

# Foundation Mathematics Units 3 & 4

## Chapter 12 — Shape, geometry and scale

This work is licensed under the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License. To view a copy of this license, visit <http://creativecommons.org/licenses/by-nc-nd/4.0/> or send a letter to Creative Commons, PO Box 1866, Mountain View, CA 94042, USA.

**Attribution:** Classbasics Pty Ltd, vwx.au

**Aboriginal and Torres Strait Islander content** has been excluded as accuracy cannot be guaranteed, and it is better to be respectful than cover the entire curriculum inaccurately.

Significant effort has been made to ensure this content is legal. Please send any areas of concern to [admin@vwx.au](mailto:admin@vwx.au) with appropriate document/legal references so errors can be verified and changes be made.

### Contents

| Section                            | Page |
|------------------------------------|------|
| 12C Shapes and objects             | 2    |
| 12D Transformations and symmetry   | 8    |
| 12E Angle properties               | 14   |
| 12F Maps and scale                 | 20   |
| 12G Reading and working with plans | 25   |
| 12H Nets and 3D from 2D            | 30   |

### Theory

Being able to name shapes and objects correctly, and to describe their properties, is the starting point for all measurement and design work. This section names the common **2D shapes** (flat) and **3D objects** (solid) and lists the properties you use to identify them.

#### Polygons.

A **polygon** is a closed 2D shape made of straight sides. Polygons are named by their number of sides: **triangle** (3), **quadrilateral** (4), **pentagon** (5), **hexagon** (6), **heptagon** (7), **octagon** (8), **nonagon** (9) and **decagon** (10). A polygon is **regular** if all its sides are equal *and* all its angles are equal (a stop sign is a regular octagon); otherwise it is **irregular**.

#### Triangles.

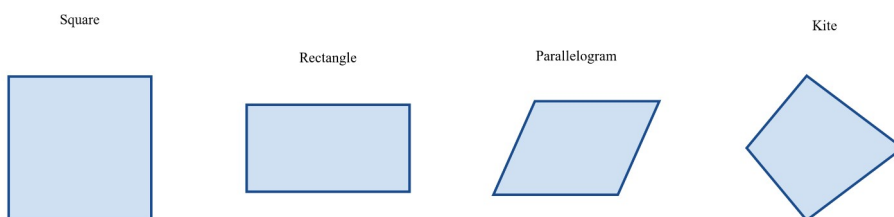
Triangles are classified two ways.

- By **sides**: **equilateral** (all three sides equal), **isosceles** (two sides equal), **scalene** (no sides equal).
- By **angles**: **acute** (all angles less than  $90^\circ$ ), **right-angled** (one  $90^\circ$  angle), **obtuse** (one angle greater than  $90^\circ$ ).

#### Quadrilaterals.

The special four-sided shapes and their key properties:

| Shape         | Key properties                         |
|---------------|----------------------------------------|
| Square        | 4 equal sides, 4 right angles          |
| Rectangle     | opposite sides equal, 4 right angles   |
| Parallelogram | opposite sides equal and parallel      |
| Rhombus       | 4 equal sides, opposite sides parallel |
| Trapezium     | exactly one pair of parallel sides     |
| Kite          | two pairs of adjacent equal sides      |



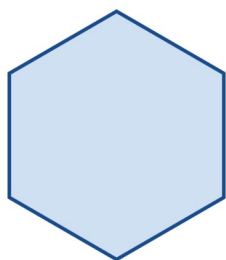
#### Interior angles of a polygon.

The interior angles of any polygon add to

$$\text{angle sum} = (n - 2) \times 180^\circ,$$

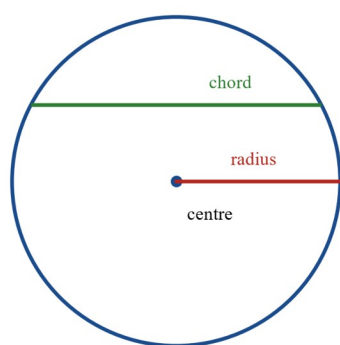
where  $n$  is the number of sides. So a triangle's angles sum to  $180^\circ$ , a quadrilateral's to  $360^\circ$ , a pentagon's to  $540^\circ$  and a hexagon's to  $720^\circ$ . For a **regular** polygon, each interior angle is the angle sum divided by  $n$  — for a regular hexagon that is  $720^\circ \div 6 = 120^\circ$ .

Regular hexagon



### Parts of a circle.

A circle has a **centre**; the **radius**  $r$  is the distance from the centre to the edge; the **diameter**  $d = 2r$  passes through the centre; the **circumference** is the distance around it. A **chord** joins two points on the circle, an **arc** is part of the circumference, a **sector** is a “pizza slice”, and a **tangent** just touches the circle at one point.

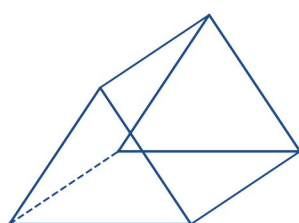


### 3D objects (solids).

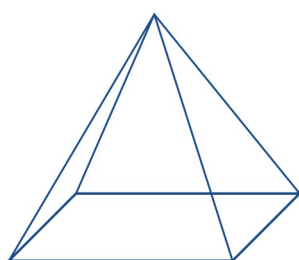
Solids are named by their faces:

- A **prism** has the same cross-section all along its length; it is named by that cross-section — a **triangular prism** (a Toblerone box), a **rectangular prism** (a shoebox), a **hexagonal prism**, and so on.
- A **pyramid** has a flat base and triangular faces meeting at an apex; it is named by its base — a **square pyramid**, a **triangular pyramid** (also called a tetrahedron).
- A **cylinder** (a can), a **cone** (an ice-cream cone) and a **sphere** (a ball) have **curved** surfaces.

