26

A connected planar graph has 8 vertices and 12 edges. Using Euler's formula, the number of faces (including the infinite outer region) is

A. 4

B. 18

C. 22

D. 6

Question 27

The adjacency matrix of a simple graph with vertices P, Q, R and S (in that order) is

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$

The number of edges in this graph is

A. 10

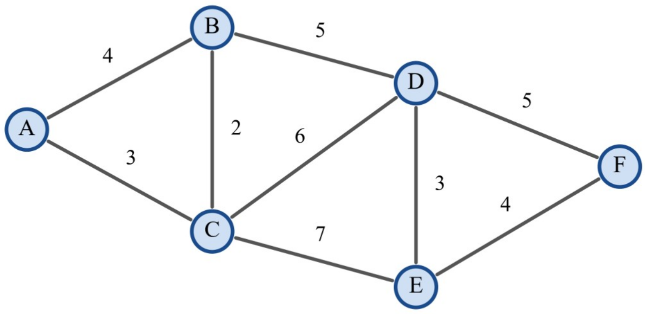
B. 4

C. 5

D. 6

Question 28

The network below shows the roads connecting six towns, A to F. The number on each edge is the length of that road, in kilometres.



The length of the shortest path from town A to town F, in kilometres, is

A. 12

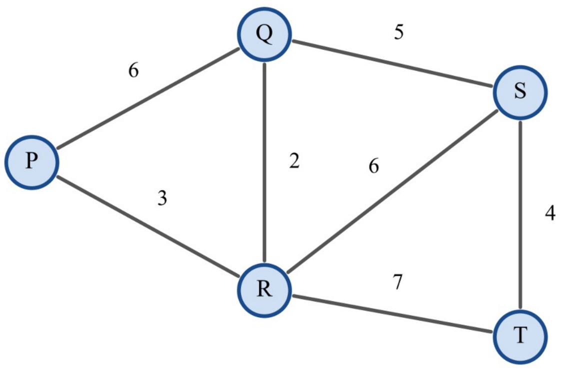
B. 14

C. 13

D. 16

Question 29

The network below shows the possible cable connections between five houses, P to T. The number on each edge is the cost, in thousands of dollars, of laying that cable.



The minimum cost of connecting all five houses (the total weight of the minimum spanning tree), in thousands of dollars, is

A. 14

B. 15

C. 16

D. 13

Question 30

The distance travelled by a train moving at a constant speed varies directly with the time travelled. The train travels 240 km in 3 hours. At the same speed, the distance it travels in 5 hours is

A. 144 km

B. 80 km

C. 400 km

D. 720 km

Question 31

The time taken to complete a landscaping job varies inversely with the number of workers. When 6 workers are used, the job takes 8 hours. The time the same job would take with 4 workers is

A. 5.33 hours

B. 12 hours

C. 16 hours

D. 32 hours

Question 32

Two variables are related by the model

$$y = 3 + 2x^2.$$

When $x = 4$, the value of y is

A. 11

B. 121

C. 67

D. 35

Question 33

Earthquake magnitude is measured on a base-10 logarithmic scale, so that each increase of 1 in magnitude corresponds to a 10-fold increase in the amplitude of the ground movement. The amplitude of a magnitude 7 earthquake is how many times the amplitude of a magnitude 4 earthquake?

A. 1000

B. 3

C. 30

D. 100

Question 34

A rectangular field measures 0.42 km by 0.25 km. The area of the field, in square metres, is

A. 1050 m²

B. 0.105 m²

C. 1.05×10^5 m²

D. 1.05×10^4 m²

Question 35

A sector of a circle has a radius of 12 cm and a sector angle of 60°. The area of the sector, in square centimetres, is closest to

A. 12.57

B. 75.40

C. 150.80

D. 452.39

Question 36

A solid sphere has a radius of 6 cm. Its volume, in cubic centimetres, is closest to

A. 452.39

B. 150.80

C. 2714.34

D. 904.78

Question 37

Two water tanks are similar in shape. Their surface areas are in the ratio $9:16$. The ratio of their volumes is

A. $9:16$

B. $3:4$

C. $27:64$

D. $81:256$

Question 38

From a point on level ground 40 m from the base of a vertical building, the angle of elevation to the top of the building is 35°. The height of the building, in metres, is closest to

A. 28.01

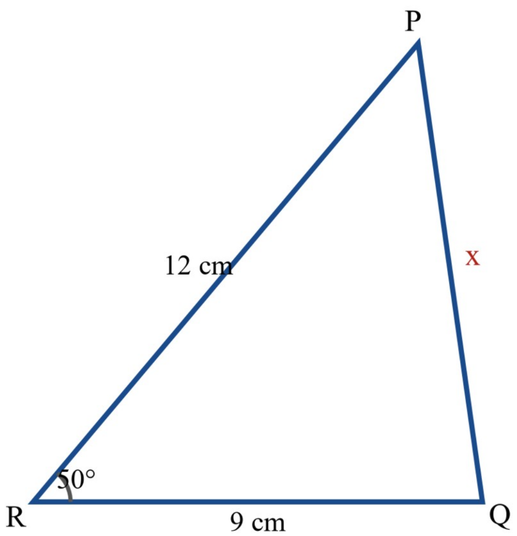
B. 22.94

C. 32.77

D. 57.13

Question 39

In the triangle PQR shown below, $RQ=9$ cm, $RP=12$ cm and the angle at R is 50°.



The length of side PQ (marked x), in centimetres, is closest to

A. 15.00

B. 9.28

C. 19.07

D. 7.94

Question 40

A triangle has two sides of length 10 cm and 14 cm, with an included angle of 68° between them. The area of the triangle, in square centimetres, is closest to

A. 70.00

B. 26.22

C. 129.81

D. 64.90

End of Exam 2

Exam 2 Formula Sheet

This Formula Sheet is provided with every examination in this booklet.

Data analysis

mean	$\bar{x}=\frac{\sum x}{n}$
sample standard deviation	$s=\sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$
interquartile range	$\text{IQR}=Q_3-Q_1$
lower fence of a boxplot	$Q_1-1.5\times\text{IQR}$
upper fence of a boxplot	$Q_3+1.5\times\text{IQR}$
line of good fit	$y=a+bx$

Sequences, recurrence and financial mathematics

first-order linear recurrence	$t_0=a,\, t_{n+1}=R\,t_n+d$
arithmetic sequence	$t_n=a+(n-1)d$
geometric sequence	$t_n=aR^{n-1}$
simple interest	$I=\frac{Prn}{100}$
flat-rate / unit-cost depreciation	$V_n=P-nD$
compound interest	$A=P\left(1+\frac{r}{100}\right)^n$
reducing-balance depreciation	$V_n=P\left(1-\frac{r}{100}\right)^n$

Functions, graphs and variation

linear function	$y=a+bx$
slope of a line	$b=\frac{y_2-y_1}{x_2-x_1}$
direct variation	$y=kx$
inverse variation	$y=\frac{k}{x}$

Matrices

determinant of a 2×2 matrix: for $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $det\ A=ad-bc$

inverse of a 2×2 matrix: $A^{-1}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $ad-bc\neq0$

transition (state) recurrence $S_{n+1}=T\,S_n$

Graphs and networks

Euler's formula (connected planar graph) $v-e+f=2$

Measurement

Heron's rule: area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \dfrac{a+b+c}{2}$

Pythagoras' theorem	$c^2 = a^2 + b^2$
circumference of a circle	$C = 2\pi r$
area of a circle	$A = \pi r^2$
arc length	$\ell = \dfrac{\theta}{360} \times 2\pi r$
area of a sector	$A = \dfrac{\theta}{360} \times \pi r^2$
area of a triangle	$A = \dfrac{1}{2} b h$
volume of a prism	$V = A_{\text{cross}} \times h$
volume of a cylinder	$V = \pi r^2 h$
volume of a pyramid	$V = \dfrac{1}{3} A_{\text{base}} h$
volume of a sphere	$V = \dfrac{4}{3} \pi r^3$
surface area of a sphere	$A = 4\pi r^2$
curved surface area of a cylinder	$A = 2\pi r h$
similar-figure scale factors	length k, area k^2, volume k^3

Trigonometry

trigonometric ratios: $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$, $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

area of a triangle $A = \dfrac{1}{2} a b \sin C$

End of Formula Sheet

Exam 2 — Answers and solutions

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	B	D	A	C	A	C	D	B	D	A	C	B	A	C	D	B	A	D	C	B

Q	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	C	A	D	B	A	D	C	B	A	C	B	D	A	C	B	D	C	A	B	D

Q1. B. The number of drinks and the amount spent are numerical; the drink ordered and whether the customer dined in are categorical, so 2 of the four variables are numerical.

Q2. D. The mean is $\bar{x}=\frac{40}{5}=8$; the sample standard deviation $s=\sqrt{\frac{(0)^2+(-3)^2+3^2+(-2)^2+2^2}{5-1}}=\sqrt{\frac{26}{4}}\approx 2.55$ (dividing by $n-1$).

Q3. A. From the boxplot $Q_1=27$ and $Q_3=41$, so $\text{IQR}=Q_3-Q_1=41-27=14$.

Q4. C. The mean ($720,000) is greater than the median ($610,000), which indicates a tail of high values, so the distribution is positively skewed.

Q5. A. $t_1=2\times 6+5=17$ and $t_2=2\times 17+5=39$.

Q6. C. This is arithmetic with $a=14$ and $d=3$, so $t_{10}=a+(10-1)d=14+9\times 3=41$ seats.

Q7. D. This is geometric with first bounce 80 cm and ratio $R=0.5$, so the fourth bounce is $80\times 0.5^3=80\times 0.125=10$ cm.

Q8. B. The terms increase by a constant 5, so the sequence is arithmetic with $t_0=3$ and $t_{n+1}=t_n+5$.

Q9. D. After a 30% discount the sale price is 70% of the original, so the original price is $\frac{91}{0.70}=\$130.00$.

Q10. A. Flat-rate depreciation is $0.15\times 24\,000=\$3600$ per year, so after 4 years the value is $24\,000-4\times 3600=\$9600$.

Q11. C. $8000\times 1.06^3=8000\times 1.191016=\$9528.13$ (to the nearest cent).

Q12. B. Brand A costs $\frac{6.00}{0.750}=\$8.00$ per kg and Brand B costs $\frac{9.00}{1.200}=\$7.50$ per kg, so Brand B is the better value, by $0.50 per kg.

Q13. A. The slope is $\frac{y_2-y_1}{x_2-x_1}=\frac{17-5}{6-2}=\frac{12}{4}=3$.

Q14. C. $C=60+45\times 3.5=60+157.5=\$217.50$.

Q15. D. Let c and m be the costs of a coffee and a muffin. Solving $3c+2m=25$ and $c+4m=20$ gives $m=3.5$ and $c=6$, so a coffee costs $6.00.

Q16. B. Only income above $18,000 is taxed: $(25\,000-18\,000)\times 0.20=7000\times 0.20=\$1400$.

Q17. A. A has 3 rows and 2 columns, so its order is 3×2; a_{31} is the entry in row 3, column 1, which is 5.

Q18. D. $PQ=\begin{bmatrix}3(2)+5(1) & 3(0)+5(5)\\ 2(2)+4(1) & 2(0)+4(5)\end{bmatrix}=\begin{bmatrix}11 & 25\\ 8 & 20\end{bmatrix}$.

Q19. C. The matrix is singular when its determinant is zero: $6k-3\times 4=6k-12=0$, so $k=2$.

Q20. B. Next year service A has $0.85\times 600+0.20\times 400=510+80=590$ subscribers.

Q21. C. Points lying close to a straight line indicate a strong, linear association, and a fall from upper left to lower right indicates a negative direction.

Q22. A. An association does not establish cause and effect; both variables are likely linked to a common cause such as town population, so causation cannot be concluded.

Q23. D. The slope is $+0.5$, so on average the printing time increases by 0.5 seconds for each additional page.

Q24. B. $y=4+0.5\times 60=4+30=34$ seconds.

Q25. A. Vertex C is joined by an edge to each of A, B, D and E, so its degree is 4.

Q26. D. Euler's formula gives $v-e+f=2$, so $f=2-v+e=2-8+12=6$.

Q27. C. The sum of all the entries of the adjacency matrix is $2+2+3+3=10$; each edge is counted twice, so the number of edges is $\frac{10}{2}=5$.

Q28. B. The shortest path is $A-C-D-F$ with length $3+6+5=14$ km (equal to $A-C-E-F=3+7+4=14$).

Q29. A. The minimum spanning tree uses edges $QR(2)$, $PR(3)$, $ST(4)$ and $QS(5)$, giving a total of $2+3+4+5=14$ thousand dollars.

Q30. C. The constant of variation is $k=\frac{240}{3}=80$ km/h, so the distance in 5 hours is $80\times 5=400$ km.

Q31. B. For inverse variation $k=6\times 8=48$, so with 4 workers the time is $\frac{48}{4}=12$ hours.

Q32. D. $y=3+2\times 4^2=3+2\times 16=3+32=35$.

Q33. A. Each unit of magnitude multiplies the amplitude by 10, so a difference of $7-4=3$ magnitudes gives $10^3=1000$ times the amplitude.

Q34. C. Converting to metres, the field is 420 m by 250 m, so its area is $420\times 250=105\,000=1.05\times 10^5$ m².

Q35. B. Area of a sector $=\frac{\theta}{360}\times\pi r^2=\frac{60}{360}\times\pi\times 12^2=24\pi\approx 75.40$ cm².

Q36. D. Volume of a sphere $=\frac{4}{3}\pi r^3=\frac{4}{3}\pi\times 6^3=288\pi\approx 904.78$ cm³.

Q37. C. The length scale factor is $\sqrt{\frac{9}{16}}=\frac{3}{4}$, so the volume ratio is $3^3:4^3=27:64$.

Q38. A. Using $\tan 35°=\frac{h}{40}$, the height is $h=40\tan 35°\approx 40\times 0.7002=28.01$ m.

Q39. B. By the cosine rule, $x^2=9^2+12^2-2\times 9\times 12\times\cos 50°=225-138.84=86.16$, so $x\approx 9.28$ cm.

Q40. D. Area $=\frac{1}{2}ab\sin C=\frac{1}{2}\times 10\times 14\times\sin 68°\approx 70\times 0.9272=64.90$ cm².

