Foundation Mathematics Units 3 & 4

Chapter 13 — Measurement: length, area, volume and error

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Chapter 13 Measurement: length, area, volume and error — 13C Measuring and measurement instruments

Theory

Almost every practical measurement task begins with two decisions: **which instrument** to use, and **which unit** to record the result in. You then have to **read the scale** correctly and know **how precise** the reading can be. This section covers all three.

Choosing an instrument and a unit.

Match the instrument and unit to the quantity and its likely size — you would not weigh a truck on kitchen scales, or measure a football field with a 30 cm ruler.

Quantity	Instrument	Common units
Length / distance	ruler, tape measure, trundle wheel, odometer	mm, cm, m, km
Mass	kitchen or bathroom scales, balance	g, kg, tonne (t)
Capacity (volume of liquid)	measuring cylinder, measuring jug	mL, L
Temperature	thermometer	degrees Celsius (°C)
Time	stopwatch, clock	seconds, minutes, hours
Angle	protractor	degrees
Speed	speedometer	km/h

Working out the size of a division.

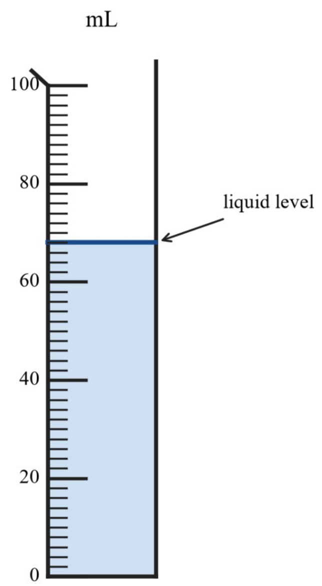
Most scales do **not** label every mark. The value of one small division is

$$\text{size of one division} = \frac{\text{value between two labelled marks}}{\text{number of equal spaces between them}}.$$

For example, if a scale is labelled every 20 mL and there are 10 equal spaces between the labels, each small division is $\frac{20}{10} = 2$ mL. Always work this out **before** reading a value.

Reading an analogue scale.

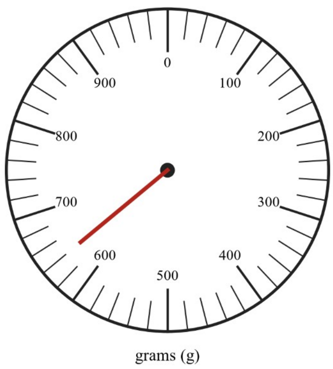
Read the scale at **eye level** to avoid a parallax error. For a liquid in a measuring cylinder, read the bottom of the curved surface (the **meniscus**).



In the cylinder above the labels are 20 mL apart with 10 divisions between them, so each division is 2 mL. The liquid sits 4 divisions above the 60 mL mark, so the reading is $60+4\times 2=68$ mL.

Reading a dial.

A dial is read the same way — find the size of a division, then count from the nearest label to the pointer.



Here the labels are 100 g apart with 5 divisions between them, so each division is $\frac{100}{5}=20$ g. The pointer is 2 divisions past 600 g, giving $600+2\times 20=640$ g.

Digital displays.

A digital instrument shows the value directly. Its precision is set by the **last shown digit**: a scale reading 3.47 kg is precise to the nearest 0.01 kg.

Precision and the limit of reading.

A measurement can only be as precise as the **smallest division** of the instrument. We record a reading to the nearest division, and its true value lies within **half a division** either side. So a cylinder with 2 mL divisions gives a reading precise to ± 1 mL: a reading of 68 mL means the true volume is between 67 mL and 69 mL. A finer scale (smaller divisions) gives a more precise measurement.

Estimating a measurement.

When you cannot measure directly, **estimate** using a known reference — a standard door is about 2 m tall, a large step about 0.5 m, a teaspoon about 5 mL. Give a sensible value and, where asked, a likely **range** around it.

Worked examples

Example 1 — scaffolded

A measuring cylinder (shown above in the theory) is labelled every 20 mL with 10 equal divisions between the labels. The liquid level is 4 divisions above the 60 mL mark. Find the volume and state the precision of the reading.

Working	Comment
Size of one division $= \frac{20}{10} = 2$ mL.	Value between labels ÷ number of spaces.
Reading $= 60 + 4 \times 2 = 68$ mL.	Count divisions up from the nearest label.
Smallest division is 2 mL, so the reading is precise to ± 1 mL.	Precision $= \pm$ half a division.
The true volume lies between 67 mL and 69 mL.	$68 - 1$ to $68 + 1$.

Example 2 — scaffolded

A kitchen dial scale (shown above) is labelled every 100 g with 5 equal divisions between the labels. The pointer is 2 divisions past the 600 g mark. Find the mass and its precision.

Working	Comment
Size of one division $= \frac{100}{5} = 20$ g.	Value between labels ÷ number of spaces.
Reading $= 600 + 2 \times 20 = 640$ g.	Count divisions from the 600 g label.
Smallest division 20 g, so precision is ± 10 g.	$\pm$ half of 20 g.
The true mass is between 630 g and 650 g.	640 ± 10.

Example 3 — unscaffolded

State the most appropriate instrument and unit for measuring each of the following: (a) the length of a netball court, (b) the mass of a letter to be posted, (c) the amount of water in a saucepan, (d) a patient's body temperature.

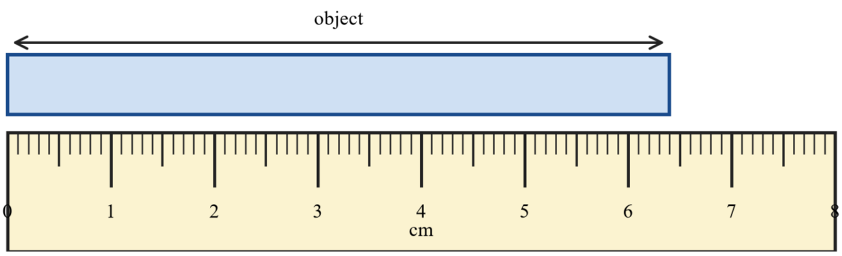
(a) A netball court is about 30 m long, so use a **tape measure** (or trundle wheel) and record in **metres (m)**. (b) A letter has a small mass, so use a **balance or kitchen scales** and record in **grams (g)**. (c) Water in a saucepan is a liquid of up to a litre or two, so use a **measuring jug** and record in **millilitres (mL)** or **litres (L)**.

(b) Body temperature needs a **thermometer**, recorded in **degrees Celsius (°C)**.

∴ tape measure/m, scales/g, measuring jug/mL, thermometer/°C.

Example 4 — unscaffolded

An object is measured against a ruler, as shown. State its length to the nearest millimetre and give the range within which its true length lies.



The ruler is marked in centimetres with 10 millimetre divisions in each centimetre, so each division is $\frac{1}{10} = 0.1$ cm. The object reaches 4 divisions past the 6 cm mark, so its length is $6 + 4 \times 0.1 = 6.4$ cm. The smallest division is 1 mm (0.1 cm), so the reading is precise to ± 0.05 cm.

∴ the object is 6.4 cm long, with its true length between 6.35 cm and 6.45 cm.

Practice questions

Scaffolded

Q1. A thermometer is labelled every 10 °C with 5 equal divisions between the labels. The liquid is 3 divisions above the 20 °C mark. (a) Find the size of one division. (b) State the temperature reading. (c) State the precision of the reading.

Q2. A car's speedometer is labelled every 20 km/h with 4 equal divisions between the labels. The needle is 2 divisions past the 60 km/h mark. (a) Find the size of one division. (b) State the speed. (c) State the precision of the reading.

Q3. State the most appropriate instrument and unit for measuring each quantity. (a) The distance from Melbourne to Geelong. (b) The mass of a bag of apples. (c) The angle in the corner of a triangle.

Q4. Convert each measurement to the unit shown. (a) 1.8 m to centimetres. (b) 0.75 L to millilitres. (c) 2.5 kg to grams.

Q5. A digital kitchen scale displays a mass of 3.47 kg. (a) State the precision (the smallest unit shown). (b) Convert the mass to grams.

Q6. Estimate a sensible value for each measurement. (a) The height of a standard interior door. (b) The capacity of a teaspoon. (c) The mass of a medium apple.

Unscaffolded

Q7. A weighing scale is labelled every 5 kg with 5 equal divisions between the labels. The pointer sits 3 divisions past the 20 kg mark. State the size of one division and the mass shown.

Q8. A measuring jug is labelled every 250 mL with 5 equal divisions between the labels. The liquid is 2 divisions above the 500 mL mark. State the volume and the precision of the reading.

Q9. A dial gauge is labelled every 50 units with 5 equal divisions between the labels. The pointer sits exactly halfway between the marks for 230 and 240. State the size of one division and the reading.

Q10. A measuring tape is labelled every 10 cm with 10 equal divisions between the labels. A benchtop is measured as 47 divisions from the zero end. State the length in centimetres, and convert it to metres.

Q11. State the most appropriate instrument and unit for measuring each quantity, and briefly say why. (a) The width of a doorway (to check a fridge will fit). (b) The amount of medicine for a single dose. (c) The time for a 100 m sprint.

Q12. A carpenter can choose between a ruler with 1 mm divisions and a tape measure with 1 cm divisions. Which instrument gives a more **precise** measurement, and what is the precision (± value) of each?

Q13. A pressure gauge is labelled every 100 units with 4 equal divisions between the labels. The needle points to the 2nd division past 300. State the size of one division and the reading.

Q14. A rain gauge is labelled every 10 mm with 5 equal divisions between the labels. The water level is 2 divisions above the 30 mm mark. State the rainfall and the precision of the reading.

Q15. A digital bathroom scale displays 72.4 kg. (a) State the precision of the display. (b) State the range within which the person's true mass lies.

Q16. Estimate a sensible value **and a likely range** for each measurement. (a) The capacity of a household bath. (b) The length of a classroom.

Q17. A length is recorded as "15 cm, to the nearest centimetre". State the range of possible true lengths.

Q18. On a scale with 1 unit divisions, a pointer sits between the 8 and 9 marks, a little closer to 8. Explain how you would record this reading and why a value such as "8.4" can be given even though there is no mark there.