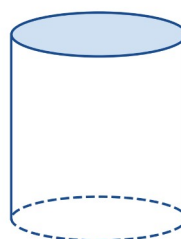
Triangular prism



Square pyramid



Cylinder



### Faces, edges and vertices; Euler's relation.

For a solid with flat faces (a **polyhedron**): a **face** is a flat surface, an **edge** is where two faces meet, and a **vertex** is a corner. These three counts are linked by **Euler's relation**:

$$F + V - E = 2.$$

For example a rectangular prism has  $F=6$ ,  $V=8$ ,  $E=12$ , and  $6+8-12=2$ . Euler's relation lets you find a missing count from the other two. It applies only to polyhedra — a cylinder, cone and sphere have curved surfaces, so it does not apply to them.

## Worked examples

### Example 1 — scaffolded

A triangle has sides 7 cm, 7 cm and 4 cm. Classify it by its sides. Then name the quadrilateral that has four equal sides and four right angles.

| Working                                                                                                    | Comment                                                                   |
|------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------|
| The triangle has two equal sides (7 cm and 7 cm), so it is <b>isosceles</b> .                              | Two equal sides = isosceles.                                              |
| It is not equilateral (the third side is 4 cm, not 7 cm) and not scalene (that needs all sides different). | Rule out the other two types.                                             |
| A quadrilateral with four equal sides <b>and</b> four right angles is a <b>square</b> .                    | Four right angles rules out a rhombus (which need not have right angles). |

### Example 2 — scaffolded

A pentagonal prism has a pentagon at each end. Find the number of faces, vertices and edges, and check Euler's relation.

| Working                                                                                    | Comment                               |
|--------------------------------------------------------------------------------------------|---------------------------------------|
| <b>Faces:</b> the two pentagons plus one rectangle for each of the 5 sides: $2+5=7$ faces. | Two ends + one face per side.         |
| <b>Vertices:</b> 5 corners on each pentagon: $2 \times 5 = 10$ vertices.                   | Corners at both ends.                 |
| <b>Edges:</b> 5 around each pentagon plus 5 joining them: $5+5+5=15$ edges.                | Top edges + bottom edges + verticals. |
| <b>Check:</b> $F+V-E=7+10-15=2$ . ✓                                                        | Euler's relation holds.               |

### Example 3 — unscaffolded

A regular stop sign is a regular octagon. Find the sum of its interior angles, and the size of each interior angle.

An octagon has  $n=8$  sides, so the interior angles sum to

$$(n-2) \times 180^\circ = (8-2) \times 180^\circ = 6 \times 180^\circ = 1080^\circ.$$

Because the sign is a **regular** octagon, all eight angles are equal, so each one is

$$1080^\circ \div 8 = 135^\circ.$$

$\therefore$  the interior angles sum to  $1080^\circ$  and each interior angle is  $135^\circ$ .

### Example 4 — unscaffolded

A solid polyhedron has 6 faces and 12 edges. Use Euler's relation to find the number of vertices, and suggest what the solid could be.

Euler's relation is  $F+V-E=2$ . Substituting  $F=6$  and  $E=12$ ,

$$6+V-12=2 \quad V=2-6+12=8.$$

So the solid has 8 vertices. A solid with 6 faces, 8 vertices and 12 edges is a **rectangular prism** (or a cube).

$\therefore V = 8$ , and the solid could be a rectangular prism (cube).

---

## Practice questions

### Scaffolded

**Q1.** Classify each triangle by its sides. (a) sides 5 cm, 5 cm, 5 cm. (b) sides 6 cm, 8 cm, 10 cm. (c) sides 9 cm, 9 cm, 5 cm.

**Q2.** Name the quadrilateral described. (a) opposite sides equal and parallel, no right angles. (b) exactly one pair of parallel sides. (c) two pairs of adjacent equal sides.

**Q3.** A polygon has 6 sides. (a) Name it. (b) Find the sum of its interior angles. (c) If it is regular, find the size of each interior angle.

**Q4.** A triangular prism. (a) How many faces does it have? (b) How many vertices and edges does it have? (c) Check Euler's relation  $F + V - E = 2$ .

**Q5.** For a circle, name the part described. (a) a straight line from the centre to the edge. (b) a straight line joining two points on the circle. (c) a "slice" bounded by two radii and an arc.

**Q6.** A regular pentagon. (a) How many sides does it have? (b) Find the sum of its interior angles. (c) Find the size of each interior angle.

### Un scaffolded

**Q7.** Classify a triangle with angles  $40^\circ$ ,  $75^\circ$  and  $65^\circ$  by its angles (acute, right-angled or obtuse). Show that the three angles are possible.

**Q8.** A give way sign is an equilateral triangle. State the size of each of its angles, and explain your reasoning.

**Q9.** Name the polygon with 8 sides, and find the sum of its interior angles.

**Q10.** A quadrilateral has three angles of  $85^\circ$ ,  $100^\circ$  and  $95^\circ$ . Find the fourth angle.

**Q11.** Find the size of each interior angle of a regular dodecagon (12 sides).

**Q12.** A hexagonal prism (a pencil shape) has a hexagon at each end. Find its number of faces, vertices and edges, and check Euler's relation.