Exam 3 — Multiple-choice (Hard)

Reading time: 15 minutes. Writing time: 1 hour 30 minutes. 40 multiple-choice questions, 40 marks. One bound reference and one approved technology (CAS calculator or software) are permitted; a scientific calculator may also be used. Choose the response that is ***correct*** *for the question. Unless otherwise indicated, diagrams are not drawn to scale.*

Section A — Multiple-choice

Question 1

The values in a data set have a mean of 20 and a standard deviation of 4. Each value in the data set is transformed by first multiplying it by 3 and then adding 5. The mean and standard deviation of the transformed data set are, respectively,

A. 65 and 12

B. 65 and 17

C. 60 and 12

D. 60 and 17

Question 2

The table below shows the number of goals scored by a netball team in each of its 25 matches this season.

Goals scored	0	1	2	3	4	5
Number of matches	3	5	8	4	3	2

The mean and the median number of goals per match are, respectively,

A. 2.5 and 2

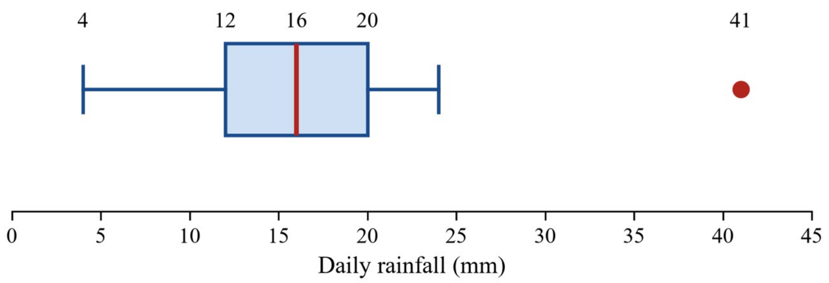
B. 2.2 and 3

C. 2.2 and 2

D. 2.2 and 2.5

Question 3

The boxplot below summarises the daily rainfall, in millimetres, recorded at a weather station on each of 15 days.



The interquartile range (in mm) and the number of values displayed as outliers are, respectively,

A. 8 and 1

B. 8 and 0

C. 12 and 1

D. 20 and 1

Question 4

Two data sets each contain seven values.

$$\text{Data set A:} 12, 15, 15, 18, 20, 22, 25$$

$$\text{Data set B:} 8, 14, 16, 19, 19, 21, 30$$

Which one of the following statements is correct?

A. The two data sets have the same median.

B. Data set B has a greater range than data set A.

C. Data set A has a greater interquartile range than data set B.

D. Data set A has a greater mean than data set B.

Question 5

In an arithmetic sequence, the 4th term is 17 and the 9th term is 42. The first term of the sequence is

A. 5

B. 7

C. -3

D. 2

Question 6

In a geometric sequence, the second term is 12 and the fifth term is 96. The eighth term of the sequence is

A. 384

B. 768

C. 192

D. 1536

Question 7

A sequence is generated by the recurrence relation

$$u_0 = 200, u_{n+1} = 0.8u_n + 30.$$

As n increases, the terms of this sequence approach a limiting value of

A. 150

B. 200

C. 230

D. 120

Question 8

The terms of a sequence are given by $t_n = 4n + 3$, for $n = 1, 2, 3, \ldots$. The smallest term of this sequence that is greater than 100 is

A. 99

B. 100

C. 103

D. 107

Question 9

An amount of $8000 is invested in an account paying interest at 6% per annum, compounding quarterly. The value of the investment after 2 years is closest to

A. $8960.00

B. $8988.80

C. $9017.28

D. $9011.94

Question 10

A machine is purchased for $45,000 and depreciates using the reducing-balance method at a rate of 15% per annum. After how many complete years does the value of the machine first fall below $20,000?

A. 4

B. 5

C. 6

D. 7

Question 11

A shop marks up the wholesale cost of a jacket by 40%. In a sale, the marked price is then reduced by 25%. If the sale price of the jacket is $126, the wholesale cost of the jacket was

A. $120.00

B. $109.57

C. $150.00

D. $105.00

Question 12

A personal loan of $6000 is charged flat (simple) interest at 9% per annum and is repaid in equal monthly instalments over 3 years. The amount of each monthly repayment is

A. $166.67

B. $195.00

C. $211.67

D. $225.00

Question 13

At a school canteen, 3 coffees and 2 muffins cost $23, while 5 coffees and 4 muffins cost $41. The cost of one muffin is

A. $3.00

B. $4.00

C. $4.50

D. $5.00

Question 14

A car park charges $6 for the first hour (or part of an hour) and $4 for each additional hour (or part of an hour). The charge for a car that is parked for 3 hours and 20 minutes is

A. $14.00

B. $16.00

C. $22.00

D. $18.00

Question 15

Income tax in a certain country is calculated using the scale below.

Taxable income ($)	Tax payable
0 – 20,000	nil
20,001 – 50,000	20% of each dollar over 20,000
50,001 and over	$6000 plus 35% of each dollar over 50,000

The tax payable on a taxable income of \$68,000 is

A. \$12,300

B. \$9600

C. \$13,600

D. \$6300

Question 16

A mobile-phone company offers two plans. Plan A charges a \$20 monthly fee plus \$0.10 for each minute of calls. Plan B charges a \$35 monthly fee plus \$0.05 for each minute of calls. The number of minutes of calls in a month for which the two plans cost the same is

A. 150

B. 200

C. 300

D. 100

Question 17

Consider the matrices

$$A=\begin{bmatrix}5 & 4\\ 2 & 2\end{bmatrix}, B=\begin{bmatrix}5 & 2\\ -1 & 3\end{bmatrix}.$$

The product AB is equal to

A. $\begin{bmatrix}29 & 24\\ 1 & 2\end{bmatrix}$

B. $\begin{bmatrix}21 & 22\\ 8 & 10\end{bmatrix}$

C. $\begin{bmatrix}25 & 8\\ -2 & 6\end{bmatrix}$

D. $\begin{bmatrix}21 & 18\\ 8 & 10\end{bmatrix}$

Question 18

The simultaneous equations $4x+3y=10$ and $5x+4y=13$ may be written as the matrix equation

$$\begin{bmatrix}4 & 3\\ 5 & 4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}10\\ 13\end{bmatrix}.$$

The solution (x, y) is

A. $(1,2)$

B. $(2,1)$

C. $(-1,2)$

D. $(1,-2)$

Question 19

Each year, households in a region subscribe to either streaming service A or streaming service B. The movement of subscribers is described by the transition matrix

$$T = \begin{array}{c} \begin{bmatrix} 0.75 & 0.5 \\ 0.25 & 0.5 \end{bmatrix} \end{array}\begin{array}{c} A \\ B \end{array}$$

in which the columns sum to 1. This year 800 households subscribe to A and 1200 subscribe to B. According to this model, the number of households subscribing to A in two years' time will be

A. 1200

B. 1250

C. 1360

D. 1300

Question 20

A population of 500 birds moves each day between two feeding sites, A and B, according to the transition matrix

$$\begin{bmatrix} 0.7 & 0.45 \\ 0.3 & 0.55 \end{bmatrix}\begin{array}{c} A \\ B \end{array}$$

in which the columns sum to 1. In the long run, the number of birds at site A each day approaches

A. 200

B. 250

C. 300

D. 350

Question 21

A gardener models the height of a seedling using the line of good fit

$$y = 4 + 2.5x,$$

where y is the height in centimetres and x is the number of weeks after planting. Which one of the following statements is correct?

A. The seedling was 2.5 cm tall when it was planted.

B. On average, the height of the seedling increases by 2.5 cm each week.

C. On average, the height of the seedling increases by 4 cm each week.

D. The seedling reaches a maximum height of 4 cm.

Question 22

The line of good fit $y = 4 + 2.5x$ (with y the height in centimetres and x the number of weeks after planting) was fitted to data collected from week 2 to week 10. Using this line, the predicted height of the seedling at week 20, and the type of prediction this represents, are respectively

A. 54 cm and an extrapolation

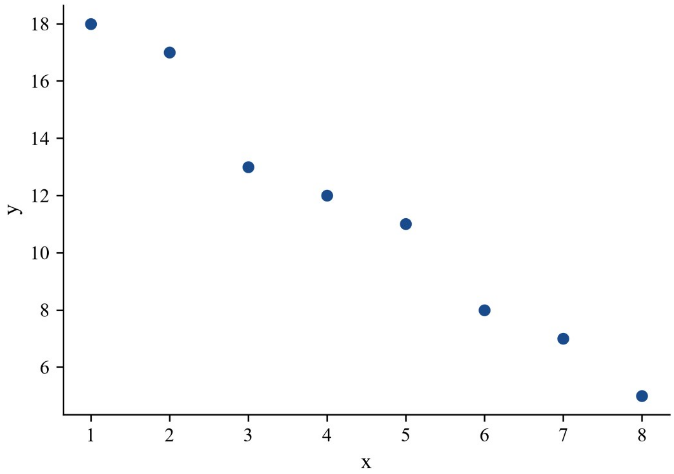
B. 54 cm and an interpolation

C. 50 cm and an extrapolation

D. 29 cm and an interpolation

Question 23

The scatterplot below shows the values of two numerical variables, x and y.



Which one of the following is most likely to be the equation of the line of good fit for these data?

A. $y = 20 + 2x$

B. $y = 4 + 2x$

C. $y = -2 + 20x$

D. $y = 20 - 2x$

Question 24

A researcher finds a strong positive association between the weekly sales of ice cream at a beach kiosk and the weekly number of swimmers rescued by lifeguards at the same beach. Which one of the following is the most appropriate conclusion?

A. Buying ice cream causes people to need rescuing.

B. Needing to be rescued causes people to buy ice cream.

C. The association is most likely explained by a common cause, such as hot weather, so it cannot be concluded that one variable causes the other.

D. Because the association is strong, one of the variables must be causing the other.

Question 25

A connected planar graph has 6 vertices, and every vertex has degree 3. Using the relationship between the sum of the degrees and the number of edges, together with Euler's formula, the number of faces (including the infinite outer region) is

A. 5

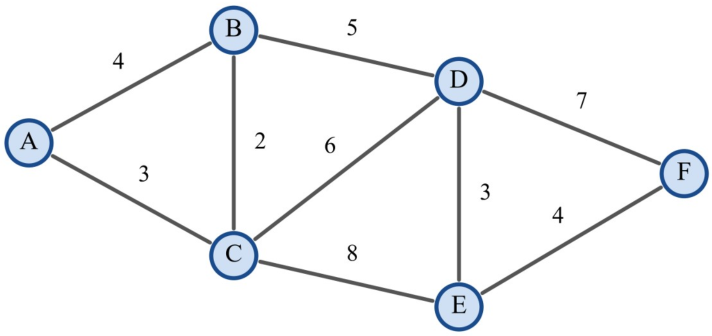
B. 3

C. 9

D. 7

Question 26

The network below shows the roads connecting six towns, A to F. The number on each edge is the length of that road, in kilometres.



The length, in kilometres, of the shortest path from town A to town F is

A. 13

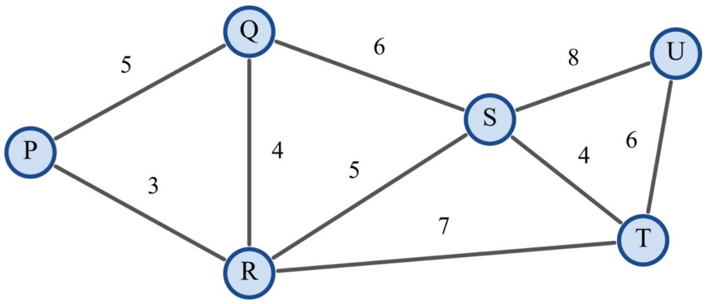
B. 15

C. 14

D. 16

Question 27

The network below shows the possible trenches for laying cable between six houses, P to U. The number on each edge is the cost, in thousands of dollars, of laying cable along that trench.



The minimum cost, in thousands of dollars, of connecting all six houses with cable is

A. 20

B. 24

C. 25

D. 22

Question 28

The adjacency matrix of a graph with vertices A, B, C and D (in that order) is

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}.$$

The number of walks of length 2 from vertex B to vertex D is

A. 0

B. 1

C. 2

D. 3

Question 29

The variable y varies directly as the square of x. When $x=4$, $y=32$. The value of y when $x=6$ is

A. 72

B. 48

C. 36

D. 12

Question 30

The time taken to complete a task varies inversely with the number of workers assigned to it. When 6 workers are assigned, the task takes 8 hours. The time taken, in hours, when 10 workers are assigned is

A. 5.0

B. 4.8

C. 6.0

D. 13.3

Question 31

For two variables x and y, a plot of y against x^2 is a straight line that passes through the point $(0,3)$ and has a slope of 2. Using this model, the value of y when $x=4$ is

A. 11

B. 19

C. 35

D. 67

Question 32

Two variables are related by a rule of the form $y=a+\frac{b}{x}$. When $x=2$, $y=11$, and when $x=4$, $y=8$. The values of a and b, respectively, are

A. $a=8, b=6$

B. $a=3, b=12$

C. $a=5, b=24$

D. $a=5, b=12$

Question 33

A window has the shape of a rectangle 2 m wide and 1.5 m high, with a semicircle of diameter 2 m on top of it. The total area of the window, in square metres, is closest to

A. 4.57

B. 6.14

C. 5.57

D. 4.14

Question 34

A solid is made up of a cylinder of radius 5 cm and height 12 cm, with a cone of radius 5 cm and height 6 cm placed on top. The total volume of the solid, in cubic centimetres, is

A. 300π

B. 350π

C. 400π

D. 250π

Question 35

A right pyramid has a square base of side length 10 cm and a vertical height of 12 cm. The total surface area of the pyramid, in square centimetres, is

A. 260

B. 100

C. 460

D. 360

Question 36

Two cylinders are mathematically similar. The ratio of their heights is $2:3$. The volume of the smaller cylinder is 240 cm³. The volume of the larger cylinder, in cubic centimetres, is

A. 360

B. 540

C. 810

D. 1080

Question 37

From the top of a vertical cliff 60 m high, the angle of depression to a boat at sea is 28°. The horizontal distance of the boat from the base of the cliff, in metres, is closest to

A. 31.9

B. 127.8

C. 128.0

D. 112.8

Question 38

In triangle ABC, angle $A = 40°$, side $a = 8$ cm and side $b = 11$ cm. To the nearest $0.1°$, the possible size or sizes of angle B is/are

A. 62.1° only

B. 62.1° or 117.9°

C. 27.9° or 152.1°

D. 55.2° or 124.8°

Question 39

In triangle PQR, $p = 7$ cm, $q = 9$ cm and the included angle $R = 110°$. The length of side r, in centimetres, is closest to

A. 11.4

B. 10.9

C. 13.2

D. 8.5

Question 40

A ship sails 25 km from a port on a bearing of 060°, then changes course and sails 40 km on a bearing of 150°. The distance of the ship from the port, in kilometres, is closest to

A. 65

B. 51

C. 32

D. 47

End of Exam 3

Exam 3 Formula Sheet

This Formula Sheet is provided with every examination in this booklet.