Answers

Q1. (a) 2 °C. (b) 26 °C. (c) ± 1 °C. **Q2.** (a) 5 km/h. (b) 70 km/h. (c) ± 2.5 km/h. **Q3.** (a) odometer / car trip meter, in kilometres (km). (b) kitchen scales, in kilograms (kg) or grams (g). (c) protractor, in degrees. **Q4.** (a) 180 cm. (b) 750 mL. (c) 2500 g. **Q5.** (a) to the nearest 0.01 kg (± 0.005 kg). (b) 3470 g. **Q6.** (a) about 2 m. (b) about 5 mL. (c) about 150 g. **Q7.** division 1 kg; mass 23 kg. **Q8.** 600 mL, precise to ± 25 mL. **Q9.** division 10; reading 235. **Q10.** 47 cm $= 0.47$ m. **Q11.** (a) tape measure, in cm/mm — the fit needs a precise length. (b) medicine cup/syringe, in mL — small liquid dose. (c) stopwatch, in seconds — short time interval. **Q12.** the ruler (1 mm divisions) is more precise: ruler ± 0.5 mm, tape ± 0.5 cm (± 5 mm). **Q13.** division 25; reading 350. **Q14.** 34 mm, precise to ± 1 mm. **Q15.** (a) to the nearest 0.1 kg (± 0.05 kg). (b) between 72.35 kg and 72.45 kg. **Q16.** (a) about 150 L (roughly 120–180 L). (b) about 7 m (roughly 6–9 m). **Q17.** between 14.5 cm and 15.5 cm. **Q18.** record it as about 8.4 by estimating the fraction of a division; see solution.

Fully-worked solutions

Q1. (a) Size of one division $= \frac{10}{5} = 2$ °C. (b) The liquid is 3 divisions above 20 °C: $20 + 3 \times 2 = 26$ °C. (c) The smallest division is 2 °C, so the reading is precise to ± 1 °C.

Q2. (a) Size of one division $= \frac{20}{4} = 5$ km/h. (b) $60 + 2 \times 5 = 70$ km/h. (c) Precision is ± half a division $= \pm 2.5$ km/h.

Q3. (a) The distance is tens of kilometres, so use a car **odometer/trip meter** and record in **kilometres (km)**. (b) A bag of apples is a few kilograms, so use **kitchen scales**, in **kilograms (kg)** (or grams). (c) An angle is measured with a **protractor**, in **degrees**.

Q4. (a) $1.8 \times 100 = 180$ cm. (b) $0.75 \times 1000 = 750$ mL. (c) $2.5 \times 1000 = 2500$ g.

Q5. (a) The last digit shown is hundredths of a kilogram, so the scale is precise to 0.01 kg (i.e. ± 0.005 kg). (b) $3.47 \times 1000 = 3470$ g.

Q6. (a) A standard interior door is about 2 m tall. (b) A teaspoon holds about 5 mL. (c) A medium apple has a mass of about 150 g. (Any sensible nearby value is acceptable.)

Q7. Size of one division $= \frac{5}{5} = 1$ kg. The pointer is 3 divisions past 20 kg, so the mass is $20 + 3 \times 1 = 23$ kg.

Q8. Size of one division $= \frac{250}{5} = 50$ mL. The level is 2 divisions above 500 mL: $500 + 2 \times 50 = 600$ mL. The smallest division is 50 mL, so the reading is precise to ± 25 mL.

Q9. Size of one division $= \frac{50}{5} = 10$ units. The pointer is halfway between 230 and 240, so the reading is 235 (estimated to half a division).

Q10. Size of one division $= \frac{10}{10} = 1$ cm. The benchtop is 47 divisions, so its length is 47 cm. Converting, $47 \div 100 = 0.47$ m.

Q11. (a) A **tape measure** in centimetres/millimetres — checking a fridge fits needs an accurate length. (b) A **medicine cup or syringe** in millilitres — the dose is a small volume of liquid. (c) A **stopwatch** in seconds — a sprint is a short time interval.

Q12. The ruler has the smaller divisions (1 mm), so it gives the more precise measurement. Precision is ± half a division: the ruler is precise to ± 0.5 mm, while the tape (with 1 cm divisions) is precise only to ± 0.5 cm $= \pm 5$ mm.

Q13. Size of one division $= \frac{100}{4} = 25$ units. The needle is 2 divisions past 300: $300 + 2 \times 25 = 350$.

Q14. Size of one division $= \frac{10}{5} = 2$ mm. The level is 2 divisions above 30 mm: $30 + 2 \times 2 = 34$ mm. The smallest division is 2 mm, so the reading is precise to ± 1 mm.

Q15. (a) The last digit shown is tenths of a kilogram, so the display is precise to 0.1 kg (i.e. ± 0.05 kg). (b) The true mass lies within half of 0.1 kg either side: between 72.35 kg and 72.45 kg.

Q16. (a) A household bath holds roughly 150 L (a range of about 120–180 L is sensible). (b) A classroom is about 7 m long (roughly 6–9 m). (Reasonable nearby estimates are acceptable.)

Q17. "To the nearest centimetre" means the true length is within half a centimetre of 15 cm, so it lies between $15 - 0.5 = 14.5$ cm and $15 + 0.5 = 15.5$ cm.

Q18. The pointer lies between the 8 and 9 marks (one division apart), closer to 8. You estimate the fraction of the division it has reached — about four-tenths — and record about 8.4. Reading between the marks like this is normal; the extra digit is an **estimate**, so the value is only reliable to about ± 0.5 of a division.

Theory

Measurements only make sense with a **unit**, and real problems often mix units — a plan in millimetres, a delivery in tonnes, a tank in litres. Being able to **convert** between units lets you compare, add and calculate with measurements that were recorded differently.

The metric system.

The metric system is built on **powers of 10**, so every conversion is a multiplication or division by 10, 100, 1000, and so on. The common units are:

- **Length:** $10\,\text{mm} = 1\,\text{cm}$, $100\,\text{cm} = 1\,\text{m}$, $1000\,\text{m} = 1\,\text{km}$.
- **Mass:** $1000\,\text{mg} = 1\,\text{g}$, $1000\,\text{g} = 1\,\text{kg}$, $1000\,\text{kg} = 1\,\text{t}$ (tonne).
- **Capacity:** $1000\,\text{mL} = 1\,\text{L}$, $1000\,\text{L} = 1\,\text{kL}$.

Converting up and down.

Going from a **small** unit to a **larger** unit, you **divide** (it takes fewer of the bigger unit); going from a **large** unit to a **smaller** unit, you **multiply**. A quick check: the *number* gets smaller when the *unit* gets bigger.

For example, $2500\,\text{m} = 2500 \div 1000 = 2.5\,\text{km}$, and $3.5\,\text{kg} = 3.5 \times 1000 = 3500\,\text{g}$.

Multi-step conversions.

If two units are more than one step apart, chain the conversions. To change $45\,000\,\text{mm}$ to metres, go $\text{mm} \rightarrow \text{cm} \rightarrow \text{m}$: $45\,000 \div 10 = 4500\,\text{cm}$, then $4500 \div 100 = 45\,\text{m}$ (equivalently, divide by 1000 at once).

Converting units of area.

Area is measured in **square** units, and here a common mistake creeps in. Because $1\,\text{m} = 100\,\text{cm}$, a $1\,\text{m}$ by $1\,\text{m}$ square is $100\,\text{cm}$ by $100\,\text{cm}$, so

$$1\,\text{m}^2 = 100 \times 100 = 100^2 = 10\,000\,\text{cm}^2.$$

The length factor is **squared**. So to convert an area from m² to cm² you multiply by $10\,000$, not by 100. Other useful area facts: $1\,\text{cm}^2 = 10^2 = 100\,\text{mm}^2$, $1\,\text{ha (hectare)} = 10\,000\,\text{m}^2$.

Converting units of volume.

Volume is measured in **cubic** units, so the length factor is **cubed**. Because $1\,\text{m} = 100\,\text{cm}$,

$$1\,\text{m}^3 = 100 \times 100 \times 100 = 100^3 = 1\,000\,000\,\text{cm}^3.$$

Similarly $1\,\text{cm}^3 = 10^3 = 1000\,\text{mm}^3$. Volume links to capacity through

$$1\,\text{cm}^3 = 1\,\text{mL}\,,\, 1000\,\text{cm}^3 = 1\,\text{L}\,,\, 1\,\text{m}^3 = 1000\,\text{L}\,.$$

Non-metric conversions.

Some everyday units are not metric but are still routine. For **time**: $60\,\text{s} = 1\,\text{min}$, $60\,\text{min} = 1\,\text{h}$, so $1\,\text{h} = 3600\,\text{s}$. **Rates** combine units — a speed in km/h can be changed to m/s by converting the kilometres to metres and the hours to seconds.

Worked examples

Example 1 — scaffolded

Convert (a) $3.5\,\mathrm{km}$ to metres, (b) $250\,\mathrm{cm}$ to metres and (c) $8\,\mathrm{mm}$ to centimetres.

Working	Comment
(a) $3.5\,\mathrm{km} = 3.5 \times 1000 = 3500\,\mathrm{m}$.	km → m is large → small, so multiply by 1000.
(b) $250\,\mathrm{cm} = 250 \div 100 = 2.5\,\mathrm{m}$.	cm → m is small → large, so divide by 100.
(c) $8\,\mathrm{mm} = 8 \div 10 = 0.8\,\mathrm{cm}$.	mm → cm, divide by 10.

Example 2 — scaffolded

Convert (a) $2.4\,\mathrm{m}^2$ to square centimetres and (b) $5\,\mathrm{L}$ to cubic centimetres.

Working	Comment
(a) $1\,\mathrm{m}^2 = 100^2 = 10\,000\,\mathrm{cm}^2$.	Area: square the length factor 100.
$2.4\,\mathrm{m}^2 = 2.4 \times 10\,000 = 24\,000\,\mathrm{cm}^2$.	Multiply by the squared factor, not by 100.
(b) $1\,\mathrm{L} = 1000\,\mathrm{cm}^3$.	Capacity–volume link.
$5\,\mathrm{L} = 5 \times 1000 = 5000\,\mathrm{cm}^3$.	Multiply by 1000.

Example 3 — unscaffolded

A rectangular concrete slab measures $3.5\,\mathrm{m}$ by $2.4\,\mathrm{m}$. Find its area in square metres and in square centimetres.

The area is $3.5 \times 2.4 = 8.4\,\mathrm{m}^2$. To convert to square centimetres, multiply by the squared length factor $100^2 = 10\,000$:

$$8.4\,\mathrm{m}^2 = 8.4 \times 10\,000 = 84\,000\,\mathrm{cm}^2.$$

∴ the slab has an area of $8.4\,\mathrm{m}^2$, which is $84\,000\,\mathrm{cm}^2$.

Example 4 — unscaffolded

A water tank holds $1.5\,\mathrm{m}^3$. Convert this capacity to litres and to millilitres.