**Q13.** A square-based pyramid. (a) Find its number of faces, vertices and edges. (b) Check Euler's relation.

**Q14.** A polyhedron has 5 vertices and 8 edges. Use Euler's relation to find its number of faces, and name a solid it could be.

**Q15.** State whether Euler's relation  $F + V - E = 2$  can be used for a cylinder, and explain why or why not.

**Q16.** A packaging designer describes a box as a solid with 6 rectangular faces, all meeting at right angles. Name the solid and state its number of edges and vertices.

**Q17.** A tiling pattern uses regular hexagons. Explain, using the interior angle of a regular hexagon, why three of them fit exactly around a point.

**Q18.** A solid has a hexagonal base and six triangular faces meeting at an apex. (a) Name the solid. (b) Find its number of faces, vertices and edges. (c) Check Euler's relation.

---

## Answers

**Q1.** (a) equilateral; (b) scalene; (c) isosceles. **Q2.** (a) parallelogram; (b) trapezium; (c) kite. **Q3.** (a) hexagon; (b)  $720^\circ$ ; (c)  $120^\circ$ . **Q4.** (a) 5 faces; (b) 6 vertices, 9 edges; (c)  $5 + 6 - 9 = 2$ . ✓ **Q5.** (a) radius; (b) chord; (c) sector. **Q6.** (a) 5; (b)  $540^\circ$ ; (c)  $108^\circ$ . **Q7.** acute (all three angles are less than  $90^\circ$ ); the angles sum to  $40 + 75 + 65 = 180^\circ$ , as required. **Q8.** each angle  $60^\circ$  (the angles of any triangle sum to  $180^\circ$ , and in an equilateral triangle they are equal, so  $180^\circ \div 3 = 60^\circ$ ). **Q9.** octagon;  $1080^\circ$ . **Q10.**  $80^\circ$ . **Q11.**  $150^\circ$ . **Q12.**  $F = 8$ ,  $V = 12$ ,  $E = 18$ ;  $8 + 12 - 18 = 2$ . ✓ **Q13.** (a)  $F = 5$ ,  $V = 5$ ,  $E = 8$ ; (b)  $5 + 5 - 8 = 2$ . ✓ **Q14.**  $F = 5$ ; a square-based pyramid. **Q15.** No — a cylinder has curved surfaces, so it is not a polyhedron and Euler's relation does not apply. **Q16.** rectangular prism (cuboid); 12 edges, 8 vertices. **Q17.** each interior angle is  $120^\circ$ , and  $3 \times 120^\circ = 360^\circ$ , a full turn, so three hexagons meet exactly at a point with no gap or overlap. **Q18.** (a) hexagonal pyramid; (b)  $F = 7$ ,  $V = 7$ ,  $E = 12$ ; (c)  $7 + 7 - 12 = 2$ . ✓

---

## Fully-worked solutions

**Q1.** (a) All three sides are equal (5 cm), so the triangle is **equilateral**. (b) All three sides are different (6, 8, 10 cm), so it is **scalene**. (c) Two sides are equal (9 cm and 9 cm), so it is **isosceles**.

**Q2.** (a) Opposite sides equal and parallel with no right angles describes a **parallelogram**. (b) Exactly one pair of parallel sides is a **trapezium**. (c) Two pairs of adjacent (next-to-each-other) equal sides is a **kite**.

**Q3.** (a) A 6-sided polygon is a **hexagon**. (b) Angle sum  $= (n - 2) \times 180^\circ = (6 - 2) \times 180^\circ = 4 \times 180^\circ = 720^\circ$ . (c) A regular hexagon has 6 equal angles, so each is  $720^\circ \div 6 = 120^\circ$ .

**Q4.** A triangular prism has a triangle at each end and three rectangular sides. (a) Faces: 2 triangles + 3 rectangles = 5 faces. (b) Vertices: 3 at each end = 6 vertices. Edges: 3 around each triangle + 3 joining them =  $3 + 3 + 3 = 9$  edges. (c)  $F + V - E = 5 + 6 - 9 = 2$ . ✓

**Q5.** (a) A line from the centre to the edge is the **radius**. (b) A line joining two points on the circle is a **chord**. (c) A slice bounded by two radii and an arc is a **sector**.

**Q6.** (a) A pentagon has 5 sides. (b) Angle sum  $= (5 - 2) \times 180^\circ = 3 \times 180^\circ = 540^\circ$ . (c) A regular pentagon has 5 equal angles, so each is  $540^\circ \div 5 = 108^\circ$ .

**Q7.** The three angles are  $40^\circ$ ,  $75^\circ$  and  $65^\circ$ . Their sum is  $40 + 75 + 65 = 180^\circ$ , which is the required angle sum for a triangle, so the triangle is possible. Since every angle is less than  $90^\circ$ , the triangle is **acute**.

**Q8.** The three angles of any triangle sum to  $180^\circ$ . An equilateral triangle has three equal angles, so each is  $180^\circ \div 3 = 60^\circ$ .

**Q9.** A polygon with 8 sides is an **octagon**. Its interior angles sum to  $(8 - 2) \times 180^\circ = 6 \times 180^\circ = 1080^\circ$ .

**Q10.** The four angles of a quadrilateral sum to  $360^\circ$ . So the fourth angle is

$$360^\circ - (85^\circ + 100^\circ + 95^\circ) = 360^\circ - 280^\circ = 80^\circ.$$

**Q11.** A dodecagon has  $n = 12$  sides, so the angle sum is  $(12 - 2) \times 180^\circ = 10 \times 180^\circ = 1800^\circ$ . As it is regular, each interior angle is  $1800^\circ \div 12 = 150^\circ$ .