Data analysis

mean	$\bar{x} = \dfrac{\sum x}{n}$
sample standard deviation	$s = \sqrt{\dfrac{\sum (x - \bar{x})^2}{n-1}}$
interquartile range	$\text{IQR} = Q_3 - Q_1$
lower fence of a boxplot	$Q_1 - 1.5 \times \text{IQR}$
upper fence of a boxplot	$Q_3 + 1.5 \times \text{IQR}$
line of good fit	$y = a + b\,x$

Sequences, recurrence and financial mathematics

first-order linear recurrence	$t_0 = a\,,\, t_{n+1} = R\,t_n + d$
arithmetic sequence	$t_n = a + (n-1)\,d$
geometric sequence	$t_n = a\,R^{n-1}$
simple interest	$I = \dfrac{P\,r\,n}{100}$
flat-rate / unit-cost depreciation	$V_n = P - n\,D$
compound interest	$A = P\left(1 + \dfrac{r}{100}\right)^n$
reducing-balance depreciation	$V_n = P\left(1 - \dfrac{r}{100}\right)^n$

Functions, graphs and variation

linear function	$y = a + b\,x$
slope of a line	$b = \dfrac{y_2 - y_1}{x_2 - x_1}$
direct variation	$y = k\,x$
inverse variation	$y = \dfrac{k}{x}$

Matrices

determinant of a 2×2 matrix: for $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $det\ A = a\,d - b\,c$

inverse of a 2×2 matrix: $A^{-1} = \dfrac{1}{a\,d - b\,c}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $a\,d - b\,c \neq 0$

transition (state) recurrence $\quad S_{n+1} = T\,S_n$

Graphs and networks

Euler’s formula (connected planar graph) $\quad v - e + f = 2$

Measurement

Heron's rule: area of a triangle $=\sqrt{s(s-a)(s-b)(s-c)}$, where $s=\dfrac{a+b+c}{2}$

Pythagoras' theorem	$c^2=a^2+b^2$
circumference of a circle	$C=2\pi r$
area of a circle	$A=\pi r^2$
arc length	$\ell=\dfrac{\theta}{360}\times 2\pi r$
area of a sector	$A=\dfrac{\theta}{360}\times \pi r^2$
area of a triangle	$A=\dfrac{1}{2}bh$
volume of a prism	$V=A_{\text{cross}}\times h$
volume of a cylinder	$V=\pi r^2 h$
volume of a pyramid	$V=\dfrac{1}{3}A_{\text{base}}h$
volume of a sphere	$V=\dfrac{4}{3}\pi r^3$
surface area of a sphere	$A=4\pi r^2$
curved surface area of a cylinder	$A=2\pi rh$
similar-figure scale factors	length k, area k^2, volume k^3

Trigonometry

trigonometric ratios: $\sin\theta=\dfrac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta=\dfrac{\text{adjacent}}{\text{hypotenuse}}$, $\tan\theta=\dfrac{\text{opposite}}{\text{adjacent}}$

sine rule: $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

cosine rule: $a^2=b^2+c^2-2bc\cos A$

area of a triangle $A=\dfrac{1}{2}ab\sin C$

End of Formula Sheet

Exam 3 — Answers and solutions

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	A	C	A	B	D	B	A	C	D	B	A	C	B	D	A	C	B	A	D	C

Q	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	B	A	D	C	A	B	D	C	A	B	C	D	A	B	D	C	D	B	C	D

Q1. A. Multiplying by 3 scales both the mean and the standard deviation by 3; adding 5 shifts the mean but leaves the spread unchanged. New mean $=3\times20+5=65$; new standard deviation $=3\times4=12$.

Q2. C. Total goals $=0(3)+1(5)+2(8)+3(4)+4(3)+5(2)=55$, so the mean $=\frac{55}{25}=2.2$. With 25 matches the median is the 13th value in order, which lies in the "2 goals" group, so the median is 2.

Q3. A. From the boxplot $Q_1=12$ and $Q_3=20$, so IQR $=20-12=8$. The upper fence is $20+1.5\times8=32$, so only the value 41 is shown as an outlier — one outlier.

Q4. B. Medians are 18 (A) and 19 (B); both interquartile ranges equal 7; both means equal $127/7$. Only the ranges differ: B has range $30-8=22$, greater than A's $25-12=13$.

Q5. D. The common difference is $d=\frac{42-17}{9-4}=5$, so the first term is $t_1=17-3\times5=2$.

Q6. B. $\frac{t_5}{t_2}=R^3=\frac{96}{12}=8$, so $R=2$ and $t_1=\frac{12}{2}=6$. Then $t_8=6\times2^7=768$.

Q7. A. At the steady state $L=0.8L+30$, so $0.2L=30$ and $L=150$ (the terms fall from 200 towards 150).

Q8. C. $t_n=4n+3>100$ gives $n>24.25$, so the smallest such n is 25, and $t_{25}=4\times25+3=103$.

Q9. D. Quarterly: rate $=6\%/4=1.5\%$ over $4\times2=8$ periods, so value $=8000\times1.015^8=\$9011.94$.

Q10. B. $V_n=45\,000\times0.85^n$. At $n=4$, $V_4=\$23\,490.28$ (still above 20,000); at $n=5$, $V_5=\$19\,966.74$ (below), so the value first falls below \$20,000 after 5 years.

Q11. A. The net multiplier is $1.40\times0.75=1.05$, so the cost $=\frac{126}{1.05}=\$120.00$.

Q12. C. Flat interest $=6000\times0.09\times3=\$1620$, so the total repaid is $\$7620$ over 36 months, giving $\frac{7620}{36}=\$211.67$ per month.

Q13. B. Let c, m be the prices. From $3c+2m=23$ and $5c+4m=41$: doubling the first gives $6c+4m=46$, so $c=5$ and then $m=\frac{23-15}{2}=4$; a muffin costs \$4.

Q14. D. First hour \$6; the remaining 2 h 20 min counts as 3 additional part-hours at \$4 each $=\$12$. Total $=6+12=\$18$.

Q15. A. Tax $=6000+0.35\times(68\,000-50\,000)=6000+0.35\times18\,000=6000+6300=\$12\,300$.

Q16. C. Set $20+0.10t=35+0.05t$, so $0.05t=15$ and $t=300$ minutes.

Q17. B. $AB=\begin{bmatrix}5(5)+4(-1) & 5(2)+4(3)\\ 2(5)+2(-1) & 2(2)+2(3)\end{bmatrix}=\begin{bmatrix}21 & 22\\ 8 & 10\end{bmatrix}$.

Q18. A. The determinant is $4(4)-3(5)=1$, so $\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}4 & -3\\ -5 & 4\end{bmatrix}\begin{bmatrix}10\\ 13\end{bmatrix}=\begin{bmatrix}1\\ 2\end{bmatrix}$, i.e. $(x,y)=(1,2)$.

Q19. D. After 1 year $A=0.75(800)+0.5(1200)=1200$, $B=800$. After 2 years $A=0.75(1200)+0.5(800)=900+400=1300$.

Q20. C. At the steady state $0.7a+0.45b=a$ with $a+b=500$, giving $b=2/3a$, so $5/3a=500$ and $a=300$.

Q21. B. In $y=4+2.5x$ the slope 2.5 is the average increase in height per additional week; the intercept 4 is the height at planting.

Q22. A. $y=4+2.5\times 20=54$ cm; because week 20 lies outside the data range (weeks 2 to 10), this is an extrapolation.

Q23. D. The points fall as x increases (negative slope), and extending the trend back to $x=0$ gives a y-intercept near 20. Only $y=20-2x$ has both a negative slope and an intercept of about 20.

Q24. C. A strong association does not establish causation; a common cause such as hot weather can drive both variables up together.

Q25. A. The sum of the degrees is $6\times 3=18$, so the number of edges is $e=9$. Euler's formula $v-e+f=2$ gives $f=2-6+9=5$.

Q26. B. The shortest path is $A-C-E-F$ with length $3+8+4=15$ km (shorter than $A-C-D-F=16$).

Q27. D. The minimum spanning tree uses edges $PR(3)$, $QR(4)$, $RS(5)$, $ST(4)$ and $TU(6)$, a total of $3+4+5+4+6=22$ thousand dollars.

Q28. C. The number of length-2 walks from B to D is the (B,D) entry of the square of the adjacency matrix, which equals 2 (via $B-A-D$ and $B-C-D$).

Q29. A. $y=kx^2$ with $32=k\times 4^2$ gives $k=2$, so at $x=6$, $y=2\times 36=72$.

Q30. B. $t=\frac{k}{n}$ with $k=6\times 8=48$, so for 10 workers $t=\frac{48}{10}=4.8$ hours.

Q31. C. The linear model in x^2 is $y=3+2x^2$; at $x=4$, $y=3+2\times 16=35$.

Q32. D. From $a+b/2=11$ and $a+b/4=8$, subtracting gives $b/4=3$, so $b=12$ and $a=8-3=5$.

Q33. A. Rectangle $2\times 1.5=3$ m² plus semicircle $1/2\pi(1)^2=\pi/2\approx 1.57$ m². Total ≈ 4.57 m².

Q34. B. Cylinder $\pi(5)^2(12)=300\pi$; cone $1/3\pi(5)^2(6)=50\pi$. Total $=350\pi$ cm³.

Q35. D. The slant height is $\sqrt{12^2+5^2}=13$ cm. Total surface area $=10^2+4\times 1/2(10)(13)=100+260=360$ cm².

Q36. C. The volume scale factor is $(3/2)^3=27/8$, so the larger volume $=240\times 27/8=810$ cm³.

Q37. D. The angle of depression equals the angle of elevation from the boat, so $\tan 28° =\frac{60}{d}$ gives

$d=\frac{60}{\tan 28°}\approx 112.8$ m.

Q38. B. By the sine rule $\sin B=\frac{11\sin 40°}{8}\approx 0.8838$, so $B\approx 62.1°$ or $B\approx 117.9°$; both are valid since $40°+117.9°<180°$ (the ambiguous case).

Q39. C. By the cosine rule $r^2 = 7^2 + 9^2 - 2(7)(9)\cos 110° \approx 173.09$, so $r \approx 13.2$ cm.

Q40. D. The two bearings differ by $150° - 60° = 90°$, so the legs meet at a right angle and the distance $= \sqrt{25^2 + 40^2} = \sqrt{2225} \approx 47$ km.

Exam 4 — Short-answer (Easy)

Reading time: 15 minutes. Writing time: 1 hour 30 minutes. Answer all questions. Total 60 marks. One bound reference and one approved technology (CAS calculator or software) are permitted; a scientific calculator may also be used. In all questions where a numerical answer is required, an exact value must be given unless otherwise specified. Where more than one mark is available, appropriate working must be shown. Unless otherwise indicated, diagrams are not drawn to scale.

Data analysis

Question 1 (6 marks)

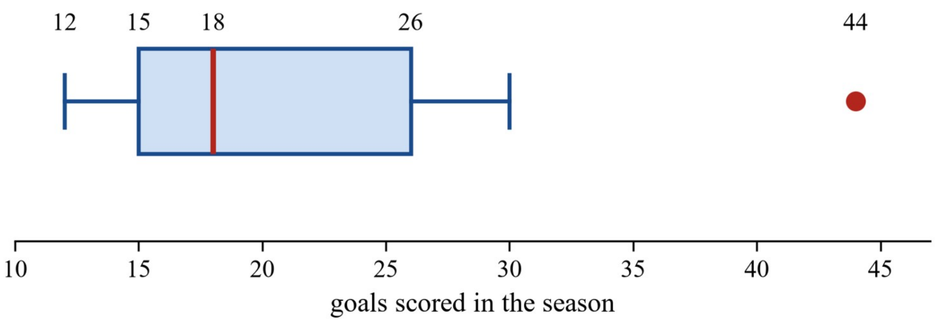
During one season, the number of goals scored by each of the 11 players in a soccer squad was recorded. The values, in order, are

$$12, 14, 15, 16, 17, 18, 20, 23, 26, 30, 44.$$

a. Write down the five-number summary for this data. 2 marks

b. Determine the interquartile range (IQR), and hence show that a player who scored 44 goals is an outlier. 2 marks

The data is displayed in the boxplot below.



c. Describe the shape of the distribution, referring to any outliers. 2 marks

Question 2 (4 marks)

A gardener recorded the number of new leaves that appeared during one week on each of 8 pot plants. The results were

$$5, 6, 8, 8, 9, 11, 12, 13.$$

a. Find the mean number of new leaves. 1 mark

b. Find the median number of new leaves. 1 mark

c. Find the range of the data. 1 mark

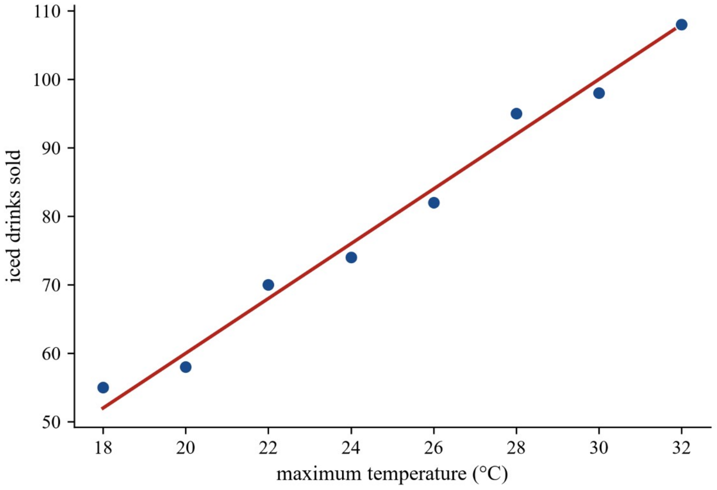
d. Find the standard deviation, correct to two decimal places. 1 mark

Question 3 (4 marks)

On 8 days a café recorded the maximum daily temperature, in degrees Celsius, and the number of iced drinks sold. The results are shown in the table.

Maximum temperature (°C)	18	20	22	24	26	28	30	32
Iced drinks sold	55	58	70	74	82	95	98	108

The data is displayed in the scatterplot below, together with a line of good fit.