Using $1\,\mathrm{m}^3 = 1000\,\mathrm{L}$,

$$1.5\,\mathrm{m}^3 = 1.5 \times 1000 = 1500\,\mathrm{L}.$$

Then, since $1\,\mathrm{L} = 1000\,\mathrm{mL}$, $1500\,\mathrm{L} = 1500 \times 1000 = 1\,500\,000\,\mathrm{mL}$.

∴ the tank holds $1500\,\mathrm{L}$, which is $1\,500\,000\,\mathrm{mL}$.

Practice questions

Scaffolded

Q1. Convert these lengths. (a) $4.7\,\mathrm{km}$ to metres. (b) $320\,\mathrm{cm}$ to metres. (c) $65\,\mathrm{mm}$ to centimetres.

Q2. Convert these masses. (a) $4.2\,\mathrm{kg}$ to grams. (b) $4500\,\mathrm{g}$ to kilograms. (c) $3\,\mathrm{t}$ to kilograms.

Q3. Convert these capacities. (a) $2.5\,\mathrm{L}$ to millilitres. (b) $750\,\mathrm{mL}$ to litres. (c) $3\,\mathrm{kL}$ to litres.

Q4. Convert these areas. (a) $3\,\mathrm{m}^2$ to square centimetres. (b) $25\,000\,\mathrm{cm}^2$ to square metres. (c) Explain why the conversion factor between m^2 and cm^2 is $10\,000$ and not 100.

Q5. Convert these volumes and capacities. (a) $2\,\mathrm{m}^3$ to litres. (b) $4500\,\mathrm{cm}^3$ to litres. (c) $0.75\,\mathrm{L}$ to cubic centimetres.

Q6. Convert these times. (a) $150\,\mathrm{s}$ to minutes. (b) $2.5\,\mathrm{h}$ to minutes. (c) $3\,\mathrm{h}$ to seconds.

Unscaffolded

Q7. A running track is $0.85\,\mathrm{km}$ long. Write this length in metres and in centimetres.

Q8. A steel beam is $2400\,\mathrm{mm}$ long. Convert its length to metres.

Q9. A truck carries $1.2\,\mathrm{t}$ of gravel. Convert this mass to kilograms and to grams.

Q10. A window has an area of $6.4\,\mathrm{m}^2$. Convert this area to square centimetres.

Q11. A tiled courtyard has an area of $90\,000\,\mathrm{cm}^2$. Convert this area to square metres.

Q12. A shipping container has a volume of $3.5\,\mathrm{m}^3$. Convert this to cubic centimetres and to litres.

Q13. A fish tank holds $250\,000\,\mathrm{cm}^3$ of water. Convert this to cubic metres and to litres.

Q14. A recipe needs $1.5\,\mathrm{L}$ of stock, measured out in a $250\,\mathrm{mL}$ cup. How many cups are required?

Q15. A path $12\,\mathrm{m}$ long is to be laid with tiles $30\,\mathrm{cm}$ long, placed end to end. How many tiles fit along the path?

Q16. (a) Convert $4500\,\mathrm{mL}$ to litres. (b) A car travels at $90\,\mathrm{km/h}$. By converting kilometres to metres and hours to seconds, find its speed in metres per second.

Q17. A room measures $5\,\mathrm{m}$ by $4\,\mathrm{m}$. Find the floor area in square metres, and convert it to square centimetres.

Q18. A tank has a volume of $0.8\,\mathrm{m}^3$. (a) Convert its capacity to litres. (b) The tank is filled at $20\,\mathrm{L}$ per minute. How long does it take to fill?

Answers

Q1. (a) $4700\,\text{m}$. (b) $3.2\,\text{m}$. (c) $6.5\,\text{cm}$. **Q2.** (a) $4200\,\text{g}$. (b) $4.5\,\text{kg}$. (c) $3000\,\text{kg}$. **Q3.** (a) $2500\,\text{mL}$. (b) $0.75\,\text{L}$. (c) $3000\,\text{L}$. **Q4.** (a) $30\,000\,\text{cm}^2$. (b) $2.5\,\text{m}^2$. (c) area uses squared units, so the length factor 100 is squared: $100^2 = 10\,000$. **Q5.** (a) $2000\,\text{L}$. (b) $4.5\,\text{L}$. (c) $750\,\text{cm}^3$. **Q6.** (a) $2.5\,\text{min}$ (2 min 30 s). (b) $150\,\text{min}$. (c) $10\,800\,\text{s}$. **Q7.** $850\,\text{m}$; $85\,000\,\text{cm}$. **Q8.** $2.4\,\text{m}$. **Q9.** $1200\,\text{kg}$; $1\,200\,000\,\text{g}$. **Q10.** $64\,000\,\text{cm}^2$. **Q11.** $9\,\text{m}^2$. **Q12.** $3\,500\,000\,\text{cm}^3$; $3500\,\text{L}$. **Q13.** $0.25\,\text{m}^3$; $250\,\text{L}$. **Q14.** 6 cups. **Q15.** 40 tiles. **Q16.** (a) $4.5\,\text{L}$. (b) $25\,\text{m/s}$. **Q17.** $20\,\text{m}^2$; $200\,000\,\text{cm}^2$. **Q18.** (a) $800\,\text{L}$. (b) $40\,\text{minutes}$.

Fully-worked solutions

Q1. (a) $4.7\,\text{km} = 4.7 \times 1000 = 4700\,\text{m}$. (b) $320\,\text{cm} = 320 \div 100 = 3.2\,\text{m}$. (c) $65\,\text{mm} = 65 \div 10 = 6.5\,\text{cm}$.

Q2. (a) $4.2\,\text{kg} = 4.2 \times 1000 = 4200\,\text{g}$. (b) $4500\,\text{g} = 4500 \div 1000 = 4.5\,\text{kg}$. (c) $3\,\text{t} = 3 \times 1000 = 3000\,\text{kg}$.

Q3. (a) $2.5\,\text{L} = 2.5 \times 1000 = 2500\,\text{mL}$. (b) $750\,\text{mL} = 750 \div 1000 = 0.75\,\text{L}$. (c) $3\,\text{kL} = 3 \times 1000 = 3000\,\text{L}$.

Q4. (a) $3\,\text{m}^2 = 3 \times 10\,000 = 30\,000\,\text{cm}^2$. (b) $25\,000\,\text{cm}^2 = 25\,000 \div 10\,000 = 2.5\,\text{m}^2$. (c) A $1\,\text{m} \times 1\,\text{m}$ square is $100\,\text{cm} \times 100\,\text{cm}$, so its area is $100 \times 100 = 10\,000\,\text{cm}^2$; the length factor 100 is **squared** because area is a squared unit.

Q5. (a) $2\,\text{m}^3 = 2 \times 1000 = 2000\,\text{L}$ (using $1\,\text{m}^3 = 1000\,\text{L}$). (b) $4500\,\text{cm}^3 = 4500 \div 1000 = 4.5\,\text{L}$. (c) $0.75\,\text{L} = 0.75 \times 1000 = 750\,\text{cm}^3$.

Q6. (a) $150\,\text{s} = 150 \div 60 = 2.5\,\text{min}$ (that is, 2 minutes 30 seconds). (b) $2.5\,\text{h} = 2.5 \times 60 = 150\,\text{min}$. (c) $3\,\text{h} = 3 \times 3600 = 10\,800\,\text{s}$.

Q7. $0.85\,\text{km} = 0.85 \times 1000 = 850\,\text{m}$. Continuing, $850\,\text{m} = 850 \times 100 = 85\,000\,\text{cm}$.

Q8. $2400\,\text{mm} = 2400 \div 1000 = 2.4\,\text{m}$ (dividing by 1000 takes mm straight to m).

Q9. $1.2\,\text{t} = 1.2 \times 1000 = 1200\,\text{kg}$, and $1200\,\text{kg} = 1200 \times 1000 = 1\,200\,000\,\text{g}$.

Q10. $6.4\,\text{m}^2 = 6.4 \times 10\,000 = 64\,000\,\text{cm}^2$ (multiply by the squared factor).

Q11. $90\,000\,\text{cm}^2 = 90\,000 \div 10\,000 = 9\,\text{m}^2$.

Q12. $3.5\,\text{m}^3 = 3.5 \times 1\,000\,000 = 3\,500\,000\,\text{cm}^3$ (the length factor 100 cubed). Since $1\,\text{m}^3 = 1000\,\text{L}$, $3.5\,\text{m}^3 = 3.5 \times 1000 = 3500\,\text{L}$.

Q13. $250\,000\,\text{cm}^3 = 250\,000 \div 1\,000\,000 = 0.25\,\text{m}^3$. Since $1000\,\text{cm}^3 = 1\,\text{L}$, $250\,000\,\text{cm}^3 = 250\,000 \div 1000 = 250\,\text{L}$.

Q14. Convert the stock to millilitres: $1.5\,\text{L} = 1500\,\text{mL}$. The number of $250\,\text{mL}$ cups is $1500 \div 250 = 6$ cups.

Q15. Convert the path length to centimetres: $12\,\text{m} = 1200\,\text{cm}$. The number of $30\,\text{cm}$ tiles is $1200 \div 30 = 40$ tiles.

Q16. (a) $4500\,\text{mL} = 4500 \div 1000 = 4.5\,\text{L}$. (b) $90\,\text{km/h} = 90\,000\,\text{m}$ per hour, and $1\,\text{h} = 3600\,\text{s}$, so the speed is $90\,000 \div 3600 = 25\,\text{m/s}$.

Q17. The floor area is $5 \times 4 = 20\,\text{m}^2$. Converting, $20\,\text{m}^2 = 20 \times 10\,000 = 200\,000\,\text{cm}^2$.

Q18. (a) $0.8\,\text{m}^3 = 0.8 \times 1000 = 800\,\text{L}$. (b) At $20\,\text{L}$ per minute, the time to fill is $800 \div 20 = 40\,\text{minutes}$.

Theory

This section finds the **perimeter** (distance around the outside) and the **area** (amount of surface) of the flat shapes you meet in everyday measurement. The area formulas for the triangle, trapezium, circle and sector are on the Foundation Mathematics formula sheet — but you must know **which** shape you are looking at, and match the measurement to the right length in the formula.

Perimeter.

The **perimeter** of any shape with straight sides is simply the **sum of the side lengths**. It is a length, so it is measured in mm, cm, m or km. For a shape with a curved edge you need the circumference or arc length below.

Circumference of a circle.