**Q12.** A hexagonal prism has a hexagon at each end. Faces: 2 hexagons + 6 rectangles = 8. Vertices: 6 at each end = 12. Edges: 6 around each hexagon + 6 joining them =  $6 + 6 + 6 = 18$ . Check:  $F + V - E = 8 + 12 - 18 = 2$ . ✓

**Q13.** A square pyramid has a square base and four triangular faces. (a) Faces: 1 square + 4 triangles = 5. Vertices: 4 on the base + 1 apex = 5. Edges: 4 around the base + 4 up to the apex = 8. (b)  $F + V - E = 5 + 5 - 8 = 2$ . ✓

**Q14.** Euler's relation is  $F + V - E = 2$ . With  $V = 5$  and  $E = 8$ ,

$$F + 5 - 8 = 2 \quad F = 2 - 5 + 8 = 5.$$

A solid with 5 faces, 5 vertices and 8 edges is a **square-based pyramid**.

**Q15.** Euler's relation applies only to **polyhedra** (solids with flat faces, straight edges and vertices). A cylinder has two curved edges and a curved surface and no vertices in the usual sense, so it is **not** a polyhedron and Euler's relation cannot be used.

**Q16.** A solid with 6 rectangular faces all meeting at right angles is a **rectangular prism** (cuboid). It has 12 edges and 8 vertices (the same counts as a cube).

**Q17.** Each interior angle of a regular hexagon is  $120^\circ$ . Around a single point the angles must total a full turn,  $360^\circ$ . Since  $3 \times 120^\circ = 360^\circ$ , exactly three hexagons meet at the point with no gap and no overlap — which is why hexagonal tiling works.

**Q18.** (a) A solid with a hexagonal base and six triangular faces to an apex is a **hexagonal pyramid**. (b) Faces: 1 hexagon + 6 triangles = 7. Vertices: 6 on the base + 1 apex = 7. Edges: 6 around the base + 6 up to the apex = 12. (c)  $F + V - E = 7 + 7 - 12 = 2$ . ✓

## Theory

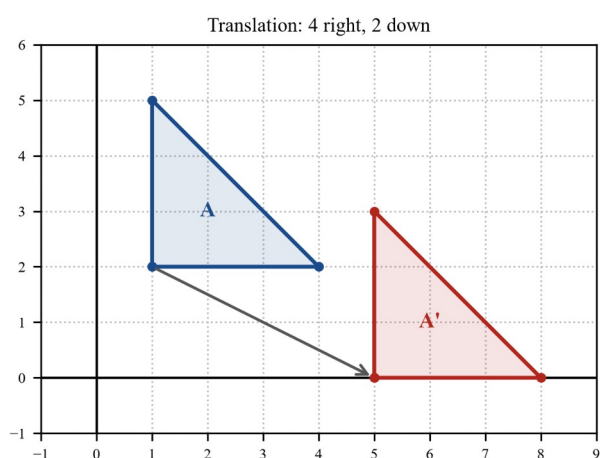
A **transformation** changes the position or orientation of a shape. The three transformations in this section — **translation**, **reflection** and **rotation** — all move a shape without changing its size or shape, so the image is always **congruent** to the original (same side lengths, same angles). The starting shape is the **object** and the moved shape is the **image**; a point  $A$  has image  $A'$  (read “A dash”).

### Translation — a slide.

A **translation** slides every point of a shape the same distance in the same direction, described as so many units **right/left** and so many units **up/down**. Moving a point  $a$  units right and  $b$  units up sends

$$(x, y) \rightarrow (x + a, y + b),$$

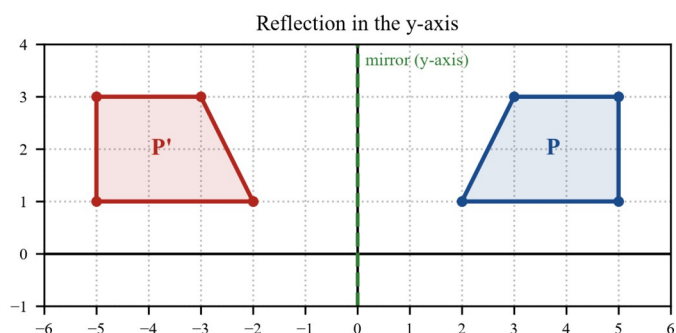
where a **left** move makes  $a$  negative and a **down** move makes  $b$  negative.



### Reflection — a flip.

A **reflection** flips a shape over a **mirror line**; each image point is the same distance behind the mirror as the object point is in front. For the common mirror lines,

in the  $x$ -axis:  $(x, y) \rightarrow (x, -y)$ , in the  $y$ -axis:  $(x, y) \rightarrow (-x, y)$ , in the line  $y = x$ :  $(x, y) \rightarrow (y, x)$ .