The equation of the line of good fit is

$$\text{drinks} = -20 + 4 \times \text{temperature}.$$

a. Describe the association between the number of iced drinks sold and the maximum temperature in terms of direction, form and strength. 2 marks

b. Use the line of good fit to predict the number of iced drinks sold on a day when the maximum temperature is 27 °C. 1 mark

c. Interpret the slope of the line of good fit in terms of temperature and iced drinks sold. 1 mark

Sequences, recurrence and financial mathematics

Question 4 (6 marks)

A sequence is defined by the recurrence relation

$$t_1 = 5, t_{n+1} = t_n + 6.$$

a. Write down the first four terms of the sequence. 2 marks

b. This sequence is arithmetic. Write down its common difference, and hence use the rule $t_n = a + (n-1)d$ to find the 15th term. 2 marks

A different sequence is geometric with first term 3 and common ratio 2.

c. Write down the first three terms of this geometric sequence, and find its 6th term. 2 marks

Question 5 (6 marks)

Leah invests $6,000 in an account that pays 4% per annum simple interest.

a. Find the interest earned after 5 years, and the total value of the investment at that time. 2 marks

b. Instead, Leah could invest the \$6,000 at 4% per annum compounding annually. Find the value of this investment after 5 years, correct to the nearest cent. 2 marks

A delivery van is purchased for \$24,000 and depreciates at 12% per annum using the reducing-balance method.

c. Find the value of the van after 3 years, correct to the nearest cent. 2 marks

Functions, graphs and variation

Question 6 (6 marks)

A plumber charges a fixed call-out fee plus an hourly rate. The cost, \$C, of a job lasting t hours is given by

$$C=80+65t.$$

a. State the value of the call-out fee, and state the hourly rate charged. 2 marks

b. Find the cost of a job that lasts 4 hours. 1 mark

c. A job cost \$470. Find how many hours the job lasted. 1 mark

A second plumber charges according to the rule $C=140+50t$.

d. Find the number of hours for which the two plumbers charge the same amount. 2 marks

Question 7 (4 marks)

a. The variable y varies directly as x. When $x=6$, $y=15$. Find the constant of variation k, and hence find the value of y when $x=10$. 2 marks

b. The time t hours taken to fill a tank varies inversely with the number of pumps p used. When 4 pumps are used the tank fills in 9 hours. Find the time taken to fill the tank when 6 pumps are used. 2 marks

Matrices

Question 8 (4 marks)

Consider the matrices

$$A=\begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}, B=\begin{bmatrix} 5 & 2 \\ 0 & 1 \end{bmatrix}.$$

a. Write down the order of matrix A. 1 mark

b. Find $A+B$. 1 mark

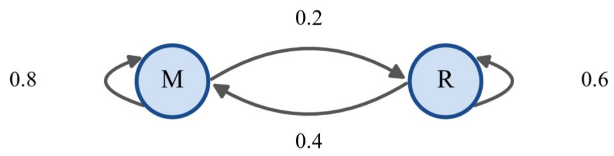
c. Find the matrix product AB. 2 marks

Question 9 (4 marks)

a. For the matrix $C=\begin{bmatrix} 6 & 4 \\ 3 & 7 \end{bmatrix}$, find $det(C)$, and state whether C has an inverse. 2 marks

A fitness chain runs two gyms, the Metro gym (M) and the Riverside gym (R). Each month, members switch between the two gyms according to the transition matrix and diagram below, where the columns and rows are taken in the order M, R.

$$T=\begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix}$$



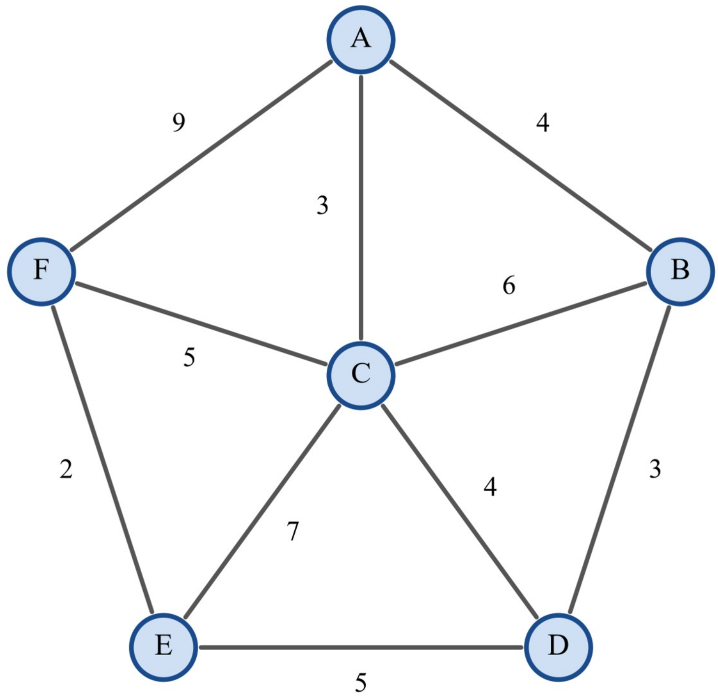
At the start of one month there are 300 members at the Metro gym and 200 members at the Riverside gym, so the initial state matrix is $S_0 = \begin{bmatrix} 300 \\ 200 \end{bmatrix}$.

b. Find the state matrix $S_1 = T\,S_0$, and hence state the number of members at each gym after one month. 2 marks

Graphs and networks

Question 10 (8 marks)

The weighted graph below shows six towns A, B, C, D, E and F, and the roads that connect them. Each number gives the length of that road, in kilometres.



a. Write down the degree of vertex C and the total number of edges in the graph. 2 marks

b. The graph is connected and planar, with $v = 6$ vertices and $e = 10$ edges. Use Euler's formula $v - e + f = 2$ to find the number of faces, f. 2 marks

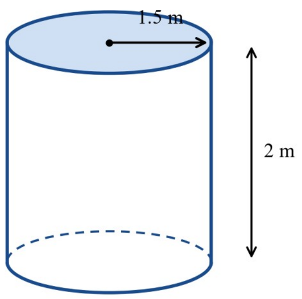
c. Find the length of the shortest path from town A to town F, and state the path. 2 marks

d. Cable is to be laid along some of the roads so that every town is connected, using the least possible total length of cable. Find the total length of the minimum spanning tree for this graph. 2 marks

Measurement and trigonometry

Question 11 (4 marks)

A water tank is in the shape of a cylinder with radius 1.5 m and height 2 m, as shown.

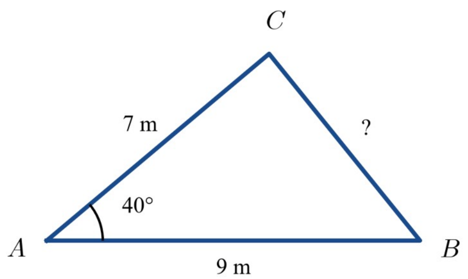


a. Find the volume of the tank, in cubic metres, correct to two decimal places. 2 marks

b. Find the curved surface area of the tank, in square metres, correct to two decimal places. 2 marks

Question 12 (4 marks)

A triangular garden bed ABC has $AB = 9$ m, $AC = 7$ m and the angle at A equal to $40°$, as shown.



a. Find the length of side BC, correct to two decimal places. 2 marks

b. Find the area of the garden bed, correct to two decimal places. 2 marks

End of Exam 4

Exam 4 Formula Sheet

This Formula Sheet is provided with every examination in this booklet.

Data analysis

mean	$\bar{x} = \frac{\sum x}{n}$
sample standard deviation	$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$
interquartile range	$\text{IQR} = Q_3 - Q_1$
lower fence of a boxplot	$Q_1 - 1.5 \times \text{IQR}$
upper fence of a boxplot	$Q_3 + 1.5 \times \text{IQR}$
line of good fit	$y = a + b x$

Sequences, recurrence and financial mathematics

first-order linear recurrence	$t_0 = a, t_{n+1} = R t_n + d$
arithmetic sequence	$t_n = a + (n-1) d$
geometric sequence	$t_n = a R^{n-1}$
simple interest	$I = \frac{P r n}{100}$
flat-rate / unit-cost depreciation	$V_n = P - n D$
compound interest	$A = P\left(1 + \frac{r}{100}\right)^n$
reducing-balance depreciation	$V_n = P\left(1 - \frac{r}{100}\right)^n$

Functions, graphs and variation

linear function	$y = a + b x$
slope of a line	$b = \frac{y_2 - y_1}{x_2 - x_1}$
direct variation	$y = k x$
inverse variation	$y = \frac{k}{x}$

Matrices

determinant of a 2×2 matrix: for $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $det\ A = a d - b c$

inverse of a 2×2 matrix: $A^{-1} = \frac{1}{a d - b c} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $a d - b c \neq 0$

transition (state) recurrence $S_{n+1} = T S_n$

Graphs and networks

Euler's formula (connected planar graph) $v - e + f = 2$

Measurement

Heron's rule: area of a triangle $=\sqrt{s(s-a)(s-b)(s-c)}$, where $s=\dfrac{a+b+c}{2}$

Pythagoras' theorem	$c^2=a^2+b^2$
circumference of a circle	$C=2\pi r$
area of a circle	$A=\pi r^2$
arc length	$\ell=\dfrac{\theta}{360}\times 2\pi r$
area of a sector	$A=\dfrac{\theta}{360}\times \pi r^2$
area of a triangle	$A=\dfrac{1}{2}bh$
volume of a prism	$V=A_{\text{cross}}\times h$
volume of a cylinder	$V=\pi r^2 h$
volume of a pyramid	$V=\dfrac{1}{3}A_{\text{base}}h$
volume of a sphere	$V=\dfrac{4}{3}\pi r^3$
surface area of a sphere	$A=4\pi r^2$
curved surface area of a cylinder	$A=2\pi r h$
similar-figure scale factors	length k, area k^2, volume k^3

Trigonometry

trigonometric ratios: $\sin\theta=\dfrac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta=\dfrac{\text{adjacent}}{\text{hypotenuse}}$, $\tan\theta=\dfrac{\text{opposite}}{\text{adjacent}}$

sine rule: $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

cosine rule: $a^2=b^2+c^2-2bc\cos A$

area of a triangle $A=\dfrac{1}{2}ab\sin C$

End of Formula Sheet

Exam 4 — Answers and solutions

Data analysis

Q1. (a) The data is already in order ($n=11$). The minimum is 12 and the maximum is 44. The median is the 6th value, $M=18$. The lower half (excluding the median) is $12, 14, 15, 16, 17$, whose median is $Q_1=15$; the upper half is $20, 23, 26, 30, 44$, whose median is $Q_3=26$. The five-number summary is

$$\min=12, Q_1=15, M=18, Q_3=26, \max=44.$$

(b) The interquartile range is $\text{IQR}=Q_3-Q_1=26-15=11$. The upper fence is

$$Q_3+1.5\times\text{IQR}=26+1.5\times 11=26+16.5=42.5.$$

Since $44>42.5$, a score of 44 goals lies beyond the upper fence and is therefore an outlier.

(c) Ignoring the outlier, the box and the whisker are longer on the right-hand (higher) side of the median, so the distribution is **positively skewed** (skewed to the right). There is one outlier, at 44 goals.

Q2. (a) $\bar{x}=\dfrac{5+6+8+8+9+11+12+13}{8}=\dfrac{72}{8}=9$ new leaves.

(b) With 8 values, the median is the average of the 4th and 5th values: $\dfrac{8+9}{2}=8.5$ new leaves.

(c) The range is $\max-\min=13-5=8$ new leaves.

(d) Using technology, the sample standard deviation is $s\approx 2.83$ (to two decimal places).

Q3. (a) As the maximum temperature increases the number of iced drinks sold also increases, so the association is **positive**. The points lie close to a straight line, so the form is **linear** and the association is **strong**.

(b) Substituting $\text{temperature}=27$:

$$\text{drinks}=-20+4\times 27=-20+108=88.$$

About 88 iced drinks are predicted.

(c) The slope is 4. This means that, on average, for each 1 °C increase in the maximum temperature, the number of iced drinks sold increases by about 4.

Sequences, recurrence and financial mathematics

Q4. (a) Starting from $t_1=5$ and adding 6 each time:

$$t_1=5, t_2=11, t_3=17, t_4=23.$$

(b) Each term is 6 more than the previous term, so the common difference is $d=6$ (with $a=5$). Then

$$t_{15}=a+(15-1)d=5+14\times 6=5+84=89.$$

(c) With first term 3 and common ratio 2, the first three terms are $3, 6, 12$. The 6th term is

$$t_6=aR^{n-1}=3\times 2^5=3\times 32=96.$$

Q5. (a) The simple interest is

$$I=\frac{Prn}{100}=\frac{6000\times 4\times 5}{100}=\$1\,200.$$

The total value after 5 years is $6000+1200=\$7\,200$.

(b) With annual compounding,

$$A=P\left(1+\frac{r}{100}\right)^{n}=6000\times(1.04)^{5}=6000\times 1.2166529\ldots=\$7\,299.92\text{ (to the nearest cent).}$$

(c) With reducing-balance depreciation,

$$V_3=P\left(1-\frac{r}{100}\right)^{n}=24000\times(0.88)^{3}=24000\times 0.681472=\$16\,355.33\text{ (to the nearest cent).}$$

Functions, graphs and variation

Q6. (a) When $t=0$, $C=80$, so the call-out fee is $\$80$. The coefficient of t is the hourly rate, $\$65$ per hour.

(b) $C=80+65\times 4=80+260=\$340$.

(c) Solve $470=80+65t$:

$$65t=470-80=390\,,\,t=\frac{390}{65}=6\text{ hours.}$$

(d) The two plumbers charge the same when $80+65t=140+50t$:

$$65t-50t=140-80\,,\,15t=60\,,\,t=4\text{ hours.}$$

(At $t=4$ both charge $80+260=\$340$.)

Q7. (a) Direct variation means $y=kx$. Using $x=6$, $y=15$: $15=k\times 6$, so $k=\frac{15}{6}=2.5$. Then when $x=10$, $y=2.5\times 10=25$.

(b) Inverse variation means $t=\frac{k}{p}$, so $k=t\times p=9\times 4=36$. When $p=6$,

$$t=\frac{36}{6}=6\text{ hours.}$$

Matrices

Q8. (a) A has 2 rows and 2 columns, so its order is 2×2.

(b) $A+B=\begin{bmatrix}4+5 & 1+2\\ 7+0 & 2+1\end{bmatrix}=\begin{bmatrix}9 & 3\\ 7 & 3\end{bmatrix}$.

(c) $AB=\begin{bmatrix}4 & 1\\ 7 & 2\end{bmatrix}\begin{bmatrix}5 & 2\\ 0 & 1\end{bmatrix}=\begin{bmatrix}4(5)+1(0) & 4(2)+1(1)\\ 7(5)+2(0) & 7(2)+2(1)\end{bmatrix}=\begin{bmatrix}20 & 9\\ 35 & 16\end{bmatrix}$.