The distance around a circle is its **circumference**. With diameter d and radius r (so $d=2r$),

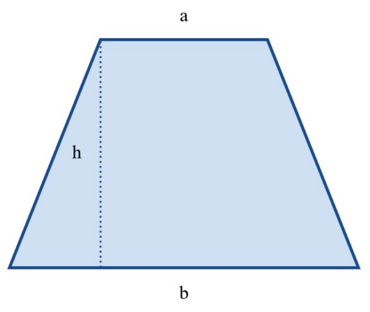
$$C=\pi d=2\pi r.$$

Keep π in your calculator all the way through and round only at the final step.

Areas of the standard shapes.

Shape	Area
Rectangle (length l, width w)	$A=lw$
Square (side s)	$A=s^2$
Triangle (base b, height h)	$A=1/2\,bh$
Parallelogram (base b, height h)	$A=bh$
Trapezium (parallel sides a, b; height h)	$A=1/2(a+b)h$
Circle (radius r)	$A=\pi r^2$

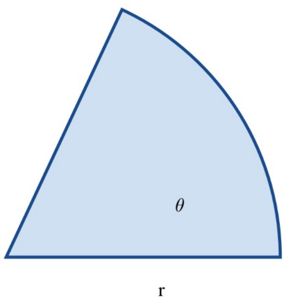
For a triangle, a parallelogram and a trapezium the **height** is the *perpendicular* distance between the base and the opposite side or vertex — not a slanting side.



Sectors and arcs.

A **sector** is a “pizza slice” of a circle — the region between two radii and the arc between them. If the angle at the centre is θ degrees, the sector is the fraction $\frac{\theta}{360}$ of the whole circle, so

$$\text{sector area}=\pi r^2\times\frac{\theta}{360},\text{arc length}=2\pi r\times\frac{\theta}{360}.$$



Composite (compound) shapes.

Many real shapes are made from the basic ones. Find their area by **adding** the parts (an arched window = rectangle + semicircle) or by **subtracting** a part (a path around a pool = big rectangle − pool). Always work in the **same units**, and state the area in square units (m^2, cm^2).

Worked examples

Example 1 — scaffolded

A rectangular garden bed is 4.5 m long and 2.8 m wide. Find its perimeter and its area.

Working	Comment
Perimeter $= 2 \times (4.5 + 2.8) = 2 \times 7.3 = 14.6$ m.	Add a length and a width, then double (two of each).
Area $= 4.5 \times 2.8 = 12.6$ m^2.	Rectangle area \$= \$ length × width.
Perimeter 14.6 m; area 12.6 m^2.	Perimeter is a length (m); area is a surface (m^2).

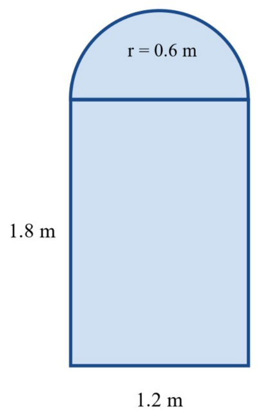
Example 2 — scaffolded

A circular pond has radius 1.4 m. Find its circumference and its area, each correct to two decimal places.

Working	Comment
$C = 2\pi r = 2 \times \pi \times 1.4 = 2.8\pi$.	Circumference formula with $r = 1.4$.
$C \approx 8.80$ m.	Keep π in the calculator, round at the end.
$A = \pi r^2 = \pi \times 1.4^2 = 1.96\pi$.	Area formula; square the radius first.
$A \approx 6.16$ m^2.	Area is in square metres.

Example 3 — unscaffolded

A window is a rectangle 1.2 m wide and 1.8 m tall, with a semicircle of radius 0.6 m on top. Find the total area of glass, correct to two decimal places.



The rectangle has area $1.2 \times 1.8 = 2.16$ m². The semicircle is half a circle of radius 0.6 m, so its area is

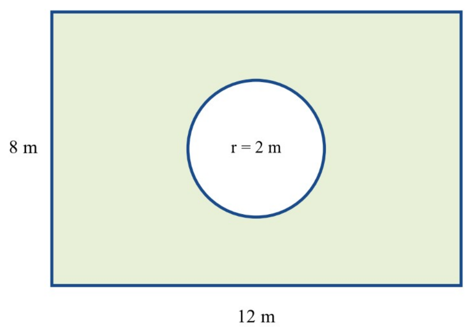
$$\frac{1}{2}\pi r^2 = \frac{1}{2} \times \pi \times 0.6^2 = 0.18\pi \approx 0.57\,\mathrm{m}^2.$$

The total area is $2.16 + 0.18\pi \approx 2.73$ m².

∴ the window has about 2.73 m² of glass.

Example 4 — unscaffolded

A rectangular lawn is 12 m by 8 m. A circular garden of radius 2 m is dug out of the middle and the rest is turfed. Turf costs $\$15$ per square metre. Find the area to be turfed and the cost, to the nearest cent.



The lawn area is $12 \times 8 = 96$ m² and the circular garden is $\pi \times 2^2 = 4\pi \approx 12.57$ m². The area to be turfed is

$$96 - 4\pi \approx 83.43\,\mathrm{m}^2.$$

At $\$15$ per m², the cost is $15 \times (96 - 4\pi) \approx \$1251.50$.

∴ about 83.43 m² is turfed, at a cost of $\$1251.50$.

Practice questions

Scaffolded

Q1. A rectangular room is 6.5 m long and 4 m wide. (a) Find the perimeter. (b) Find the area.

Q2. A triangle has base 8 cm and perpendicular height 5 cm. (a) Write down the area formula for a triangle. (b) Find the area.

Q3. A trapezium has parallel sides 6 m and 10 m, and a perpendicular height of 4 m. (a) Write down the trapezium area formula. (b) Substitute the values. (c) Find the area.

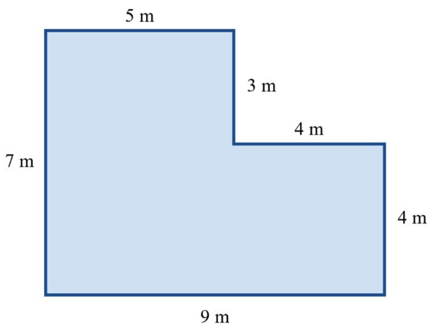
Q4. A circle has radius 3.5 cm. (a) Find the circumference, correct to two decimal places. (b) Find the area, correct to two decimal places.

Q5. A parallelogram has base 7 m, perpendicular height 4 m and slant side 5 m. (a) Find the area. (b) Find the perimeter.

Q6. A sector of a circle has radius 6 cm and an angle of 60° at the centre. (a) Find the area of the sector, correct to two decimal places. (b) Find the length of the arc, correct to two decimal places.

Unscaffolded

Q7. Find the perimeter and the area of the L-shaped block below.



Q8. A bicycle wheel has diameter 68 cm. Find the circumference of the wheel, correct to two decimal places, and hence the distance travelled (in metres) in one full turn.

Q9. A rectangular floor is 5.4 m by 3.5 m. Carpet costs $\$45$ per square metre. Find the area of the floor and the cost of carpeting it.

Q10. A triangular sail has base 4 m and perpendicular height 6 m. Find its area.

Q11. A block of land is a trapezium with parallel sides 40 m and 28 m, and a width (perpendicular distance between them) of 25 m. Find its area.

Q12. A circular pond has radius 2.5 m. Find its area, correct to two decimal places.

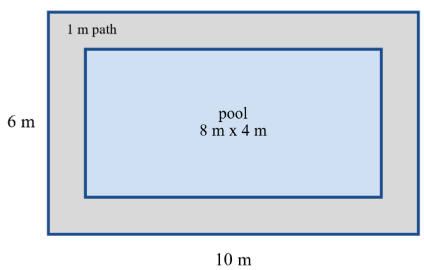
Q13. A parallelogram has base 12 cm and perpendicular height 7 cm. Find its area.

Q14. A slice of pizza is a sector of radius 15 cm with an angle of 45° at the centre. Find the area of the slice, correct to two decimal places.

Q15. A curved garden edge is an arc of a circle of radius 10 m, through an angle of 90°. Find the length of the edge, correct to two decimal places.

Q16. A room is L-shaped and can be split into a 6 m by 4 m rectangle and a 3 m by 2 m rectangle. Find the total floor area.

Q17. A rectangular swimming pool 8 m by 4 m is surrounded by a path 1 m wide, as shown. Find the area of the path.



Q18. A wall is 4 m long and 2.5 m high. It has a window 1.2 m by 0.8 m that is not painted. (a) Find the area to be painted. (b) One litre of paint covers 8 m². How many whole litres must be bought?

Answers

Q1. (a) 21 m. (b) 26 m². **Q2.** (a) $A=1/2\,b\,h$. (b) 20 cm². **Q3.** (a) $A=1/2(a+b)h$. (b) $1/2(6+10)(4)$. (c) 32 m². **Q4.** (a) 21.99 cm. (b) 38.48 cm². **Q5.** (a) 28 m². (b) 24 m. **Q6.** (a) 18.85 cm². (b) 6.28 cm. **Q7.** perimeter 32 m; area 51 m². **Q8.** 213.63 cm ≈ 2.14 m per turn. **Q9.** area 18.9 m²; cost $\$850.50$. **Q10.** 12 m². **Q11.** 850 m². **Q12.** 19.63 m². **Q13.** 84 cm². **Q14.** 88.36 cm². **Q15.** 15.71 m. **Q16.** 30 m². **Q17.** 28 m². **Q18.** (a) 9.04 m². (b) 2 litres.

Fully-worked solutions

Q1. (a) Perimeter $=2\times(6.5+4)=2\times 10.5=21$ m. (b) Area $=6.5\times 4=26$ m².

Q2. (a) The area of a triangle is $A=1/2\,b\,h$. (b) $A=1/2\times 8\times 5=20$ cm².

Q3. (a) $A=1/2(a+b)h$. (b) Substituting $a=6$, $b=10$, $h=4$ gives $A=1/2(6+10)(4)$. (c) $A=1/2\times 16\times 4=32$ m².

Q4. (a) $C=2\pi r=2\times\pi\times 3.5=7\pi\approx 21.99$ cm. (b) $A=\pi r^2=\pi\times 3.5^2=12.25\pi\approx 38.48$ cm².

Q5. (a) Area $=b\,h=7\times 4=28$ m² (the height, not the slant side, is used). (b) Perimeter $=2\times(7+5)=24$ m (the two bases and the two slant sides).

Q6. (a) Sector area $=\pi r^2\times\dfrac{\theta}{360}=\pi\times 6^2\times\dfrac{60}{360}=36\pi\times\dfrac{1}{6}=6\pi\approx 18.85$ cm². (b) Arc length $=2\pi r\times\dfrac{\theta}{360}=2\pi\times 6\times\dfrac{60}{360}=12\pi\times\dfrac{1}{6}=2\pi\approx 6.28$ cm.