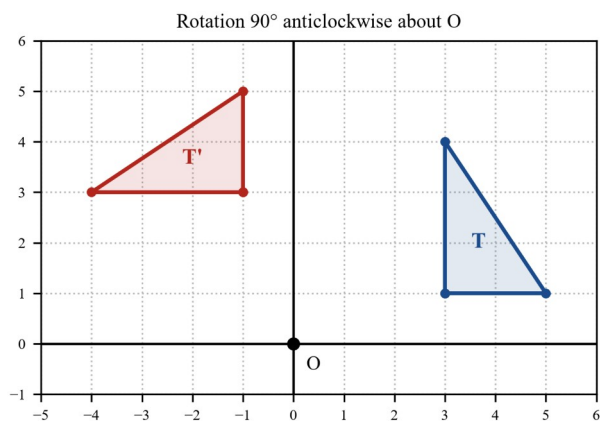


### Rotation — a turn.

A **rotation** turns a shape about a fixed **centre** (here the origin  $O$ ) through an angle, **anticlockwise** unless stated otherwise. About the origin,

$90^\circ$  anticlockwise:  $(x, y) \rightarrow (-y, x)$ ,  $180^\circ$ :  $(x, y) \rightarrow (-x, -y)$ ,  $90^\circ$  clockwise:  $(x, y) \rightarrow (y, -x)$ .

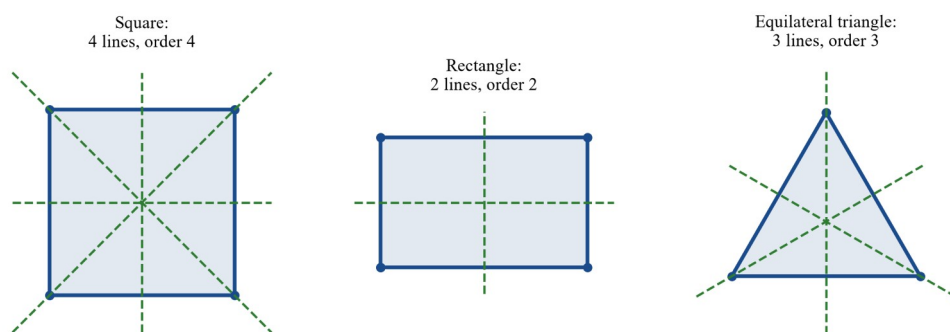
A  $90^\circ$  clockwise turn is the same as a  $270^\circ$  anticlockwise turn.



Because translation, reflection and rotation all preserve lengths and angles, the object and image are **congruent** — these are called **congruence transformations**.

### Line symmetry.

A shape has a **line (axis) of symmetry** if reflecting it in that line maps the shape exactly onto itself. The number of such lines varies with the shape.



### Rotational symmetry.

A shape has **rotational symmetry** if it maps onto itself when turned about its centre by less than a full turn. The **order** of rotational symmetry is the number of times the shape fits onto itself in one full  $360^\circ$  turn (every shape fits at least once, so the smallest order is 1, meaning “no rotational symmetry”).

| Shape                       | Lines of symmetry | Order of rotational symmetry |
|-----------------------------|-------------------|------------------------------|
| Equilateral triangle        | 3                 | 3                            |
| Square                      | 4                 | 4                            |
| Rectangle                   | 2                 | 2                            |
| Rhombus                     | 2                 | 2                            |
| Parallelogram (non-special) | 0                 | 2                            |
| Regular pentagon            | 5                 | 5                            |
| Regular hexagon             | 6                 | 6                            |
| Circle                      | infinitely many   | infinite                     |

### Tessellation.

A **tessellation** is a pattern of one or more shapes that covers a flat surface with no gaps and no overlaps, using translations, reflections and rotations of the shape. At every point where corners meet, the angles must add to  $360^\circ$ . Of the regular polygons, only the **equilateral triangle** ( $60^\circ$ ), the **square** ( $90^\circ$ ) and the **regular hexagon** ( $120^\circ$ ) tessellate on their own, because 360 is a whole-number multiple of each of those angles. A regular pentagon ( $108^\circ$ ) does not.

---

## Worked examples

### Example 1 — scaffolded

Triangle  $A$  has vertices  $(1, 2)$ ,  $(4, 2)$  and  $(1, 5)$ . It is translated 4 units to the right and 2 units down. Find the coordinates of the image  $A'$ .

| Working                                                                            | Comment                                              |
|------------------------------------------------------------------------------------|------------------------------------------------------|
| The translation is 4 right and 2 down, so<br>$(x, y) \rightarrow (x + 4, y - 2)$ . | Right is $+x$ ; down is $-y$ .                       |
| $(1, 2) \rightarrow (1 + 4, 2 - 2) = (5, 0)$ .                                     | Add 4 to each $x$ , subtract 2 from each $y$ .       |
| $(4, 2) \rightarrow (8, 0)$ .                                                      | Same rule on the second vertex.                      |
| $(1, 5) \rightarrow (5, 3)$ .                                                      | And the third vertex.                                |
| $A'$ has vertices $(5, 0)$ , $(8, 0)$ , $(5, 3)$ .                                 | The image is congruent to $A$ — same size and shape. |

### Example 2 — scaffolded

Quadrilateral  $P$  has vertices  $(2, 1)$ ,  $(5, 1)$ ,  $(5, 3)$  and  $(3, 3)$ . Find its image under (a) a reflection in the  $y$ -axis and (b) a reflection in the  $x$ -axis.

| Working                                                                                                                        | Comment                                  |
|--------------------------------------------------------------------------------------------------------------------------------|------------------------------------------|
| (a) Reflection in the $y$ -axis: $(x, y) \rightarrow (-x, y)$ .                                                                | Change the sign of each $x$ -coordinate. |
| $(2, 1) \rightarrow (-2, 1)$ , $(5, 1) \rightarrow (-5, 1)$ , $(5, 3) \rightarrow (-5, 3)$ ,<br>$(3, 3) \rightarrow (-3, 3)$ . | Apply to every vertex.                   |
| (b) Reflection in the $x$ -axis: $(x, y) \rightarrow (x, -y)$ .                                                                | Change the sign of each $y$ -coordinate. |
| $(2, 1) \rightarrow (2, -1)$ , $(5, 1) \rightarrow (5, -1)$ , $(5, 3) \rightarrow (5, -3)$ ,<br>$(3, 3) \rightarrow (3, -3)$ . | Apply to every vertex.                   |

### Example 3 — unscaffolded

Triangle  $T$  has vertices  $(3, 1)$ ,  $(5, 1)$  and  $(3, 4)$ . It is rotated  $90^\circ$  anticlockwise about the origin. Find the coordinates of the image.