Q9. (a) $det(C)=6\times 7-4\times 3=42-12=30$. Since $det(C)=30\neq 0$, the matrix C **has an inverse**.

(b) $S_1=TS_0=\begin{bmatrix}0.8 & 0.4\\ 0.2 & 0.6\end{bmatrix}\begin{bmatrix}300\\ 200\end{bmatrix}=\begin{bmatrix}0.8(300)+0.4(200)\\ 0.2(300)+0.6(200)\end{bmatrix}=\begin{bmatrix}320\\ 180\end{bmatrix}$.

After one month there are 320 members at the Metro gym and 180 members at the Riverside gym.

Graphs and networks

Q10. (a) Vertex C is the central town, joined to A,B,D,E and F, so its degree is 5. The degrees of A,B,C,D,E,F are $3,3,5,3,3,3$; their sum is 20, which is twice the number of edges, so the graph has $20\div 2=10$ edges.

(b) Euler's formula gives $v-e+f=2$, so

$$6-10+f=2\,,\,f=2-6+10=6\,.$$

The graph has 6 faces (including the outer region).

(c) By inspection, the shortest path from A to F is $A-C-F$, with length

$$3+5=8\,\text{km}.$$

(Other routes are longer: for example the direct road $A-F=9$ km, and $A-C-E-F=3+7+2=12$ km.)

(d) Choosing the shortest roads that connect all towns without forming a cycle gives the minimum spanning tree $E-F(2), A-C(3), B-D(3), A-B(4), C-F(5)$. These five roads connect all six towns, with total length

$$2+3+3+4+5=17\,\text{km}.$$

Measurement and trigonometry

Q11. (a) The volume of a cylinder is $V=\pi r^2 h$:

$$V=\pi\times 1.5^2\times 2=4.5\pi\approx 14.14\,\text{m}^3.$$

(b) The curved surface area of a cylinder is $A=2\pi r h$:

$$A=2\pi\times 1.5\times 2=6\pi\approx 18.85\,\text{m}^2.$$

Q12. (a) Using the cosine rule with $b=AC=7$, $c=AB=9$ and the included angle $A=40°$:

$$BC^2=b^2+c^2-2bc\cos A=7^2+9^2-2(7)(9)\cos 40°=130-126\cos 40°=33.4784\ldots,$$

so $BC=\sqrt{33.4784\ldots}\approx 5.79$ m.

(b) Using the area rule $A=1/2\,bc\sin A$:

$$A=1/2\times 7\times 9\times\sin 40°=31.5\sin 40°\approx 20.25\,\text{m}^2.$$

Exam 5 — Short-answer (Medium)

Reading time: 15 minutes. Writing time: 1 hour 30 minutes. Answer all questions. Total 60 marks. One bound reference and one approved technology (CAS calculator or software) are permitted; a scientific calculator may also be used. In all questions where a numerical answer is required, an exact value must be given unless otherwise specified. Where more than one mark is available, appropriate working must be shown. Unless otherwise indicated, diagrams are not drawn to scale.

Data analysis

Question 1 (8 marks)

Over one season, the number of points scored by basketball player A in each of her 15 games was recorded. The values, arranged in ascending order, are

$$6\ 8\ 9\ 11\ 12\ 12\ 14\ 15\ 16\ 18\ 19\ 20\ 22\ 24\ 39$$

The mean number of points scored is 16.33 (correct to two decimal places).

a. Determine the five-number summary for player A's scores. 2 marks

b. Using the $1.5 \times \mathrm{IQR}$ rule, show that 39 is an outlier, and state whether the data set has any other outliers. 2 marks

c. By comparing the mean with the median, describe the shape of the distribution of player A's scores. 1 mark

Player B also played 15 games. Her scores have the five-number summary

$$\mathrm{min} = 10\,, Q_1 = 14\,, \mathrm{median} = 17\,, Q_3 = 21\,, \mathrm{max} = 28\,.$$

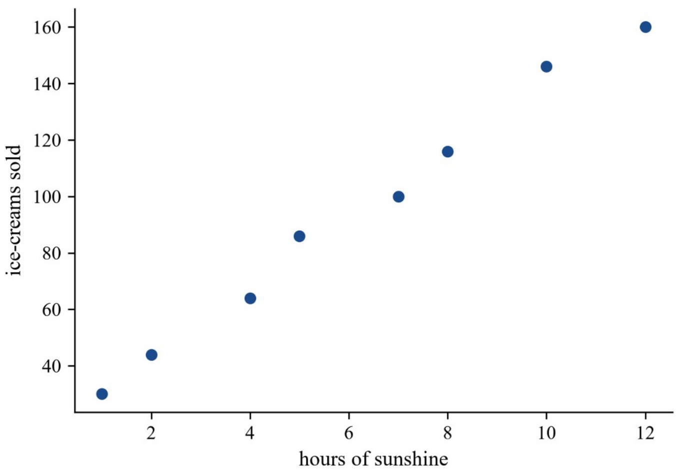
d. Compare the scoring of the two players in terms of centre and spread, referring to the median and the interquartile range. 2 marks

e. State whether the mean or the median better represents player A's typical score, and justify your choice. 1 mark

Question 2 (6 marks)

On 8 days during summer, a beachside kiosk recorded the number of hours of sunshine and the number of ice-creams sold. The data are shown in the table and scatterplot below, with hours of sunshine as the explanatory variable.

Hours of sunshine	1	2	4	5	7	8	10	12
Ice-creams sold	30	44	64	86	100	116	146	160



a. Describe the association between the number of ice-creams sold and the hours of sunshine in terms of direction, form and strength. 1 mark

b. A line of good fit is drawn by eye. It passes through the data points $(2,44)$ and $(8,116)$. Determine the equation of this line, in the form $y=a+bx$, where y is the number of ice-creams sold and x is the hours of sunshine. 2 marks

c. Interpret the slope of the line of good fit in the context of this data. 1 mark

d. Use the line of good fit to predict the number of ice-creams sold on a day with 6 hours of sunshine, and state whether this prediction is an interpolation or an extrapolation. 1 mark

e. Explain why using this line to predict the number of ice-creams sold on a day with 20 hours of sunshine would not be reliable. 1 mark

Sequences, recurrence and financial mathematics

Question 3 (6 marks)

A newly opened sports club records its membership at the end of each month. At the end of the first month there are 40 members, and the club predicts that the number of members will grow by 15% each month. Let t_n be the predicted membership at the end of month n, so that $t_1=40$.

a. Write down a recurrence relation, including the value of t_1, that models the predicted membership. 1 mark

b. Use the recurrence relation to find the predicted membership at the end of month 4, correct to the nearest whole number. 2 marks

c. Write down a rule for t_n in terms of n, and hence determine the first month at the end of which the predicted membership exceeds 100. 2 marks

d. Describe the long-term behaviour of this model, and give one reason why it is not realistic. 1 mark

Question 4 (6 marks)

Mia has \$8000 to invest for 3 years.

a. One account pays simple interest at 4.5% per annum. Find the value of the investment after 3 years. 2 marks

b. A second account pays compound interest at 4.5% per annum, compounding annually. Find the value of the investment after 3 years, correct to the nearest cent. 2 marks

c. Determine how much more the compound-interest account is worth than the simple-interest account after 3 years. 1 mark

d. Mia also buys equipment for \$12,000 that depreciates by 20% per annum using the reducing-balance method. Find the value of the equipment after 3 years. 1 mark

Functions, graphs and variation

Question 5 (6 marks)

Two car-hire companies charge for a one-day hire according to the distance d (in kilometres) driven.

- Company A charges a fixed fee of \$40 plus \$0.25 per kilometre.
- Company B charges a fixed fee of \$70 plus \$0.15 per kilometre.

a. Write down a rule for the cost C (in dollars) in terms of d for each company. 1 mark

b. Find the distance for which the two companies charge the same amount, and state that amount. 2 marks

c. For a one-day hire in which 500 km is driven, determine which company is cheaper and by how much. 1 mark

A third company, Company C, charges a flat \$60 for any hire of up to 200 km, and then \$0.30 for each kilometre driven beyond 200 km.

d. Write down a piecewise rule for Company C's cost C (in dollars) for a hire of d km, and use it to find the cost of a hire in which 350 km is driven. 2 marks

Question 6 (4 marks)

The braking distance d (in metres) of a car varies directly with the square of its speed v (in km/h). A car travelling at 30 km/h has a braking distance of 18 m.

a. Find the constant of variation, and write down a rule for d in terms of v. 2 marks

b. Find the braking distance of a car travelling at 50 km/h. 1 mark

c. Find the speed of a car whose braking distance is 72 m. 1 mark

Matrices

Question 7 (5 marks)

A cafe records its sales of coffees and muffins over two days in the matrix

$$Q=\begin{bmatrix}20 & 12\\ 26 & 9\end{bmatrix},$$

where the columns give the number of coffees and muffins sold (in that order) and row 1 is Monday, row 2 is Tuesday. A coffee sells for \$5 and a muffin for \$4, so the prices are stored in

$$P=\begin{bmatrix}5\\ 4\end{bmatrix}.$$

a. Evaluate the matrix product QP, and explain what its entries represent. 2 marks

A delivery of mystery crates is weighed. Five small crates and three large crates together weigh 19 kg, while three small crates and two large crates together weigh 12 kg. Let x be the mass (in kg) of a small crate and y the mass of a large crate, so that

$$\begin{bmatrix}5 & 3\\ 3 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}19\\ 12\end{bmatrix}.$$

b. Find the determinant and the inverse of $A=\begin{bmatrix}5 & 3\\ 3 & 2\end{bmatrix}$. 2 marks

c. Hence solve the matrix equation for x and y. 1 mark

Question 8 (3 marks)

Each week, commuters in a town travel to work either by car (C) or by public transport (P). From one week to the next, 90% of car users continue to drive and 10% switch to public transport, while 5% of public-transport users switch to the car and 95% continue with public transport. This is modelled by the transition matrix

$$T=\begin{bmatrix}0.9 & 0.05\\ 0.1 & 0.95\end{bmatrix}$$

acting on the state vector $\begin{bmatrix}C\\ P\end{bmatrix}$. In one particular week, 600 commuters travel by car and 400 by public transport, so $S_0=\begin{bmatrix}600\\ 400\end{bmatrix}$.

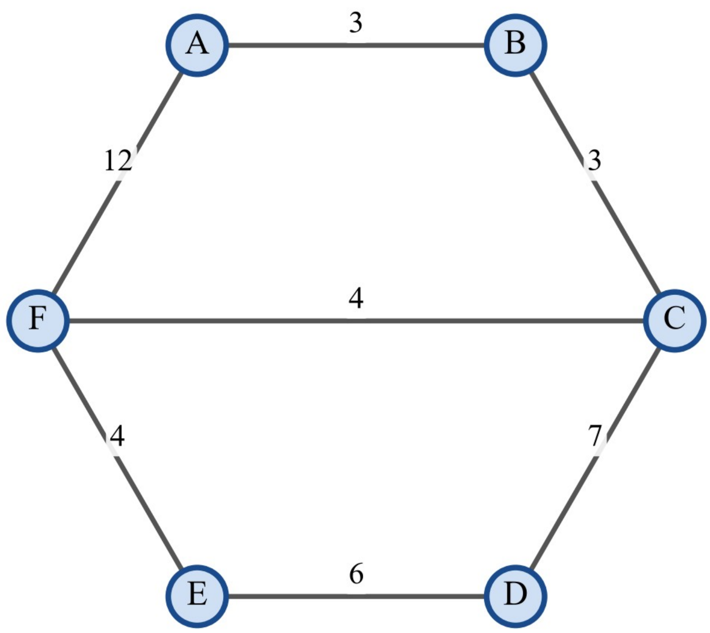
a. Interpret the entry 0.05 of the transition matrix T in this context. 1 mark

b. Find the number of commuters travelling by each mode in the following week. 2 marks

Graphs and networks

Question 9 (8 marks)

The network below shows the roads connecting six towns, A to F. The number on each edge is the length of that road, in kilometres.



a. Write down the degree of town C, and hence, by considering the sum of the degrees of all the towns, determine the number of roads in the network. 2 marks

b. The graph is planar. Use Euler's formula to determine the number of faces of the graph. 2 marks

c. Find the length of the shortest path from town A to town F, and state the route taken. 2 marks

d. A fibre cable is to be laid along the roads to connect all six towns, using the least possible total length of cable (a minimum spanning tree). Determine the total length of cable required. 2 marks

Measurement and trigonometry

Question 10 (4 marks)

A pavilion at a sports ground is in the shape of a square-based prism with base edge 8 m and vertical walls 5 m high, topped by a square-based pyramid roof of height 3 m.

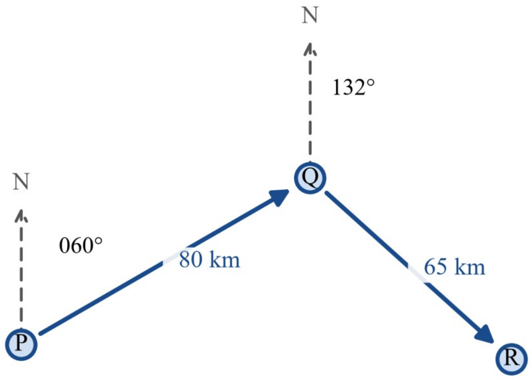
a. Find the total volume of the pavilion, in cubic metres. 2 marks

b. Show that the slant height of each triangular face of the roof is 5 m, and hence find the total area of the four faces of the roof, in square metres. 1 mark

c. A larger pavilion is built to the same design, with every length 1.5 times the corresponding length of the original pavilion. Find the total volume of the larger pavilion, in cubic metres. 1 mark

Question 11 (4 marks)

A ship leaves port P and sails 80 km to point Q on a bearing of $060\,°$T. At Q it changes course and sails 65 km to point R on a bearing of $132\,°$T. The path is shown below.



a. Show that the angle PQR is $108\,°$, and hence find the distance PR, correct to one decimal place. 2 marks

b. Find the bearing of R from P, correct to the nearest degree. 2 marks

End of Exam 5

Exam 5 Formula Sheet

This Formula Sheet is provided with every examination in this booklet.