Q7. Going around the L-shape, the sides are $9, 7, 5, 3, 4$ and 4 m, so the perimeter is $9+7+5+3+4+4=32$ m. For the area, take the full 9 m by 7 m rectangle and subtract the 4 m by 3 m notch: $9\times 7-4\times 3=63-12=51$ m².

Q8. $C=\pi d=\pi\times 68\approx 213.63$ cm. In metres this is $213.63\div 100\approx 2.14$ m, so the wheel travels about 2.14 m in one turn.

Q9. Area $=5.4\times 3.5=18.9$ m². Cost $=18.9\times\$45=\$850.50$.

Q10. $A=1/2\,b\,h=1/2\times 4\times 6=12$ m².

Q11. $A=1/2(a+b)h=1/2(40+28)(25)=1/2\times 68\times 25=850$ m².

Q12. $A=\pi r^2=\pi\times 2.5^2=6.25\pi\approx 19.63$ m².

Q13. $A=b\,h=12\times 7=84$ cm².

Q14. Sector area $=\pi r^2\times\dfrac{\theta}{360}=\pi\times 15^2\times\dfrac{45}{360}=225\pi\times\dfrac{1}{8}\approx 88.36$ cm².

Q15. Arc length $=2\pi r\times\dfrac{\theta}{360}=2\pi\times 10\times\dfrac{90}{360}=20\pi\times\dfrac{1}{4}=5\pi\approx 15.71$ m.

Q16. The two rectangles have areas $6\times 4=24$ m² and $3\times 2=6$ m², so the total floor area is $24+6=30$ m².

Q17. With a 1 m path all around, the outer rectangle is $(8+2)$ by $(4+2)$, i.e. 10 m by 6 m, with area 60 m². The pool has area $8\times 4=32$ m². The path is the difference: $60-32=28$ m².

Q18. (a) The wall is $4\times 2.5=10$ m² and the window is $1.2\times 0.8=0.96$ m², so the area to paint is $10-0.96=9.04$ m². (b) $9.04\div 8=1.13$, and paint is sold in whole litres, so 2 litres must be bought (one litre would cover only 8 m²).

Theory

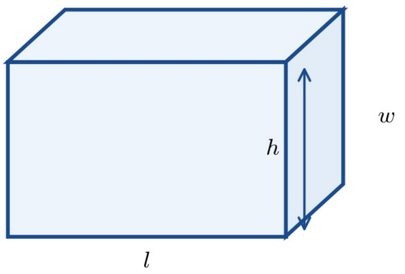
Volume is the amount of space a solid occupies. It is measured in **cubic units** — cubic centimetres (cm^3), cubic metres (m^3) and so on. **Capacity** is the amount a container can *hold* (of liquid, sand, grain), measured in millilitres (mL) and litres (L). This section finds the volume of the common solids, converts a volume into a capacity, and uses **density** to link volume with mass.

Volume of a prism.

A **prism** has the same cross-section all the way along its length. Its volume is

$$V = \text{area of base} \times \text{height},$$

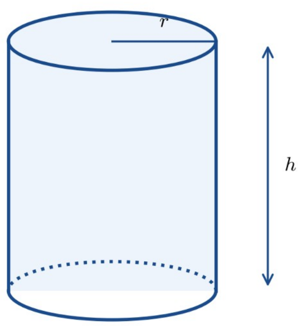
where the "base" is the uniform cross-section and the "height" is the length of the prism. For a **rectangular prism** (a box) with length l, width w and height h, $V = l w h$.



Volume of a cylinder.

A cylinder is a prism with a circular base of radius r, so its volume is the base area πr^2 times the height h:

$$V = \pi r^2 h.$$

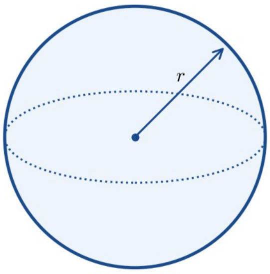


Volume of a cone, a sphere and a pyramid.

These three formulas are on the Foundation Mathematics formula sheet:

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h, V_{\text{sphere}} = \frac{4}{3}\pi r^3, V_{\text{pyramid}} = \frac{1}{3} \times \text{area of base} \times \text{height}.$$

A cone holds exactly one-third of the cylinder with the same base and height, and a pyramid holds one-third of the matching prism.



Keep the value of π in your calculator through the whole calculation and round **only at the final step**, to the accuracy the question asks for.

Capacity — converting a volume into litres.

A volume in cubic centimetres converts straight into a capacity:

$$1\,\mathrm{cm}^3 = 1\,\mathrm{mL}, 1000\,\mathrm{cm}^3 = 1\,\mathrm{L}, 1\,\mathrm{m}^3 = 1000\,\mathrm{L}.$$

So a tank of volume $1.5\,\mathrm{m}^3$ holds $1.5 \times 1000 = 1500$ L.

Density.

Density measures how much mass is packed into each unit of volume — for example grams per cubic centimetre (g/cm³) or kilograms per cubic metre (kg/m³):

$$\text{density} = \frac{\text{mass}}{\text{volume}}.$$

Rearranging gives $\text{mass} = \text{density} \times \text{volume}$ and $\text{volume} = \dfrac{\text{mass}}{\text{density}}$. Water has a density of about $1\,\mathrm{g/cm}^3$, so 1 L of water has a mass of about 1 kg.

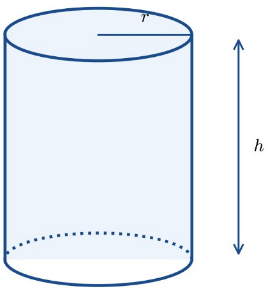
Compound solids.

A **compound solid** is built from basic solids. Find its volume by **adding** the volumes of the parts (a silo = cylinder + dome) or **subtracting** a removed part (a block with a hole drilled through it = box − cylinder).

Worked examples

Example 1 — scaffolded

A cylindrical water tank has radius 0.6 m and height 1.2 m. Find its volume, and hence its capacity in litres, correct to the nearest litre.

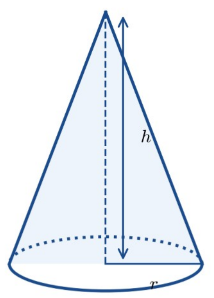


Working	Comment
$V = \pi r^2 h = \pi \times 0.6^2 \times 1.2.$	Use the cylinder formula with $r = 0.6$, $h = 1.2$.

Working	Comment
$= \pi \times 0.36 \times 1.2 = 0.432\pi = 1.3572\,\mathrm{m}^3$ (to 4 d.p.).	Keep π in the calculator; do not round yet.
Capacity $= 0.432\pi \times 1000 = 432\pi \approx 1357.17$ L.	$1\,\mathrm{m}^3 = 1000$ L; use the stored value, not the rounded one.
≈ 1357 L.	Round to the nearest litre only at the end.

Example 2 — scaffolded

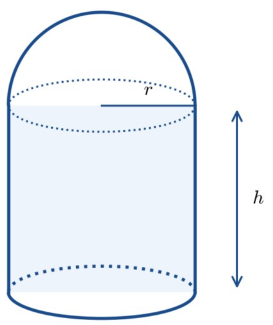
An ice-cream cone has radius 3.5 cm and height 12 cm. Find its volume, correct to one decimal place.



Working	Comment
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 3.5^2 \times 12.$	Use the cone formula.
$= \frac{1}{3}\pi \times 12.25 \times 12 = \frac{1}{3}\pi \times 147 = 49\pi.$	$3.5^2 = 12.25$ and $12.25 \times 12 = 147$; then $1/3 \times 147 = 49$.
$= 153.938\ldots \approx 153.9\,\mathrm{cm}^3.$	Evaluate 49π on the calculator and round to 1 d.p.

Example 3 — unscaffolded

A grain silo is a cylinder of radius 5 m and height 12 m with a hemispherical dome (radius 5 m) on top. Find its total volume, correct to the nearest cubic metre.



The silo is a compound solid, so add the cylinder and the hemisphere. The cylinder has

$$V_{\text{cyl}} = \pi r^2 h = \pi \times 5^2 \times 12 = 300\pi \approx 942.48\,\mathrm{m}^3.$$

A hemisphere is half a sphere, so

$$V_{\text{hemi}} = \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi \times 5^3 = \frac{250\pi}{3} \approx 261.80\,\mathrm{m}^3.$$

Adding, $V = 300\pi + \frac{250\pi}{3} = \frac{1150\pi}{3} \approx 1204.28\,\mathrm{m}^3.$

∴ the silo holds about $1204\,\text{m}^3$.

Example 4 — unscaffolded

A solid steel ball bearing has radius 1.5 cm. Steel has a density of $7.8\,\text{g/cm}^3$. Find the mass of the ball bearing, correct to one decimal place.

First find the volume of the sphere:

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 1.5^3 = \frac{4}{3}\pi \times 3.375 = 4.5\pi \approx 14.1372\,\text{cm}^3.$$

Then $\text{mass} = \text{density} \times \text{volume} = 7.8 \times 14.1372 = 110.27\ldots$ g.

∴ the ball bearing has a mass of about 110.3 g.

Practice questions

Scaffolded

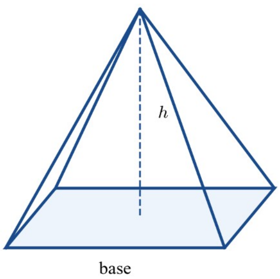
Q1. A shipping carton is a rectangular prism 40 cm long, 30 cm wide and 25 cm high. (a) Find its volume in cm^3. (b) Find its capacity in litres. (c) How many 2-litre containers could be filled from a full carton of liquid?

Q2. A drink can is a cylinder of radius 3 cm and height 12 cm. (a) Find its volume, correct to the nearest cm^3. (b) State its capacity in millilitres. (c) A can of these dimensions is labelled “375 mL”. Explain why this label cannot be correct.

Q3. A conical paper cup has radius 6 cm and height 15 cm. (a) Write down the formula for the volume of a cone. (b) Find the volume, correct to two decimal places. (c) State the volume correct to the nearest millilitre of capacity.

Q4. A solid rubber ball has radius 7 cm. (a) Write down the formula for the volume of a sphere. (b) Find the volume, correct to two decimal places.

Q5. A square-based pyramid has a base of side 8 cm and a height of 9 cm.

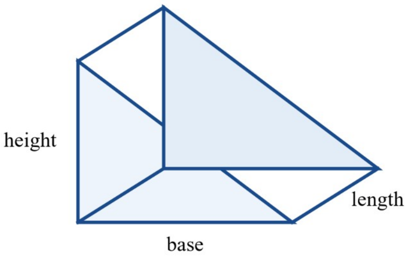


(a) Find the area of the base.
(b) Find the volume of the pyramid.
(c) A second pyramid has the same base but twice the height. State its volume.