A  $90^\circ$  anticlockwise rotation about the origin uses the rule  $(x, y) \rightarrow (-y, x)$ . Applying it to each vertex:

$$(3, 1) \rightarrow (-1, 3), (5, 1) \rightarrow (-1, 5), (3, 4) \rightarrow (-4, 3).$$

$\therefore$  the image has vertices  $(-1, 3)$ ,  $(-1, 5)$  and  $(-4, 3)$ .

### Example 4 — unscaffolded

State the number of lines of symmetry and the order of rotational symmetry of (a) a rectangle and (b) a regular hexagon.

- (a) A rectangle can be folded onto itself along a vertical line and a horizontal line through its centre, so it has 2 lines of symmetry. Turning it  $180^\circ$  maps it onto itself, and a full turn returns it, so it fits onto itself twice in  $360^\circ$  — order 2.
- (b) A regular hexagon has 6 equal sides. It has a line of symmetry through each pair of opposite vertices and through each pair of opposite edge-midpoints — 6 lines in all — and it maps onto itself after every  $60^\circ$  turn, so its order of rotational symmetry is 6.

$\therefore$  rectangle: 2 lines, order 2; regular hexagon: 6 lines, order 6.

## Practice questions

### Scaffolded

**Q1.** The point  $P(3, -2)$  is translated 5 units left and 4 units up. (a) Write the translation as a rule  $(x, y) \rightarrow (\dots, \dots)$ . (b) Find the coordinates of the image  $P'$ .

**Q2.** The point  $(4, 3)$  is reflected in each axis. (a) Find its image after a reflection in the  $x$ -axis. (b) Find its image after a reflection in the  $y$ -axis.

**Q3.** The point  $(2, 5)$  is rotated about the origin. (a) Find its image after a  $90^\circ$  anticlockwise rotation. (b) Find its image after a  $180^\circ$  rotation.

**Q4.** A shape with a vertex at  $A(1, 1)$  is transformed so that this vertex moves to  $B(1, -1)$ , and the shape keeps its size and orientation apart from being flipped. (a) Which single transformation maps  $A$  to  $B$ ? (b) State the mirror line.

**Q5.** State the number of lines of symmetry of (a) a square; (b) an equilateral triangle; (c) a parallelogram (not a rectangle or rhombus).

**Q6.** State the order of rotational symmetry of (a) a square; (b) a rectangle; (c) a regular pentagon.

### Un scaffolded

**Q7.** Translate the point  $(-1, 3)$  by 3 units right and 5 units down.

**Q8.** Triangle with vertices  $(0, 0)$ ,  $(2, 0)$  and  $(0, 3)$  is translated 3 units right and 1 unit up. Find the vertices of the image.

**Q9.** Reflect the point  $(5, -2)$  in the  $x$ -axis.

**Q10.** Reflect the point  $(-3, 6)$  in the  $y$ -axis.

**Q11.** Reflect the point  $(2, 7)$  in the line  $y = x$ .

**Q12.** Rotate the point  $(3, 2)$  by  $90^\circ$  anticlockwise about the origin.

**Q13.** Rotate the point  $(4, -1)$  by  $180^\circ$  about the origin.

**Q14.** A square has vertices  $(1, 1)$ ,  $(4, 1)$ ,  $(4, 4)$  and  $(1, 4)$ . Find the vertices of its image after a reflection in the  $y$ -axis.

**Q15.** A shape is transformed so that the point  $(2, 3)$  maps to  $(2, -3)$ , and the image is a mirror image of the object. Describe the single transformation fully.

**Q16.** Under a rotation about the origin, the point  $(1, 2)$  maps to  $(-2, 1)$ . State the angle and direction of the rotation.

**Q17.** State the number of lines of symmetry of (a) a regular hexagon; (b) a rectangle; (c) a rhombus; (d) a scalene triangle.

**Q18.** A tiler wants to cover a floor using identical regular tiles of one shape, with no gaps or overlaps. State which of the following can be used on their own — regular triangle, square, regular pentagon, regular hexagon — and briefly justify your answer using angles.

## Answers

**Q1.** (a)  $(x, y) \rightarrow (x - 5, y + 4)$ . (b)  $(-2, 2)$ . **Q2.** (a)  $(4, -3)$ . (b)  $(-4, 3)$ . **Q3.** (a)  $(-5, 2)$ . (b)  $(-2, -5)$ . **Q4.** (a) a reflection. (b) the  $x$ -axis. **Q5.** (a) 4. (b) 3. (c) 0. **Q6.** (a) 4. (b) 2. (c) 5. **Q7.**  $(2, -2)$ . **Q8.**  $(3, 1), (5, 1), (3, 4)$ . **Q9.**  $(5, 2)$ . **Q10.**  $(3, 6)$ . **Q11.**  $(7, 2)$ . **Q12.**  $(-2, 3)$ . **Q13.**  $(-4, 1)$ . **Q14.**  $(-1, 1), (-4, 1), (-4, 4), (-1, 4)$ . **Q15.** A reflection in the  $x$ -axis. **Q16.**  $90^\circ$  anticlockwise. **Q17.** (a) 6. (b) 2. (c) 2. (d) 0. **Q18.** The regular triangle, square and regular hexagon (their angles  $60^\circ, 90^\circ, 120^\circ$  divide  $360^\circ$  exactly); the regular pentagon cannot ( $108^\circ$  does not divide  $360^\circ$ ).