Data analysis

mean	$\bar{x} = \dfrac{\sum x}{n}$
sample standard deviation	$s = \sqrt{\dfrac{\sum (x - \bar{x})^2}{n - 1}}$
interquartile range	$\text{IQR} = Q_3 - Q_1$
lower fence of a boxplot	$Q_1 - 1.5 \times \text{IQR}$
upper fence of a boxplot	$Q_3 + 1.5 \times \text{IQR}$
line of good fit	$y = a + b\,x$

Sequences, recurrence and financial mathematics

first-order linear recurrence	$t_0 = a,\, t_{n+1} = R\,t_n + d$
arithmetic sequence	$t_n = a + (n - 1)\,d$
geometric sequence	$t_n = a\,R^{n-1}$
simple interest	$I = \dfrac{P\,r\,n}{100}$
flat-rate / unit-cost depreciation	$V_n = P - n\,D$
compound interest	$A = P\left(1 + \dfrac{r}{100}\right)^n$
reducing-balance depreciation	$V_n = P\left(1 - \dfrac{r}{100}\right)^n$

Functions, graphs and variation

linear function	$y = a + b\,x$
slope of a line	$b = \dfrac{y_2 - y_1}{x_2 - x_1}$
direct variation	$y = k\,x$
inverse variation	$y = \dfrac{k}{x}$

Matrices

determinant of a 2×2 matrix: for $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = a\,d - b\,c$

inverse of a 2×2 matrix: $A^{-1} = \dfrac{1}{a\,d - b\,c}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $a\,d - b\,c \neq 0$

transition (state) recurrence $S_{n+1} = T\,S_n$

Graphs and networks

Euler's formula (connected planar graph) $v - e + f = 2$

Measurement

Heron's rule: area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \dfrac{a+b+c}{2}$

Pythagoras' theorem	$c^2 = a^2 + b^2$
circumference of a circle	$C = 2\pi r$
area of a circle	$A = \pi r^2$
arc length	$\ell = \dfrac{\theta}{360} \times 2\pi r$
area of a sector	$A = \dfrac{\theta}{360} \times \pi r^2$
area of a triangle	$A = \dfrac{1}{2} b h$
volume of a prism	$V = A_{\text{cross}} \times h$
volume of a cylinder	$V = \pi r^2 h$
volume of a pyramid	$V = \dfrac{1}{3} A_{\text{base}} h$
volume of a sphere	$V = \dfrac{4}{3} \pi r^3$
surface area of a sphere	$A = 4\pi r^2$
curved surface area of a cylinder	$A = 2\pi r h$
similar-figure scale factors	length k, area k^2, volume k^3

Trigonometry

trigonometric ratios: $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$, $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

area of a triangle $A = \dfrac{1}{2} ab\sin C$

End of Formula Sheet

Exam 5 — Answers and solutions

Data analysis

Question 1

(a) With $n=15$, the median is the 8th value, so the median is 15. The lower half is the first 7 values, whose median is $Q_1=11$; the upper half is the last 7 values, whose median is $Q_3=20$. The five-number summary is

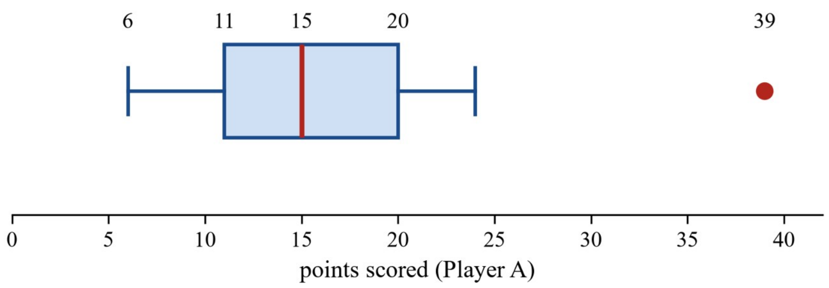
$$\text{min}=6\,,Q_1=11\,,\text{median}=15\,,Q_3=20\,,\text{max}=39\,.$$

(b) $\text{IQR}=Q_3-Q_1=20-11=9$. The upper fence is

$$Q_3+1.5\times\text{IQR}=20+1.5\times9=20+13.5=33.5\,.$$

Since $39>33.5$, the value 39 is an outlier. The lower fence is $Q_1-1.5\times\text{IQR}=11-13.5=-2.5$, and the smallest value (6) is greater than -2.5, so 39 is the **only** outlier.

(c) The median is 15 points and the mean is 16.33 points, so the mean is greater than the median. The mean is pulled up by the single high value (39), so the distribution is **positively skewed** (skewed to the right). A boxplot confirms this, with 39 shown as a separate outlier above the upper whisker.



(d) Player B has the higher median (17 compared with 15), so on average player B scores more points per game. Player B is also more consistent: her interquartile range is $Q_3-Q_1=21-14=7$, which is smaller than player A's interquartile range of 9, so the middle half of player B's scores is less spread out.

(e) The **median** better represents player A's typical score. The single unusually high score of 39 is an outlier that inflates the mean, whereas the median is not affected by such an extreme value, so the median (15) is a more typical figure than the mean (16.33).

Question 2

(a) As the number of hours of sunshine increases, the number of ice-creams sold increases, and the points lie close to a straight line. The association is **positive, linear and strong**.

(b) The line of good fit passes through $(2\,,44)$ and $(8\,,116)$. Its slope is

$$b=\frac{116-44}{8-2}=\frac{72}{6}=12\,.$$

Using the point $(2\,,44)$, $44=a+12\times2$, so $a=44-24=20$. The equation of the line of good fit is

$$y=20+12\,x\,.$$

(c) The slope is 12. This means that, on average, each additional hour of sunshine is associated with about 12 **more ice-creams sold**.

(d) At $x=6$,

$$y=20+12\times6=20+72=92\,.$$

The model predicts about 92 ice-creams sold. Because 6 hours lies within the range of the data (1 to 12 hours), this prediction is an **interpolation**.

(e) The largest number of sunshine hours in the data is 12. Predicting for 20 hours requires extending the line well beyond the range of the data (an extrapolation), where there is no evidence that the linear pattern continues, so the prediction would not be reliable.

Sequences, recurrence and financial mathematics

Question 3

(a) Growth of 15% each month multiplies the membership by $1+0.15=1.15$. With $t_1=40$,

$$t_1=40, t_{n+1}=1.15t_n.$$

(b) Applying the rule,

$$t_2=1.15\times 40=46, t_3=1.15\times 46=52.9, t_4=1.15\times 52.9=60.835.$$

To the nearest whole number, the predicted membership at the end of month 4 is 61 **members**.

(c) The sequence is geometric with first term 40 and common ratio 1.15, so

$$t_n=40\times 1.15^{n-1}.$$

Evaluating terms: $t_7=40\times 1.15^6\approx 92.5$ and $t_8=40\times 1.15^7\approx 106.4$. Since $t_7<100<t_8$, the predicted membership first exceeds 100 at the end of **month** 8.

(d) Because the common ratio is greater than 1, the membership increases without bound, growing by a larger amount each month. This is not realistic: a club cannot grow forever, as it will eventually be limited by the size of the local population or the club's facilities.

Question 4

(a) Simple interest: $I=\frac{Prn}{100}=\frac{8000\times 4.5\times 3}{100}=1080$. The value after 3 years is

$$8000+1080=\$9080.$$

(b) Compound interest: $A=P\left(1+\frac{r}{100}\right)^n=8000\times(1.045)^3$. Now $1.045^3=1.141166125$, so

$$A=8000\times 1.141166125=9129.329=\$9129.33 \text{ (to the nearest cent).}$$

(c) The difference after 3 years is

$$9129.33-9080=\$49.33,$$

so the compound-interest account is worth \$49.33 more than the simple-interest account.

(d) Reducing-balance depreciation at 20% per annum multiplies the value by $1-0.20=0.80$ each year:

$$V_3=12\,000\times(0.80)^3=12\,000\times 0.512=\$6144.00.$$

Functions, graphs and variation

Question 5

(a) Company A: $C=40+0.25d$. Company B: $C=70+0.15d$.

(b) The charges are equal when

$$40+0.25\,d=70+0.15\,d.$$

Then $0.25\,d-0.15\,d=70-40$, so $0.10\,d=30$ and $d=300$ km. The common charge is

$$C=40+0.25\times 300=40+75=\$\,115.$$

(c) For $d=500$: Company A charges $40+0.25\times 500=\$\,165$, and Company B charges $70+0.15\times 500=\$\,145$. Company B is cheaper, by $165-145=\$\,20$.

(d) For a hire of d km, Company C's cost is

$$C=\begin{cases} 60, & 0\le d\le 200, \\ 60+0.30\left(d-200\right), & d>200. \end{cases}$$

For $d=350$ (which is greater than 200),

$$C=60+0.30\times\left(350-200\right)=60+0.30\times 150=60+45=\$\,105.$$

Question 6

(a) Direct variation with the square of the speed means $d=k\,v^2$ for a constant k. Substituting $v=30$ and $d=18$,

$$18=k\times 30^2=900\,k\Rightarrow k=\frac{18}{900}=0.02.$$

So $d=0.02\,v^2$ (equivalently $d=\frac{v^2}{50}$).

(b) At $v=50$,

$$d=0.02\times 50^2=0.02\times 2500=50\,\text{m}.$$

(c) Setting $d=72$,

$$72=0.02\,v^2\Rightarrow v^2=\frac{72}{0.02}=3600\Rightarrow v=60\,\text{km/h}.$$

Matrices

Question 7

(a) $QP=\begin{bmatrix}20 & 12\\ 26 & 9\end{bmatrix}\begin{bmatrix}5\\ 4\end{bmatrix}=\begin{bmatrix}20\times 5+12\times 4\\ 26\times 5+9\times 4\end{bmatrix}=\begin{bmatrix}148\\ 166\end{bmatrix}.$

The entries give the cafe's total takings, in dollars: **\$148 on Monday** and **\$166 on Tuesday**.

(b) $det\left(A\right)=5\times 2-3\times 3=10-9=1$. Since $det\left(A\right)=1\neq 0$, the inverse exists:

$$A^{-1}=\begin{bmatrix}2 & -3\\ -3 & 5\end{bmatrix}.$$

(c) Multiplying both sides of $A\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}19\\ 12\end{bmatrix}$ on the left by A^{-1},

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}2 & -3\\ -3 & 5\end{bmatrix}\begin{bmatrix}19\\ 12\end{bmatrix}=\begin{bmatrix}38-36\\ -57+60\end{bmatrix}=\begin{bmatrix}2\\ 3\end{bmatrix}.$$

Hence $x=2$ and $y=3$: a small crate has mass 2 kg and a large crate 3 kg. (Check: $5\times 2+3\times 3=19$ and $3\times 2+2\times 3=12$.)

Question 8

(a) The entry 0.05 is in row C (car), column P (public transport). It means that each week 5% of public-transport users switch to travelling by car.

(b) After one week,

$$S_1 = T\,S_0 = \begin{bmatrix} 0.9 & 0.05 \\ 0.1 & 0.95 \end{bmatrix}\begin{bmatrix} 600 \\ 400 \end{bmatrix} = \begin{bmatrix} 0.9 \times 600 + 0.05 \times 400 \\ 0.1 \times 600 + 0.95 \times 400 \end{bmatrix} = \begin{bmatrix} 540 + 20 \\ 60 + 380 \end{bmatrix} = \begin{bmatrix} 560 \\ 440 \end{bmatrix}.$$

In the following week, 560 commuters travel by car and 440 by public transport.

Graphs and networks

Question 9

(a) Town C is joined to B, D and F, so its degree is 3. The degrees of the six towns are $A\,2, B\,2, C\,3, D\,2, E\,2, F\,3$, with sum $2+2+3+2+2+3=14$. Since the sum of the degrees equals twice the number of edges, the number of roads is $14 \div 2 = 7$.

(b) For a connected planar graph, Euler's formula is $v - e + f = 2$. Here $v = 6$ and $e = 7$, so

$$f = 2 - v + e = 2 - 6 + 7 = 3.$$

The graph has 3 faces (two bounded regions together with the outer region).

(c) Checking the possible routes, the shortest is

$$A \to B \to C \to F : 3 + 3 + 4 = 10\,\text{km}.$$

(For comparison, the direct road $A \to F = 12$ km and $A \to B \to C \to D \to E \to F = 3+3+7+6+4 = 23$ km, both longer.) The shortest path from A to F has length 10 **km**, along the route $A - B - C - F$.

(d) Building the minimum spanning tree by choosing the shortest edges that do not form a cycle gives

$$A - B(3), B - C(3), C - F(4), E - F(4), D - E(6),$$

which connect all six towns with 5 edges. The least total length of cable required is

$$3 + 3 + 4 + 4 + 6 = 20\,\text{km}.$$

Measurement and trigonometry

Question 10

(a) The walls form a square-based prism with cross-sectional area $8^2 = 64$ m² and height 5 m, so

$$V_{\text{prism}} = A_{\text{cross}} \times h = 64 \times 5 = 320\,\text{m}^3.$$

The roof is a square-based pyramid standing on the same 8 m by 8 m base, with height 3 m, so

$$V_{\text{pyramid}} = \frac{1}{3} A_{\text{base}} h = \frac{1}{3} \times 64 \times 3 = 64\,\text{m}^3.$$

The total volume of the pavilion is

$$320 + 64 = 384\,\text{m}^3.$$

(b) The slant height ℓ runs from the apex of the roof to the midpoint of a base edge. It is the hypotenuse of a right-angled triangle whose other two sides are the height of the pyramid (3 m) and half the base edge ($1/2 \times 8 = 4$ m), so by Pythagoras' theorem

$$\ell = \sqrt{3^2 + 4^2} = \sqrt{25} = 5\,\text{m}.$$

Each of the four faces of the roof is a triangle with base 8 m and perpendicular height $\ell=5$ m, so the total area of the roof is

$$4\times\frac{1}{2}\times 8\times 5=80\,\mathrm{m}^2.$$

(c) For similar solids, the volume scales by the cube of the length scale factor. With a scale factor of 1.5,

$$V_{\text{large}}=384\times 1.5^3=384\times 3.375=1296\,\mathrm{m}^3.$$

Question 11

(a) At Q, the bearing back to P is $060°+180°=240°$T, and the bearing on to R is $132°$T. The angle PQR between these two directions is

$$240°-132°=108°.$$

By the cosine rule,

$$PR^2=PQ^2+QR^2-2\times PQ\times QR\times\cos(108°)=80^2+65^2-2\times 80\times 65\times\cos(108°).$$

This gives $PR^2=6400+4225-10400\times(-0.309017)=10625+3213.78=13838.78$, so

$$PR=\sqrt{13838.78}\approx 117.6\,\mathrm{km}.$$

(b) Let $\angle QPR$ be the angle at P. By the sine rule,

$$\frac{\sin(\angle QPR)}{QR}=\frac{\sin(\angle PQR)}{PR}\Rightarrow\sin(\angle QPR)=\frac{65\times\sin(108°)}{117.6}=\frac{65\times 0.951057}{117.6}\approx 0.5255.$$

Hence $\angle QPR=\sin^{-1}(0.5255)\approx 31.7°$. The bearing of R from P is measured clockwise from north at P:

$$060°+31.7°=091.7°\approx 092°\,\mathrm{T}.$$

Exam 6 — Short-answer (Hard)

Reading time: 15 minutes. Writing time: 1 hour 30 minutes. Answer all questions. Total 60 marks. One bound reference and one approved technology (CAS calculator or software) are permitted; a scientific calculator may also be used. In all questions where a numerical answer is required, an exact value must be given unless otherwise specified. Where more than one mark is available, appropriate working must be shown. Unless otherwise indicated, diagrams are not drawn to scale.