Q6. A container holds 2.5 litres of cooking oil. (a) Find the volume of oil in cm^3. (b) The oil has a density of $0.92\,\text{g/cm}^3$. Find the mass of the oil, to the nearest gram. (c) Give the mass in kilograms, correct to two decimal places.

Unscaffolded

Q7. A chocolate bar is a triangular prism. Its cross-section is a right-angled triangle with the two shorter sides 5 cm and 12 cm, and the bar is 25 cm long. Find its volume.



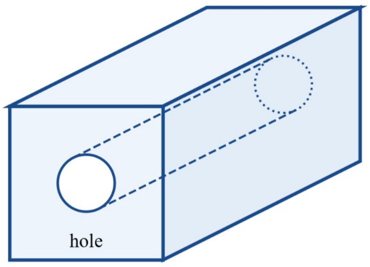
Q8. A water pipe is a cylinder of radius 4 cm and length 2 m. Find the volume of water it holds when full, correct to the nearest cm^3, and state this capacity in litres (to two decimal places).

Q9. Sand is poured into a cone-shaped pile of radius 1.5 m and height 0.8 m. Find the volume of sand, correct to one decimal place.

Q10. A basketball has radius 12 cm. Find its volume correct to one decimal place, and state its capacity in litres (to two decimal places).

Q11. A monument is a rectangular-based pyramid with a base 6 m by 4 m and a height of 9 m. Find its volume.

Q12. A solid timber block is a rectangular prism 10 cm by 10 cm by 20 cm. A cylindrical hole of radius 2 cm is drilled straight through its 20 cm length. Find the volume of timber remaining, correct to two decimal places.



Q13. A fish tank is a rectangular prism 60 cm long, 30 cm wide and 40 cm deep. How many litres of water does it hold when full?

Q14. A swimming pool holds 50 000 litres of water. A hose fills the pool at 25 litres per minute. How long, in hours and minutes, will it take to fill the pool?

Q15. A metal object has a mass of 540 g and a volume of 200 cm^3. Find its density in g/cm^3.

Q16. Concrete has a density of $2400\,kg/m^3$. Find the mass of a concrete block of volume $0.5\,m^3$.

Q17. Gold has a density of $19.3\,g/cm^3$. A gold bar has a mass of 386 g. Find the volume of the bar.

Q18. A cylindrical vase has a volume of 500 cm^3 and a base radius of 5 cm. Find its height, correct to two decimal places.

Answers

Q1. (a) $30\,000$ cm³. (b) 30 L. (c) 15. **Q2.** (a) 339 cm³. (b) 339 mL. (c) 339 mL <375 mL, so the can cannot hold 375 mL — and a 375 mL can needs a capacity above 375 mL to allow head-space, so the dimensions are too small. **Q3.** (a) $V=1/3\,\pi r^2 h$. (b) 565.49 cm³. (c) 565 mL. **Q4.** (a) $V=4/3\,\pi r^3$. (b) 1436.76 cm³. **Q5.** (a) 64 cm². (b) 192 cm³. (c) 384 cm³. **Q6.** (a) 2500 cm³. (b) 2300 g. (c) 2.30 kg. **Q7.** 750 cm³. **Q8.** $10\,053$ cm³ $=10.05$ L. **Q9.** 1.9 m³. **Q10.** 7238.2 cm³ $=7.24$ L. **Q11.** 72 m³. **Q12.** 1748.67 cm³. **Q13.** 72 L. **Q14.** 2000 minutes $=33$ hours 20 minutes. **Q15.** 2.7 g/cm³. **Q16.** 1200 kg. **Q17.** 20 cm³. **Q18.** 6.37 cm.

Fully-worked solutions

Q1. (a) $V=lwh=40\times30\times25=30\,000$ cm³. (b) $30\,000\div1000=30$ L. (c) $30\div2=15$ containers.

Q2. (a) $V=\pi r^2 h=\pi\times3^2\times12=108\pi\approx339$ cm³. (b) $339\,\text{cm}^3=339$ mL. (c) The can's volume is 339 mL, which is less than the labelled 375 mL, so the can cannot hold the stated liquid. In fact a 375 mL can needs a capacity a little greater than 375 mL, to leave head-space above the liquid — so these dimensions are too small.

Q3. (a) $V=\frac{1}{3}\pi r^2 h$. (b) $V=\frac{1}{3}\pi\times6^2\times15=\frac{1}{3}\pi\times36\times15=180\pi\approx565.49$ cm³. (c) $565.49\,\text{cm}^3\approx565$ mL.

Q4. (a) $V=\frac{4}{3}\pi r^3$. (b) $V=\frac{4}{3}\pi\times7^3=\frac{4}{3}\pi\times343=\frac{1372\pi}{3}\approx1436.76$ cm³.

Q5. (a) Base area $=8\times8=64$ cm². (b) $V=\frac{1}{3}\times64\times9=\frac{1}{3}\times576=192$ cm³. (c) Doubling the height doubles the volume: $2\times192=384$ cm³.

Q6. (a) $2.5\,\text{L}=2.5\times1000=2500$ cm³. (b) $\text{mass}=\text{density}\times\text{volume}=0.92\times2500=2300$ g. (c) $2300\,\text{g}=2.30$ kg.

Q7. The cross-section is a right-angled triangle with area $\frac{1}{2}\times5\times12=30$ cm². The prism is 25 cm long, so $V=\text{base area}\times\text{length}=30\times25=750$ cm³.

Q8. Convert the length: $2\,\text{m}=200$ cm. Then $V=\pi r^2 h=\pi\times4^2\times200=3200\pi\approx10\,053$ cm³. In litres, $10\,053\div1000\approx10.05$ L.

Q9. $V=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi\times1.5^2\times0.8=\frac{1}{3}\pi\times2.25\times0.8=0.6\pi\approx1.885$, so $V\approx1.9$ m³.

Q10. $V=\frac{4}{3}\pi r^3=\frac{4}{3}\pi\times12^3=\frac{4}{3}\pi\times1728=2304\pi\approx7238.2$ cm³. In litres, $7238.2\div1000\approx7.24$ L.

Q11. Base area $=6\times4=24$ m², so $V=\frac{1}{3}\times24\times9=\frac{1}{3}\times216=72$ m³.

Q12. The whole block has volume $10\times10\times20=2000$ cm³. The drilled hole is a cylinder of radius 2 cm and length 20 cm: $V_{\text{hole}}=\pi\times2^2\times20=80\pi\approx251.33$ cm³. The remaining timber is $2000-251.33=1748.67$ cm³.

Q13. $V=60\times30\times40=72\,000$ cm³ $=72\,000\div1000=72$ L.

Q14. Time $=50\,000\div25=2000$ minutes. Converting, $2000\div60=33$ remainder 20, so 2000 minutes $=33$ hours and 20 minutes.

Q15. $\text{density}=\frac{\text{mass}}{\text{volume}}=\frac{540}{200}=2.7$ g/cm³.

Q16. $\text{mass}=\text{density}\times\text{volume}=2400\times0.5=1200$ kg.

Q17. $\text{volume} = \dfrac{\text{mass}}{\text{density}} = \dfrac{386}{19.3} = 20$ cm³.

Q18. From $V = \pi r^2 h$, $h = \dfrac{V}{\pi r^2} = \dfrac{500}{\pi \times 5^2} = \dfrac{500}{25\pi} = \dfrac{20}{\pi} \approx 6.37$ cm.

Chapter 13 Measurement: length, area, volume and error — 13G Measurement error, accuracy, precision and tolerance

Theory

No measurement is ever exact. Every ruler, scale or measuring cylinder can only be read to a certain fineness, and the person reading it adds a little uncertainty too. This section is about describing and working with that uncertainty — how **accurate** and **precise** a measurement is, the **error** in it, and whether it falls within an allowed **tolerance**.

Accuracy and precision.

These two words are often confused, but they mean different things.

- **Accuracy** is how close a measurement is to the **true** value. A scale that reads 500 g for an object that really is 500 g is accurate.
- **Precision** is how **finely** and **consistently** a measurement is made. A scale that reads to the nearest 0.1 g is more precise than one that reads to the nearest 10 g.

A measurement can be precise but not accurate (a scale that always reads 30 g too high gives finely-detailed but wrong values), or accurate but not precise (a rough estimate that happens to be about right).

The limit of reading.

The **smallest division** on an instrument sets how precisely you can read it. Because you round to the nearest marking, any single measurement could be out by up to **half of the smallest division** — this is the **limit of reading**. A ruler marked in millimetres has a smallest division of 1 mm, so a length read from it is uncertain by ± 0.5 mm. A measurement of 24.6 cm on that ruler is really “24.6 ± 0.05 cm”, meaning the true length lies between 24.55 cm and 24.65 cm.

Absolute error and percentage error.

When the true (actual) value is known, the **absolute error** is how far the measurement is from it:

$$\text{absolute error} = |\text{measured} - \text{actual}|.$$

The vertical bars mean the size of the difference, so the error is never negative. To judge how *serious* an error is, compare it with the actual value as a **percentage error**:

$$\text{percentage error} = \frac{|\text{measured} - \text{actual}|}{\text{actual}} \times 100\,\%.$$

An absolute error of 2 g matters far more on a 10 g mass ($20\,\%$) than on a 500 g mass ($0.4\,\%$).

Tolerance.

In manufacturing and trade, a measurement does not have to be perfect — it only has to fall within an agreed range called the **tolerance**. A tolerance is written as a nominal value **plus or minus** an allowed amount, for example $250\,\text{mL} \pm 5\,\text{mL}$. This means any value from

$$250 - 5 = 245\,\text{mL to}\,250 + 5 = 255\,\text{mL}$$

is acceptable. A measured value is **within tolerance** if it lies between the lower and upper limits, and **outside tolerance** (rejected) if it does not.

Upper and lower bounds from rounding.

When a measurement is given rounded — say a length of 8.4 m “to the nearest 0.1 m” — the true value lies within half a unit of the stated figure. The **lower bound** is 8.35 m and the **upper bound** is 8.45 m, so the maximum possible error is 0.05 m. Recognising these bounds matters when rounded measurements are used in further calculations.

In short: every measurement carries uncertainty. State it with its limit of reading, judge its error as a percentage, and check it against the tolerance the job requires.

Worked examples

Example 1 — scaffolded

A metal rod is measured as 24.6 cm using a ruler marked in millimetres. State the limit of reading, write the measurement with its uncertainty, and give the range in which the true length lies.