---

## Fully-worked solutions

**Q1.** (a) “5 left” means  $x$  decreases by 5 and “4 up” means  $y$  increases by 4, so the rule is  $(x, y) \rightarrow (x - 5, y + 4)$ . (b)  $P(3, -2) \rightarrow (3 - 5, -2 + 4) = (-2, 2)$ .

**Q2.** (a) Reflection in the  $x$ -axis changes the sign of  $y$ :  $(4, 3) \rightarrow (4, -3)$ . (b) Reflection in the  $y$ -axis changes the sign of  $x$ :  $(4, 3) \rightarrow (-4, 3)$ .

**Q3.** (a) A  $90^\circ$  anticlockwise rotation about  $O$  uses  $(x, y) \rightarrow (-y, x)$ , so  $(2, 5) \rightarrow (-5, 2)$ . (b) A  $180^\circ$  rotation uses  $(x, y) \rightarrow (-x, -y)$ , so  $(2, 5) \rightarrow (-2, -5)$ .

**Q4.** (a) The point flips from above the  $x$ -axis to the same distance below it, and the shape is a mirror image, so the transformation is a **reflection**. (b)  $A(1, 1)$  and  $B(1, -1)$  are mirror images in the horizontal axis, so the mirror line is the  $x$ -axis.

**Q5.** (a) A square folds onto itself along both diagonals and both midlines: 4 lines. (b) An equilateral triangle folds along the line from each vertex to the opposite midpoint: 3 lines. (c) A general parallelogram has no line of symmetry: 0.

**Q6.** (a) A square maps onto itself every  $90^\circ$ , so order 4. (b) A rectangle maps onto itself every  $180^\circ$ , so order 2. (c) A regular pentagon maps onto itself every  $72^\circ$  (five times in  $360^\circ$ ), so order 5.

**Q7.** “3 right, 5 down” is  $(x, y) \rightarrow (x + 3, y - 5)$ , so  $(-1, 3) \rightarrow (-1 + 3, 3 - 5) = (2, -2)$ .

**Q8.** The rule is  $(x, y) \rightarrow (x + 3, y + 1)$ :  $(0, 0) \rightarrow (3, 1), (2, 0) \rightarrow (5, 1), (0, 3) \rightarrow (3, 4)$ .

**Q9.** Reflection in the  $x$ -axis:  $(x, y) \rightarrow (x, -y)$ , so  $(5, -2) \rightarrow (5, 2)$ .

**Q10.** Reflection in the  $y$ -axis:  $(x, y) \rightarrow (-x, y)$ , so  $(-3, 6) \rightarrow (3, 6)$ .

**Q11.** Reflection in  $y = x$  swaps the coordinates:  $(x, y) \rightarrow (y, x)$ , so  $(2, 7) \rightarrow (7, 2)$ .

**Q12.** A  $90^\circ$  anticlockwise rotation about  $O$  uses  $(x, y) \rightarrow (-y, x)$ , so  $(3, 2) \rightarrow (-2, 3)$ .

**Q13.** A  $180^\circ$  rotation about  $O$  uses  $(x, y) \rightarrow (-x, -y)$ , so  $(4, -1) \rightarrow (-4, 1)$ .

**Q14.** Reflection in the  $y$ -axis changes the sign of each  $x$ -coordinate:  $(1, 1) \rightarrow (-1, 1), (4, 1) \rightarrow (-4, 1), (4, 4) \rightarrow (-4, 4), (1, 4) \rightarrow (-1, 4)$ .

**Q15.** The  $x$ -coordinate is unchanged and the  $y$ -coordinate changes sign ( $(2, 3) \rightarrow (2, -3)$ ), matching  $(x, y) \rightarrow (x, -y)$ . This is a **reflection in the  $x$ -axis**.

**Q16.** Testing a  $90^\circ$  anticlockwise rotation  $(x, y) \rightarrow (-y, x)$  on  $(1, 2)$  gives  $(-2, 1)$ , which matches. So the rotation is  $90^\circ$  **anticlockwise** about the origin.

**Q17.** (a) A regular hexagon has 6 lines of symmetry. (b) A rectangle has 2. (c) A rhombus has 2 (its two diagonals). (d) A scalene triangle has none: 0.

**Q18.** Tiles meet corner-to-corner, and the angles around each meeting point must total  $360^\circ$ . A regular triangle has  $60^\circ$  angles ( $360 \div 60 = 6$  tiles meet), a square has  $90^\circ$  ( $360 \div 90 = 4$ ) and a regular hexagon has  $120^\circ$  ( $360 \div 120 = 3$ ), so each tessellates on its own. A regular pentagon has  $108^\circ$  angles, and  $360 \div 108$  is not a whole number, so pentagons leave gaps and cannot tessellate alone.

### Theory

Angles are everywhere in the built world — in the pitch of a roof, the lean of a ramp, the corner of a road sign. This section uses the **angle properties** that let you find an unknown angle from ones you already know, and to give a **reason** for each step.