Data analysis

Question 1 (8 marks)

At the Ashby district show, the mass, in kilograms, of each of the 15 pumpkins entered in the produce competition was recorded. The values, arranged in ascending order, are

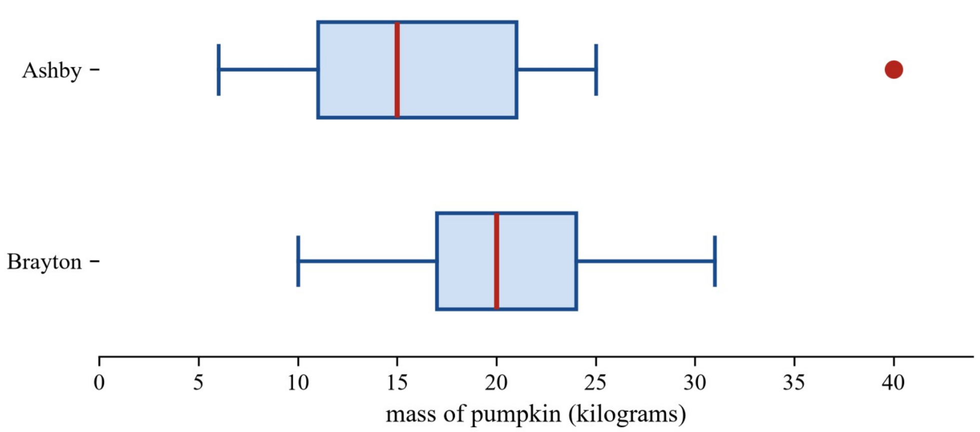
$$6\ 8\ 9\ 11\ 12\ 12\ 14\ 15\ 16\ 18\ 19\ 21\ 23\ 25\ 40$$

a. Determine the five-number summary for the Ashby masses. 2 marks

b. Using the $1.5 \times$ IQR rule, determine whether the Ashby data set contains any outliers. Show the value of the relevant fence in your working. 2 marks

c. The mean of the Ashby masses is 16.6 kg. By comparing the mean with the median, describe the shape of the distribution and explain what feature of the data produces this shape. 1 mark

At the Brayton district show, held on the same weekend, the masses of the 15 pumpkins entered were also recorded. The masses at the two shows are displayed in the parallel boxplots below.

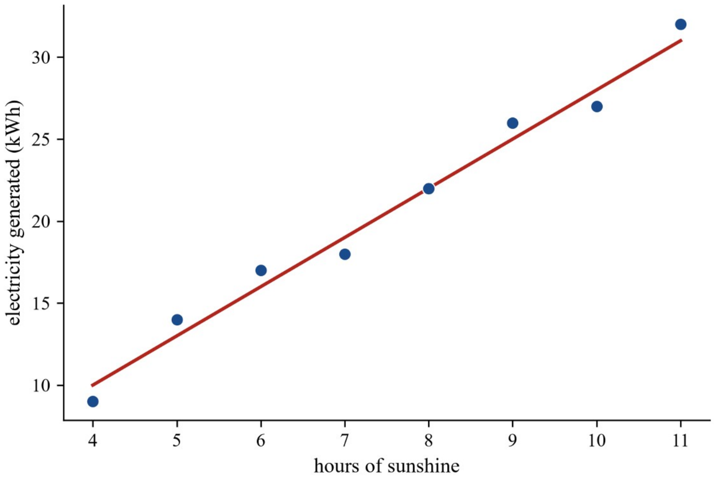


d. Use the parallel boxplots to compare the masses at the two shows in terms of both centre and spread. Refer to appropriate statistical values in your answer. 3 marks

Question 2 (6 marks)

A household with rooftop solar panels records the number of hours of sunshine and the electricity generated (in kWh) on each of eight days. The data are shown in the table and scatterplot below, with hours of sunshine as the explanatory variable.

Hours of sunshine	4	5	6	7	8	9	10	11
Electricity generated (kWh)	9	14	17	18	22	26	27	32



A line of good fit has been drawn on the scatterplot by eye. It passes through the two points $(5, 13)$ and $(10, 28)$.

a. Describe the association between the electricity generated and the number of hours of sunshine in terms of direction, form and strength. 2 marks

b. Determine the equation of the line of good fit shown, in terms of the electricity generated and the hours of sunshine. 2 marks

c. Interpret the slope of the line of good fit in the context of this data. 1 mark

d. Use the line of good fit to predict the electricity generated on a day with 13 hours of sunshine, and comment on the reliability of this prediction. 1 mark

Sequences, recurrence and financial mathematics

Question 3 (6 marks)

A lake is stocked with trout. Each year, 80% of the trout in the lake survive to the following year, and the fisheries authority then releases another 200 trout into the lake. Let P_n be the number of trout in the lake at the start of year n, with $P_0 = 1500$.

a. Write down a recurrence relation, including the value of P_0, that models the number of trout in the lake. 1 mark

b. Find the number of trout in the lake at the start of year 1 and at the start of year 2. 2 marks

c. State whether the trout population is increasing or decreasing over the first few years, and describe the long-term behaviour of the population. 2 marks

d. Determine the long-term (steady-state) number of trout in the lake. 1 mark

Question 4 (6 marks)

Priya buys a new commercial dishwasher for her café. The dishwasher is advertised at \$2400, plus 10% GST.

a. Calculate the total price Priya pays for the dishwasher, including GST. 1 mark

b. Starting from the total price paid in part **a.**, the dishwasher is depreciated using the reducing-balance method at 18% per annum. Find the value of the dishwasher after 3 years, correct to the nearest cent. 2 marks

c. Suppose that, instead of buying the dishwasher, Priya had invested the same total amount (from part **a.**) at 5.2% per annum, compounding quarterly. Find the value of the investment after 3 years, and hence the total interest earned, each correct to the nearest cent. 2 marks

d. Over the same 3 years, the price of commercial dishwashers rises with inflation at an average of 3% per annum. Find the price of an identical new dishwasher after 3 years, correct to the nearest cent. 1 mark

Functions, graphs and variation

Question 5 (6 marks)

A school is having its yearbook printed and has two quotes. **Copperplate Print** charges a set-up fee of \$450 plus \$12 for each copy printed. **Ardent Press** charges a set-up fee of \$1200 plus \$7 for each copy printed. Let C be the total cost, in dollars, of printing n copies.

a. Write down a rule for the total cost C in terms of n for each of Copperplate Print and Ardent Press. 2 marks

b. Find the number of copies for which the two printers charge the same amount, and state that common cost. 2 marks

c. The school decides to print 400 copies. Determine which printer is cheaper for this order, and by how much. 2 marks

Question 6 (4 marks)

Water is supplied to an irrigation system through pipes of different sizes, all at the same pressure. The flow rate R, in litres per minute, varies directly with the square of the internal diameter d, in millimetres, of the pipe. A pipe of internal diameter 20 mm has a flow rate of 24 litres per minute.

a. Find the constant of variation k, and write down the rule for R in terms of d. 2 marks

b. Find the flow rate through a pipe of internal diameter 50 mm. 1 mark

c. When R is plotted against d^2, the points lie on a straight line passing through the origin. State the slope of this line and what it represents. 1 mark

Matrices

Question 7 (4 marks)

A theatre sells adult tickets at \$$a$ each and concession tickets at \$$c$ each. Ticket sales over two evenings lead to the matrix equation

$$\begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} a \\ c \end{bmatrix}=\begin{bmatrix} 80 \\ 71 \end{bmatrix}.$$

a. Find the determinant of $M=\begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}$, and hence write down M^{-1}. 2 marks

b. Use the inverse matrix to solve the equation for a and c, and hence state the price of an adult ticket and of a concession ticket. 2 marks

Question 8 (4 marks)

A regional library service has 9000 regular borrowers. In any month, a borrower takes out either printed books (P) or e-books (E). From one month to the next:

- $90\,\%$ of the printed-book borrowers take out printed books again, and $10\,\%$ move to e-books;
- $20\,\%$ of the e-book borrowers move to printed books, and $80\,\%$ take out e-books again.

This is modelled by the transition matrix T acting on the state vector $\begin{bmatrix} P \\ E \end{bmatrix}$, where

$$T=\begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}.$$

In the current month, 5000 borrowers take out printed books and 4000 take out e-books, so $S_0=\begin{bmatrix} 5000 \\ 4000 \end{bmatrix}$.

a. Explain what the entry 0.2 of the transition matrix T tells the librarian. 1 mark

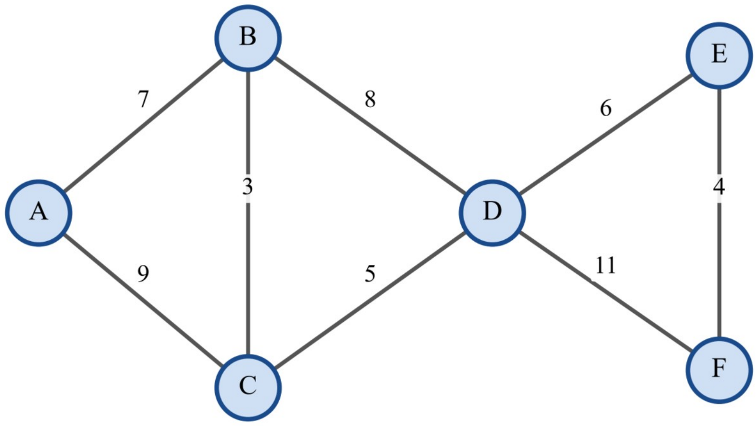
b. Find the number of borrowers taking out each format after one month. 1 mark

c. Determine the long-term (steady-state) number of borrowers taking out each format. 2 marks

Graphs and networks

Question 9 (8 marks)

Six wineries, A to F, in a Victorian wine region are joined by the country roads shown in the network below. The number on each road is its length, in kilometres.



a. Write down the degree of winery D. Hence, by considering the sum of the degrees of all six wineries, determine the total number of roads in the network. 2 marks

b. The network is a connected planar graph. Using Euler's formula, determine the number of faces in the network as drawn, and explain what a face represents in this context. 2 marks

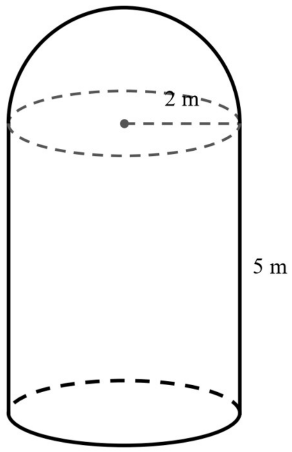
c. A tour bus starts at winery A and must reach winery F. Find the length of the shortest route, and state the route taken. 2 marks

d. A shared-use cycling path is to be built beside some of these roads so that all six wineries are linked, using the least possible total length of path (a minimum spanning tree). Determine this minimum total length, and list the roads beside which the path is built. 2 marks

Measurement and trigonometry

Question 10 (4 marks)

A bulk fuel tank at a transport depot is in the shape of a cylinder of radius 2 m and height 5 m, topped by a hemisphere of radius 2 m, as shown below.

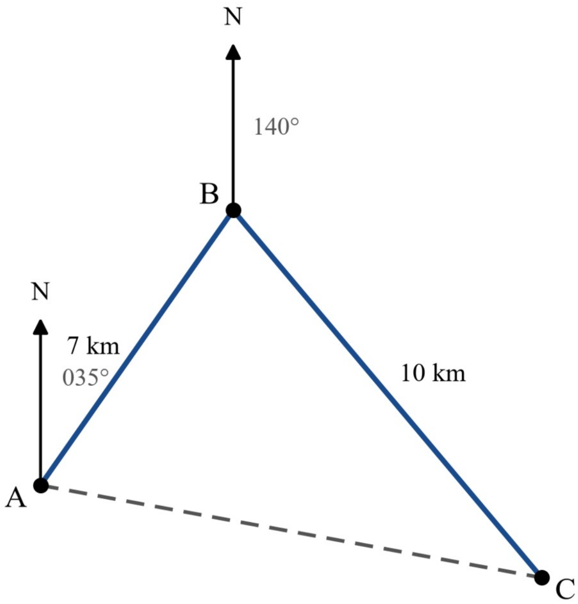


a. Show that the volume of the cylindrical part is 20π m³, and hence find the total volume of the tank, correct to two decimal places. 2 marks

b. A larger tank is geometrically similar to the first, with every linear dimension 1.2 times as large. Using the appropriate scale factor, find the total volume of the larger tank, correct to two decimal places. 2 marks

Question 11 (4 marks)

A hot-air balloon lifts off from launch site A and drifts 7 km on a bearing of $035°$ to point B. The wind then changes, and the balloon drifts a further 10 km on a bearing of $140°$ before landing at point C, as shown below.



a. Show that the size of angle ABC is $75°$. 1 mark

b. Find the straight-line distance AC, correct to one decimal place. 1 mark

c. The ground crew must drive from the landing site C straight back to the launch site A. Find the bearing on which they must travel, correct to the nearest degree. 2 marks

End of Exam 6

Exam 6 Formula Sheet

This Formula Sheet is provided with every examination in this booklet.