Working	Comment
The smallest division is 1 mm $=0.1$ cm.	Read off the finest marking on the ruler.
Limit of reading $=1/2\times0.1=0.05$ cm.	Uncertainty is half the smallest division.
Measurement $=24.6\pm0.05$ cm.	Nominal value plus or minus the limit of reading.
True length lies between $24.6-0.05=24.55$ cm and $24.6+0.05=24.65$ cm.	Lower and upper bounds.

Example 2 — scaffolded

A kitchen scale reads 498 g for a weight that is known to be exactly 500 g. Find the absolute error and the percentage error of the scale's reading.

Working	Comment
Absolute error $=\lvert 498-500\rvert=2$ g.	Size of the difference from the true value.
Percentage error $=\dfrac{2}{500}\times100\,\%$.	Error as a fraction of the actual value.
$=0.4\,\%$.	A small percentage error — the scale is accurate.

Example 3 — unscaffolded

A factory makes bolts with a required diameter of $12.00\,\text{mm}\pm0.05\,\text{mm}$. Four bolts are measured: 11.97 mm, 12.04 mm, 12.08 mm and 11.93 mm. State the acceptable range and determine which bolts pass inspection.

The tolerance 12.00 ± 0.05 mm gives an acceptable range from $12.00-0.05=11.95$ mm to $12.00+0.05=12.05$ mm. Checking each bolt against $11.95\le d\le12.05$: the bolt of 11.97 mm passes, 12.04 mm passes, 12.08 mm is above 12.05 so it fails, and 11.93 mm is below 11.95 so it fails.

∴ the acceptable range is 11.95 mm to 12.05 mm; the 11.97 mm and 12.04 mm bolts pass, while the 12.08 mm and 11.93 mm bolts are rejected.

Example 4 — unscaffolded

A garden bed is measured as 8.4 m, correct to the nearest 0.1 m. Find the lower and upper bounds of its true length and the maximum possible error.

Because the length is rounded to the nearest 0.1 m, the true value is within half of 0.1 m — that is, within 0.05 m — of 8.4 m. The lower bound is $8.4-0.05=8.35$ m and the upper bound is $8.4+0.05=8.45$ m.

∴ the true length lies between 8.35 m and 8.45 m, with a maximum possible error of 0.05 m.

Practice questions

Scaffolded

Q1. A measuring cylinder is marked in divisions of 5 mL, and a liquid reads 65 mL. (a) State the smallest division. (b) State the limit of reading. (c) Give the range in which the true volume lies.

Q2. A person estimates a parcel's mass as 45 kg. It is then weighed accurately at 48 kg. (a) Find the absolute error of the estimate. (b) State whether the estimate was too high or too low.

Q3. A tailor measures a length of fabric as 152 cm. The true length is 150 cm. (a) Find the absolute error. (b) Find the percentage error, correct to two decimal places.

Q4. A bottle is labelled $500\,\text{mL} \pm 10\,\text{mL}$. (a) Find the lowest acceptable volume. (b) Find the highest acceptable volume. (c) State the acceptable range as an inequality.

Q5. A snack packet must contain $250\,\text{g} \pm 5\,\text{g}$. A packet is measured at 254 g. (a) State the acceptable range. (b) State whether this packet is within tolerance.

Q6. Two thermometers are used to measure water known to be at 50°C. Thermometer P reads 50°C, 50°C, 50°C. Thermometer Q reads 47°C, 52°C, 49°C. (a) Which thermometer is more accurate? (b) Which is more precise (consistent)? (c) Explain the difference between accuracy and precision using this example.

Unscaffolded

Q7. A pencil is measured as 8.7 cm using a ruler marked in millimetres. Write the measurement with its limit of reading, and state the bounds of the true length.

Q8. A digital thermometer reads 21.5°C when the true temperature is 22.0°C. Find the absolute error and the percentage error, correct to two decimal places.

Q9. A set of scales reads a baby's mass as 2.35 kg. The true mass is 2.5 kg. Find the percentage error of the scales.

Q10. A machine part must be $1.50\,\text{m} \pm 0.02\,\text{m}$. State the acceptable range of lengths.

Q11. A drink bottle must hold $750\,\text{mL} \pm 15\,\text{mL}$. Three bottles are filled to 738 mL, 742 mL and 767 mL. State the acceptable range and determine which bottles are within tolerance.

Q12. A mass is given as 340 g, correct to the nearest 10 g. Find the lower and upper bounds of the true mass.

Q13. A length is stated as 6.2 cm, correct to the nearest millimetre. Find the lower and upper bounds of the true length.

Q14. A jug is meant to hold 20 L but actually holds 19.6 L. Find the percentage error, correct to two decimal places.

Q15. A bracket must measure $80\,\text{mm} \pm 1.5\,\text{mm}$. Three brackets measure 78.6 mm, 80.2 mm and 81.9 mm. State the acceptable range and state which brackets pass inspection.

Q16. A fence panel is measured as 45 cm, correct to the nearest centimetre. (a) State the maximum possible error. (b) Find the lower and upper bounds of the true length.

Q17. A runner's time is recorded as 9.9 s, but the true time was 10.0 s. Find the percentage error of the recorded time.

Q18. A water tank is designed to hold 5000 L with a tolerance of $\pm 2\%$. (a) Find the tolerance in litres. (b) State the acceptable range of capacities.

Answers

Q1. (a) 5 mL; (b) 2.5 mL; (c) between 62.5 mL and 67.5 mL. **Q2.** (a) 3 kg; (b) too low. **Q3.** (a) 2 cm; (b) 1.33%. **Q4.** (a) 490 mL; (b) 510 mL; (c) $490 \le V \le 510$ (mL). **Q5.** (a) 245 g to 255 g; (b) within tolerance (254 lies in the range). **Q6.** (a) thermometer P; (b) thermometer P; (c) P is both accurate (correct value) and precise (same each time); Q is neither — its readings scatter around and away from the true value. **Q7.** 8.7 ± 0.05 cm; between 8.65 cm and 8.75 cm. **Q8.** absolute error 0.5°C; percentage error 2.27%. **Q9.** 6%. **Q10.** 1.48 m to 1.52 m. **Q11.** range 735 mL to 765 mL; 738 mL and 742 mL pass, 767 mL fails. **Q12.** 335 g to 345 g. **Q13.** 6.15 cm to 6.25 cm. **Q14.** 2%. **Q15.** range 78.5 mm to 81.5 mm; 78.6 mm and 80.2 mm pass, 81.9 mm fails. **Q16.** (a) 0.5 cm; (b) 44.5 cm to 45.5 cm. **Q17.** 1%. **Q18.** (a) 100 L; (b) 4900 L to 5100 L.

Fully-worked solutions

Q1. (a) The cylinder is marked every 5 mL, so the smallest division is 5 mL. (b) The limit of reading is half the smallest division: $1/2 \times 5 = 2.5$ mL. (c) The true volume is 65 ± 2.5 mL, i.e. between 62.5 mL and 67.5 mL.

Q2. (a) Absolute error $= |45 - 48| = 3$ kg. (b) Since $45 < 48$, the estimate was too **low**.

Q3. (a) Absolute error $= |152 - 150| = 2$ cm. (b) Percentage error $= \dfrac{2}{150} \times 100\% = 1.33\%$ (to 2 d.p.).

Q4. (a) $500 - 10 = 490$ mL. (b) $500 + 10 = 510$ mL. (c) Acceptable if $490 \le V \le 510$ (mL).

Q5. (a) 250 ± 5 g gives 245 g to 255 g. (b) Since $245 \le 254 \le 255$, the packet is **within tolerance**.

Q6. (a) Thermometer P reads the true value 50°C every time, so it is more **accurate**. (b) P gives the same reading each time, so it is more **precise** (consistent). (c) Accuracy is about being close to the true value; precision is about being consistent. P is both accurate and precise; Q's readings $(47, 52, 49)$ are scattered (not precise) and average away from 50 (not accurate).

Q7. A millimetre ruler has smallest division 1 mm $= 0.1$ cm, so the limit of reading is 0.05 cm. The measurement is 8.7 ± 0.05 cm, so the true length lies between 8.65 cm and 8.75 cm.

Q8. Absolute error $= |21.5 - 22.0| = 0.5^\circ$C. Percentage error $= \dfrac{0.5}{22.0} \times 100\% = 2.27\%$ (to 2 d.p.).

Q9. Percentage error $= \dfrac{|2.35 - 2.5|}{2.5} \times 100\% = \dfrac{0.15}{2.5} \times 100\% = 6\%$.

Q10. 1.50 ± 0.02 m gives $1.50 - 0.02 = 1.48$ m to $1.50 + 0.02 = 1.52$ m.

Q11. The tolerance 750 ± 15 mL gives an acceptable range of 735 mL to 765 mL. Checking each: 738 mL and 742 mL lie in $[735, 765]$ so they pass; $767 > 765$, so that bottle fails.

Q12. "340 g to the nearest 10 g" means the true mass is within 5 g of 340 g (half of 10 g), so it lies between 335 g and 345 g.

Q13. "6.2 cm to the nearest millimetre" means within 0.05 cm (half of 0.1 cm) of 6.2 cm, so between 6.15 cm and 6.25 cm.

Q14. Percentage error $= \dfrac{|19.6 - 20|}{20} \times 100\% = \dfrac{0.4}{20} \times 100\% = 2\%$.

Q15. The tolerance 80 ± 1.5 mm gives an acceptable range of 78.5 mm to 81.5 mm. Checking: 78.6 mm and 80.2 mm lie in $[78.5, 81.5]$ so they pass; $81.9 > 81.5$, so that bracket fails.

Q16. (a) "45 cm to the nearest centimetre" means the true length is within half a centimetre, so the maximum possible error is 0.5 cm. (b) The bounds are $45 - 0.5 = 44.5$ cm and $45 + 0.5 = 45.5$ cm.

Q17. Percentage error $= \dfrac{|9.9 - 10.0|}{10.0} \times 100\,\% = \dfrac{0.1}{10} \times 100\,\% = 1\,\%$.

Q18. (a) Tolerance $= 2\,\%$ of $5000 = \dfrac{2}{100} \times 5000 = 100$ L. (b) The acceptable range is 5000 ± 100 L, i.e. 4900 L to 5100 L.

Topic 4 Test A — Multiple-choice

Topic 4: Space and measurement (Chapters 12–13). Materials: one scientific calculator and one bound reference. Section A: 15 multiple-choice questions (15 marks). Time: about 25 minutes. Unless otherwise indicated, diagrams are not drawn to scale.