Data analysis

mean	$\bar{x}=\dfrac{\sum x}{n}$
sample standard deviation	$s=\sqrt{\dfrac{\sum(x-\bar{x})^2}{n-1}}$
interquartile range	$\text{IQR}=Q_3-Q_1$
lower fence of a boxplot	$Q_1-1.5\times\text{IQR}$
upper fence of a boxplot	$Q_3+1.5\times\text{IQR}$
line of good fit	$y=a+bx$

Sequences, recurrence and financial mathematics

first-order linear recurrence	$t_0=a, t_{n+1}=Rt_n+d$
arithmetic sequence	$t_n=a+(n-1)d$
geometric sequence	$t_n=aR^{n-1}$
simple interest	$I=\dfrac{Prn}{100}$
flat-rate / unit-cost depreciation	$V_n=P-nD$
compound interest	$A=P\left(1+\dfrac{r}{100}\right)^n$
reducing-balance depreciation	$V_n=P\left(1-\dfrac{r}{100}\right)^n$

Functions, graphs and variation

linear function	$y=a+bx$
slope of a line	$b=\dfrac{y_2-y_1}{x_2-x_1}$
direct variation	$y=kx$
inverse variation	$y=\dfrac{k}{x}$

Matrices

determinant of a 2×2 matrix: for $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $det\ A=ad-bc$

inverse of a 2×2 matrix: $A^{-1}=\dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $ad-bc\neq0$

transition (state) recurrence $S_{n+1}=TS_n$

Graphs and networks

Euler's formula (connected planar graph) $v-e+f=2$

Measurement

Heron's rule: area of a triangle $=\sqrt{s(s-a)(s-b)(s-c)}$, where $s=\dfrac{a+b+c}{2}$

Pythagoras' theorem	$c^2=a^2+b^2$
circumference of a circle	$C=2\pi r$
area of a circle	$A=\pi r^2$
arc length	$\ell=\dfrac{\theta}{360}\times 2\pi r$
area of a sector	$A=\dfrac{\theta}{360}\times \pi r^2$
area of a triangle	$A=\dfrac{1}{2}bh$
volume of a prism	$V=A_{\text{cross}}\times h$
volume of a cylinder	$V=\pi r^2 h$
volume of a pyramid	$V=\dfrac{1}{3}A_{\text{base}}h$
volume of a sphere	$V=\dfrac{4}{3}\pi r^3$
surface area of a sphere	$A=4\pi r^2$
curved surface area of a cylinder	$A=2\pi r h$
similar-figure scale factors	length k, area k^2, volume k^3

Trigonometry

trigonometric ratios: $\sin\theta=\dfrac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta=\dfrac{\text{adjacent}}{\text{hypotenuse}}$, $\tan\theta=\dfrac{\text{opposite}}{\text{adjacent}}$

sine rule: $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

cosine rule: $a^2=b^2+c^2-2bc\cos A$

area of a triangle $A=\dfrac{1}{2}ab\sin C$

End of Formula Sheet

Exam 6 — Answers and solutions

Data analysis

Question 1

(a) With $n=15$, the median is the 8th value, $Q_2=15$. The lower half is the first 7 values ($6, 8, 9, 11, 12, 12, 14$), with median $Q_1=11$; the upper half is the last 7 values ($16, 18, 19, 21, 23, 25, 40$), with median $Q_3=21$. The five-number summary is

$$\min=6, Q_1=11, Q_2=15, Q_3=21, \max=40.$$

(b) $\text{IQR}=Q_3-Q_1=21-11=10$. The upper fence is

$$Q_3+1.5\times\text{IQR}=21+1.5\times10=21+15=36.$$

Since $40>36$, the value 40 is an outlier. The lower fence is $Q_1-1.5\times\text{IQR}=11-15=-4$, and the smallest value (6) is greater than -4, so 40 is the **only** outlier.

(c) The mean (16.6 kg) is greater than the median (15 kg). When the mean exceeds the median the distribution is **positively skewed** (skewed to the right). Here the single large value 40 (the outlier) pulls the mean up above the median.

(d) **Centre:** the Brayton median (20 kg) is higher than the Ashby median (15 kg), so the pumpkins entered at Brayton are typically heavier than those entered at Ashby. **Spread:** Ashby is more variable — its interquartile range ($21-11=10$ kg) is larger than Brayton's ($24-17=7$ kg), and its range ($40-6=34$ kg) is larger than Brayton's ($31-10=21$ kg). Ashby is also positively skewed with one outlier (40 kg), whereas Brayton is roughly symmetric with no outliers.

Question 2

(a) The association is **positive** (as the number of hours of sunshine increases, the electricity generated increases), **linear** in form, and **strong** (the points lie very close to a straight line).

(b) The line of good fit passes through $(5, 13)$ and $(10, 28)$, so its slope is

$$b=\frac{28-13}{10-5}=\frac{15}{5}=3.$$

Using the point $(5, 13)$: $13=a+3\times5$, so $a=13-15=-2$. The equation of the line of good fit is

$$\text{electricity generated}=3\times\text{hours of sunshine}-2.$$

(c) The slope is 3: on average, for each additional hour of sunshine, the electricity generated **increases by about** 3 **kWh**.

(d) For 13 hours of sunshine,

$$\text{electricity generated}=3\times13-2=39-2=37.$$

The model predicts about 37 kWh. However, 13 hours lies outside the range of the data (4 to 11 hours), so this is an **extrapolation** and the prediction is **not reliable**.

Sequences, recurrence and financial mathematics

Question 3

(a) Each year 80% of the trout survive (a multiplication by 0.8) and then 200 are added, so

$$P_0=1500, P_{n+1}=0.8P_n+200.$$

(b) Applying the rule,

$$P_1 = 0.8 \times 1500 + 200 = 1200 + 200 = 1400,$$

$$P_2 = 0.8 \times 1400 + 200 = 1120 + 200 = 1320.$$

There are 1400 trout at the start of year 1 and 1320 trout at the start of year 2.

(c) The values $1500, 1400, 1320, \ldots$ are **decreasing**. Because the survival factor 0.8 is less than 1, the yearly change becomes smaller and smaller, so in the long term the population levels off, approaching a **limiting (steady-state) value**.

(d) Continue iterating the recurrence (a CAS table, or repeated use of $0.8 \times \text{Ans} + 200$):

$$1500, 1400, 1320, 1256, 1204.8, \ldots, 1053.7(n=10), \ldots, 1005.8(n=20), \ldots, 1000.6(n=30)$$

The terms fall by less and less and settle on 1000. The long-term number of trout in the lake is 1000. (Once the terms reach 1000 they stop changing: $0.8 \times 1000 + 200 = 1000$.)

Question 4

(a) Adding 10% GST multiplies the price by 1.1:

$$2400 \times 1.1 = 2640.$$

Priya pays **$2640.00**.

(b) Reducing-balance depreciation at 18% per annum multiplies the value by $1 - 0.18 = 0.82$ each year. Starting from the $2640 paid, after 3 years

$$V_3 = 2640 \times 0.82^3 = 2640 \times 0.551368 = 1455.61.$$

The dishwasher is worth **$1455.61** (to the nearest cent).

(c) Compounding quarterly at 5.2% per annum gives a quarterly rate of $\frac{5.2\%}{4} = 1.3\%$, over $3 \times 4 = 12$ quarters:

$$A = 2640 \times \left(1 + \frac{0.052}{4}\right)^{12} = 2640 \times 1.013^{12} = 3082.60.$$

The investment is worth **$3082.60**, so the total interest earned is $3082.60 - 2640 = $ **$442.60** (each to the nearest cent).

(d) Inflation of 3% per annum multiplies the price by 1.03 each year. After 3 years,

$$2640 \times 1.03^3 = 2640 \times 1.092727 = 2884.80.$$

An identical new dishwasher would cost about **$2884.80**.

Functions, graphs and variation

Question 5

(a) For Copperplate Print, $C = 450 + 12n$. For Ardent Press, $C = 1200 + 7n$.

(b) The two printers charge the same when

$$450 + 12n = 1200 + 7n.$$

Then $12n - 7n = 1200 - 450$, so $5n = 750$ and $n = 150$. The common cost is $C = 450 + 12 \times 150 = 450 + 1800 = 2250$. The two printers charge the same for 150 **copies**, at a cost of **$2250**.

(c) For $n = 400$:

$$\text{Copperplate:} C = 450 + 12 \times 400 = 450 + 4800 = 5250,$$

$$\text{Ardent}: C = 1200 + 7 \times 400 = 1200 + 2800 = 4000.$$

Ardent Press is cheaper, by $5250 - 4000 = $ **$1250**.

Question 6

(a) Direct variation with the square of the diameter means $R = k\,d^2$. Substituting $d = 20$, $R = 24$:

$$24 = k \times 20^2 = 400\,k\,, k = \frac{24}{400} = 0.06.$$

The rule is $R = 0.06\,d^2$.

(b) At $d = 50$,

$$R = 0.06 \times 50^2 = 0.06 \times 2500 = 150.$$

The flow rate is 150 **litres per minute**.

(c) Writing $R = 0.06 \times d^2$ as $R = 0.06 \times \left(d^2\right)$ shows that the graph of R against d^2 is a straight line through the origin with slope 0.06. This slope is the constant of variation k.

Matrices

Question 7

(a) $det\left(M\right) = 2 \times 1 - 3 \times 3 = 2 - 9 = -7$. Since $det\left(M\right) = -7 \neq 0$, the inverse exists:

$$M^{-1} = \frac{1}{-7}\begin{bmatrix} 1 & -3 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -1/7 & 3/7 \\ 3/7 & -2/7 \end{bmatrix}.$$

(b) Multiplying both sides of $M\begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 80 \\ 71 \end{bmatrix}$ on the left by M^{-1},

$$\begin{bmatrix} a \\ c \end{bmatrix} = \frac{1}{-7}\begin{bmatrix} 1 & -3 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 80 \\ 71 \end{bmatrix} = \frac{1}{-7}\begin{bmatrix} 80 - 213 \\ -240 + 142 \end{bmatrix} = \frac{1}{-7}\begin{bmatrix} -133 \\ -98 \end{bmatrix} = \begin{bmatrix} 19 \\ 14 \end{bmatrix}.$$

So $a = 19$ and $c = 14$: an adult ticket costs **$19** and a concession ticket costs **$14**. (Check: $2 \times 19 + 3 \times 14 = 80$ and $3 \times 19 + 14 = 71$.)

Question 8

(a) The entry 0.2 lies in row P, column E. It tells the librarian that from one month to the next, $20\,\%$ of the e-book borrowers change to printed books.

(b) After one month,

$$S_1 = T\,S_0 = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}\begin{bmatrix} 5000 \\ 4000 \end{bmatrix} = \begin{bmatrix} 4500 + 800 \\ 500 + 3200 \end{bmatrix} = \begin{bmatrix} 5300 \\ 3700 \end{bmatrix}.$$

So 5300 borrowers take out printed books and 3700 take out e-books.

(c) Keep applying $S_{n+1} = T\,S_n$ (or compute $T^n\,S_0$ on a CAS for larger n) and watch where the state vector settles:

$$S_1 = \begin{bmatrix} 5300 \\ 3700 \end{bmatrix}, S_2 = \begin{bmatrix} 5510 \\ 3490 \end{bmatrix}, S_3 = \begin{bmatrix} 5657 \\ 3343 \end{bmatrix}, \ldots, S_{10} \approx \begin{bmatrix} 5971.8 \\ 3028.2 \end{bmatrix}, \ldots, S_{30} \approx \begin{bmatrix} 6000.0 \\ 3000.0 \end{bmatrix}.$$

The elements change by less and less and settle on 6000 and 3000, so in the long term 6000 **borrowers take out printed books and** 3000 **take out e-books**. (A further month leaves this unchanged: $0.9 \times 6000 + 0.2 \times 3000 = 6000$, and the total is still 9000.)

Graphs and networks

Question 9

(a) Winery D is joined to B, C, E and F, so its degree is 4. The degrees are $A\,2, B\,3, C\,3, D\,4, E\,2, F\,2$, with sum $2+3+3+4+2+2=16$. Since the sum of the degrees equals twice the number of edges, the number of roads is $16\div 2=8$.

(b) Euler's formula for a connected planar graph is $v-e+f=2$. With $v=6$ vertices and $e=8$ edges,

$$f=2-v+e=2-6+8=4.$$

There are 4 **faces**. Each face is a region bounded by roads: three are enclosed areas of countryside surrounded by roads (the region inside $A-B-C$, the region inside $B-C-D$, and the region inside $D-E-F$), and the fourth is the single unbounded region outside the whole network.

(c) Applying a shortest-path search from A gives shortest distances $A\,0, B\,7, C\,9, D\,14, E\,20, F\,24$. The shortest route from A to F has length 24 **km**, along $A-C-D-E-F$ ($9+5+6+4=24$).

(d) Using a minimum-spanning-tree (greedy) algorithm, the shortest edges are added while avoiding cycles: $B-C(3)$, $E-F(4)$, $C-D(5)$, $D-E(6)$ and $A-B(7)$. These five edges link all six wineries, so the least total length of path is

$$3+4+5+6+7=25\,\text{km}.$$

The path is built beside the roads $B-C$, $E-F$, $C-D$, $D-E$ and $A-B$.

Measurement and trigonometry

Question 10

(a) The cylindrical part has radius 2 m and height 5 m, so its volume is

$$V_{\text{cyl}}=\pi r^2 h=\pi\times 2^2\times 5=\pi\times 4\times 5=20\pi\,\text{m}^3.$$

The hemisphere of radius 2 m has volume $2/3\pi r^3=2/3\pi\times 2^3=2/3\pi\times 8=16/3\pi$ m^3. The total volume is

$$20\pi+\frac{16}{3}\pi=\frac{76}{3}\pi\approx 79.59\,\text{m}^3.$$

(b) For similar solids, if every length is multiplied by the linear scale factor $k=1.2$, the volume is multiplied by $k^3=1.2^3=1.728$. Hence the larger tank has volume

$$\frac{76}{3}\pi\times 1.728=43.776\pi\approx 137.53\,\text{m}^3.$$

Question 11

(a) At B, the direction back to A has bearing $035°+180°=215°$. The balloon then leaves B on a bearing of $140°$. The angle between these two directions is

$$\angle ABC=215°-140°=75°.$$

(b) Using the cosine rule in triangle ABC with $AB=7$, $BC=10$ and $\angle ABC=75°$,

$$AC^2=7^2+10^2-2\times 7\times 10\times\cos 75°=49+100-140\times 0.258819=112.7653,$$

so $AC=\sqrt{112.7653}\approx 10.6$ km.

(c) Using the sine rule to find $\angle BAC$,

$$\frac{\sin(\angle BAC)}{BC} = \frac{\sin(\angle ABC)}{AC}, \sin(\angle BAC) = \frac{10 \times \sin 75°}{10.6191} \approx 0.9096,$$

so $\angle BAC \approx 65.45°$. The bearing of B from A is $035°$, and C lies a further $65.45°$ clockwise from AB, so the bearing of C from A is $035° + 65.45° = 100.45°$. The crew drive in the opposite direction, so their bearing is the back bearing

$$100.45° + 180° = 280.45° \approx 280°.$$