Section A — Multiple-choice questions

Choose the response that is correct for the question.

Question 1

A rectangular prism (a box) has how many edges?

A. 6

B. 8

C. 12

D. 24

Question 2

The size of each interior angle of a regular hexagon is

A. 120°

B. 108°

C. 135°

D. 144°

Question 3

The point $(3,2)$ is reflected in the x-axis. The coordinates of the image are

A. $(-3,2)$

B. $(2,3)$

C. $(-3,-2)$

D. $(3,-2)$

Question 4

The order of rotational symmetry of a square is

A. 2

B. 4

C. 8

D. 1

Question 5

Two parallel lines are cut by a transversal, forming a pair of co-interior (allied) angles. If one of them is 125°, the other is

A. 55°

B. 125°

C. 90°

D. 180°

Question 6

In a triangle, two of the angles are 47° and 68°. The third angle is

A. 55°

B. 75°

C. 65°

D. 115°

Question 7

On a map with scale $1:50\,000$, two towns are 6 cm apart. The real distance between them is

A. 0.3 km

B. 3 km

C. 30 km

D. 300 km

Question 8

A shape is enlarged by a scale factor of 3. Its area is increased by a factor of

A. 3

B. 6

C. 9

D. 27

Question 9

An area of 2 m² is equal to

A. 200 cm²

B. 2000 cm²

C. $200\,000$ cm²

D. $20\,000$ cm²

Question 10

A distance of 3.5 km is equal to

A. 350 m

B. 3500 m

C. 35 000 m

D. 0.0035 m

Question 11

The circumference of a circle of radius 7 cm, to the nearest centimetre, is

A. 44 cm

B. 22 cm

C. 14 cm

D. 154 cm

Question 12

A metal block has a mass of 240 g and a volume of 30 cm^3. Its density is

A. 6 g/cm^3

B. 7200 g/cm^3

C. 8 g/cm^3

D. 0.125 g/cm^3

Question 13

A cylinder has radius 3 cm and height 10 cm. Its volume, to the nearest cubic centimetre, is

A. 90 cm^3

B. 188 cm^3

C. 848 cm^3

D. 283 cm^3

Question 14

A triangle has base 8 cm and perpendicular height 5 cm. Its area is

A. 40 cm^2

B. 20 cm^2

C. 13 cm^2

D. 26 cm^2

Question 15

A length is measured as 4.8 m when its true length is 5.0 m. The percentage error is

A. 4%

B. 2%

C. 5%

D. 0.2%

End of Section A

Topic 4 Test A — Solutions

Question	Answer	Working
1	**C**	A box has 12 edges (4 top, 4 bottom, 4 vertical).
2	**A**	Interior angle $=\frac{(6-2)\times 180^{\circ}}{6}=\frac{720^{\circ}}{6}=120^{\circ}$.
3	**D**	Reflecting in the x-axis keeps x, negates y: $(3,-2)$.
4	**B**	A square maps onto itself 4 times in a full turn.
5	**A**	Co-interior angles are supplementary: $180^{\circ}-125^{\circ}=55^{\circ}$.
6	**C**	$180^{\circ}-47^{\circ}-68^{\circ}=65^{\circ}$.
7	**B**	$6\times 50\,000=300\,000$ cm $=3$ km.
8	**C**	Area scales by the square of the length factor: $3^2=9$.
9	**D**	1 m² $=100^2=10\,000$ cm², so 2 m² $=20\,000$ cm².
10	**B**	$3.5\times 1000=3500$ m.
11	**A**	$C=2\pi r=2\pi\times 7=43.98\ldots\approx 44$ cm.
12	**C**	density $=\frac{\text{mass}}{\text{volume}}=\frac{240}{30}=8$ g/cm³.
13	**D**	$V=\pi r^2 h=\pi\times 3^2\times 10=282.7\ldots\approx 2$ cm³.
14	**B**	$A=\frac{1}{2}bh=\frac{1}{2}\times 8\times 5=20$ cm².
15	**A**	$\frac{\lvert 4.8-5.0\rvert}{5.0}\times 100\,\%=\frac{0.2}{5.0}\times 100\,\%=$.

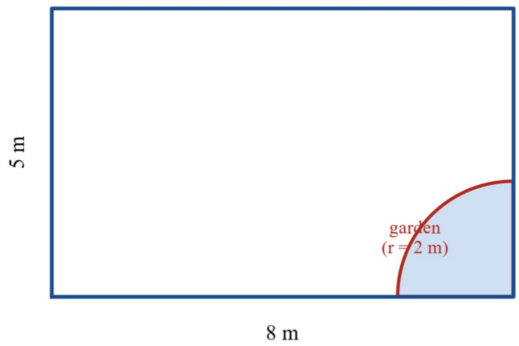
Topic 4 Test B — Short answer

Topic 4: Space and measurement. Materials: one scientific calculator and one bound reference. Section B: 6 questions (30 marks). Time: about 40 minutes. Answer all questions in the spaces provided. In questions where more than one mark is available, appropriate working must be shown. Round only when instructed.

Section B

Question 1 (5 marks)

A rectangular courtyard measures 8 m by 5 m. A quarter-circle garden bed of radius 2 m is cut from one corner, as shown. The rest of the courtyard is paved.



a. Find the area of the full 8 m by 5 m rectangle. 1 mark

b. Show that the area of the quarter-circle garden bed is π m². 2 marks

c. Find the paved area, correct to one decimal place. 2 marks

Question 2 (5 marks)

A cylindrical water tank has radius 0.8 m and height 1.5 m.

a. Find the volume of the tank, in cubic metres, correct to two decimal places. 2 marks

b. Using 1 m³ = 1000 L, find the capacity of the tank in litres, to the nearest litre. (Use the unrounded volume.) 1 mark

c. Water flows into the empty tank at 20 litres per minute. Find the time to fill the tank, to the nearest minute. 2 marks

Question 3 (5 marks)

On a floor plan drawn to a scale of $1:100$, a rectangular room measures 4 cm by 3 cm.

a. Find the real length and width of the room, in metres. 2 marks

b. Find the real floor area of the room. 1 mark

c. Carpet costs $\$45$ per square metre. Find the cost of carpeting the room. 2 marks

Question 4 (5 marks)

This question is about angle relationships. Give a reason with each answer.

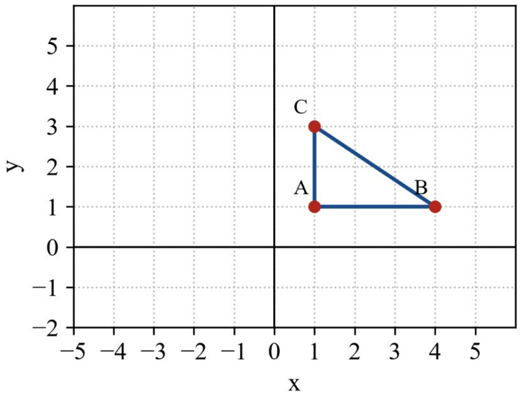
a. Two angles lie on a straight line. One of them is 125°. Find the other angle. 1 mark

b. In a triangle, two of the angles are 40° and 75°. Find the third angle, giving a reason. 2 marks

c. A transversal crosses two parallel lines. One co-interior angle is 110°. Find the other co-interior angle, giving a reason. 2 marks

Question 5 (5 marks)

Triangle ABC has vertices $A(1,1)$, $B(4,1)$ and $C(1,3)$, as shown.



a. The triangle is translated 2 units right and 1 unit up. Write down the coordinates of the images A', B' and C'. 2 marks

b. The original triangle ABC is reflected in the y-axis. Write down the coordinates of the three image vertices. 2 marks

c. State the order of rotational symmetry of a regular pentagon. 1 mark

Question 6 (5 marks)

A factory makes bolts with a specified diameter of 12.00 mm and a tolerance of ± 0.05 mm.

a. State the smallest and the largest acceptable diameters. 2 marks

b. A bolt is measured as 12.08 mm. Show that this bolt is outside the tolerance. 1 mark

c. A machinist measures a steel rod as 248 mm, but its true length is 250 mm. Find the percentage error of the measurement, correct to two decimal places. 2 marks

End of Section B

Topic 4 Test B — Solutions

Question 1.

a. Area of the rectangle $=8\times 5=40$ m².

b. A quarter-circle is one quarter of a full circle of radius 2 m:

$$A=\frac{1}{4}\pi r^2=\frac{1}{4}\times\pi\times 2^2=\frac{1}{4}\times 4\pi=\pi\ \mathrm{m}^2.$$

So the garden bed has area π m², as required.

c. Paved area = rectangle − garden $=40-\pi=36.858\ldots\approx 36.9$ m² (to 1 d.p.).

Question 2.

a. $V=\pi r^2 h=\pi\times 0.8^2\times 1.5=\pi\times 0.64\times 1.5=3.0159\ldots\approx 3.02$ m³.

b. Keeping the unrounded volume, capacity $=3.0159\ldots\times 1000=3015.9\ldots\approx 3016$ L.

c. Time $=\dfrac{3015.9\ldots}{20}=150.8\ldots\approx 151$ minutes.

Question 3.

a. Each 1 cm on the plan is 100 cm $=1$ m in real life. So 4 cm $\rightarrow$ 4 m and 3 cm $\rightarrow$ 3 m: the room is 4 m by 3 m.

b. Real floor area $=4\times 3=12$ m².

c. Cost $=12\times\$45=\$540$.

Question 4.

a. Angles on a straight line add to 180°, so the other angle $=180^\circ-125^\circ=55^\circ$.

b. The angle sum of a triangle is 180°, so the third angle $=180^\circ-40^\circ-75^\circ=65^\circ$.

c. Co-interior (allied) angles between parallel lines are supplementary (add to 180°), so the other angle $=180^\circ-110^\circ=70^\circ$.

Question 5.

a. Add 2 to each x-coordinate and 1 to each y-coordinate: $A'(3,2)$, $B'(6,2)$, $C'(3,4)$.

b. Reflecting in the y-axis negates each x-coordinate and keeps y: $(-1,1)$, $(-4,1)$, $(-1,3)$.

c. A regular pentagon maps onto itself 5 times in a full turn, so its order of rotational symmetry is 5.

Question 6.

a. Smallest $=12.00-0.05=11.95$ mm; largest $=12.00+0.05=12.05$ mm.

b. The acceptable range is 11.95 mm to 12.05 mm. Since $12.08>12.05$, the bolt is larger than the largest acceptable diameter, so it is outside the tolerance.

c. Percentage error $=\dfrac{|248-250|}{250}\times 100\%=\dfrac{2}{250}\times 100\%=0.80\%$.