#### Naming angles by size.

- An **acute** angle is less than  $90^\circ$ .
- A **right** angle is exactly  $90^\circ$  (a square corner).
- An **obtuse** angle is between  $90^\circ$  and  $180^\circ$ .
- A **straight** angle is exactly  $180^\circ$ .
- A **reflex** angle is between  $180^\circ$  and  $360^\circ$ .

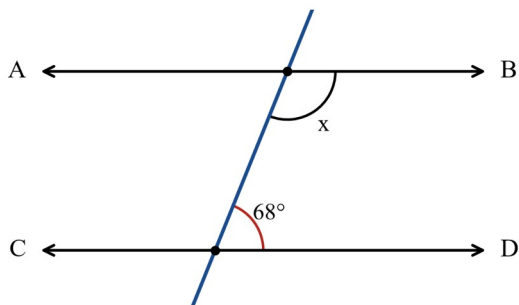
#### Angles that add to a total.

- **Complementary** angles add to  $90^\circ$ .
- **Supplementary** angles add to  $180^\circ$ . Angles on a **straight line** are supplementary.
- Angles **at a point** (all the way around) add to  $360^\circ$ .
- **Vertically opposite** angles — formed where two straight lines cross — are **equal**.

#### Parallel lines and a transversal.

When a **transversal** crosses two **parallel** lines, three useful pairs of angles appear:

- **Corresponding** angles (in matching positions) are **equal**.
- **Alternate** angles (the “Z” shape, between the lines on opposite sides of the transversal) are **equal**.
- **Co-interior** angles (the “C” or “U” shape, between the lines on the same side of the transversal) are **supplementary** — they add to  $180^\circ$ .



In the diagram above,  $AB$  is parallel to  $CD$ . The marked angle of  $68^\circ$  and the angle  $x$  are **co-interior** (both between the lines, same side of the transversal), so  $x = 180^\circ - 68^\circ = 112^\circ$ .

#### Angles in polygons.

- The three angles of a **triangle** add to  $180^\circ$ .
- The four angles of a **quadrilateral** add to  $360^\circ$ .
- The **exterior angle** of a triangle equals the **sum of the two opposite interior angles**.

When you find an unknown angle, always **state the reason** — for example “angles on a straight line” or “co-interior angles,  $AB \parallel CD$ ”. In an exam, the reason earns its own mark.

## Worked examples

### Example 1 — scaffolded

A drafting square is set so that two angles together form a right angle, and a hinged sign opens so that two angles together form a straight line. (a) Find the complement of  $34^\circ$ . (b) Find the supplement of  $145^\circ$ .

Working

Comment

(a) Complement =  $90^\circ - 34^\circ = 56^\circ$ .

Complementary angles add to  $90^\circ$ .

(b) Supplement =  $180^\circ - 145^\circ = 35^\circ$ .

Supplementary angles add to  $180^\circ$ .

### Example 2 — scaffolded

In the parallel-lines diagram in the Theory above,  $AB \parallel CD$  and the transversal makes an angle of  $68^\circ$  with  $CD$ . Find the corresponding angle at  $A$ , and the co-interior angle  $x$ .

Working

Comment

The corresponding angle at  $A$  is also  $68^\circ$ .

Corresponding angles are equal ( $AB \parallel CD$ ).

$x$  and  $68^\circ$  are co-interior angles, so  $x + 68^\circ = 180^\circ$ .

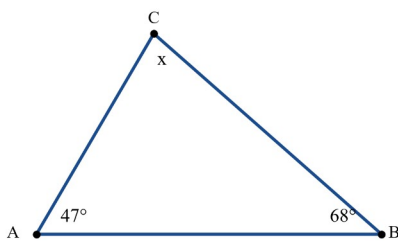
Co-interior angles are supplementary.

$x = 180^\circ - 68^\circ = 112^\circ$ .

Solve for  $x$ .

### Example 3 — unscaffolded

A triangular road sign has two of its angles equal to  $47^\circ$  and  $68^\circ$ . Find the third angle  $x$ .



The three angles of a triangle add to  $180^\circ$ , so

$$47^\circ + 68^\circ + x = 180^\circ.$$

Then  $x = 180^\circ - 47^\circ - 68^\circ = 180^\circ - 115^\circ = 65^\circ$ .

$\therefore x = 65^\circ$ .

### Example 4 — unscaffolded

A support beam  $PQ$  is parallel to the floor  $RS$ . A brace crosses both, and the angle it makes at the floor is  $72^\circ$ . This same  $72^\circ$  appears as one angle of a triangular bracket (they are **alternate** angles,  $PQ \parallel RS$ ). A second angle of the bracket is  $53^\circ$ . Find the third angle  $x$  of the bracket.

Because alternate angles are equal, one angle of the triangle is  $72^\circ$ . The angles of a triangle add to  $180^\circ$ , so

$$x = 180^\circ - 72^\circ - 53^\circ = 180^\circ - 125^\circ = 55^\circ.$$

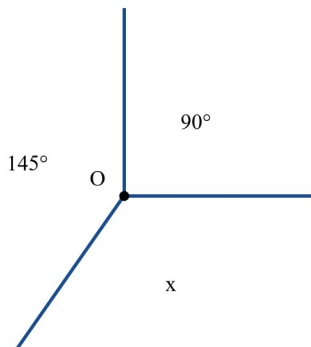
$\therefore x = 55^\circ$ .

## Practice questions

### Scaffolded

**Q1.** Find each angle. (a) The complement of  $32^\circ$ . (b) The supplement of  $105^\circ$ . (c) An angle on a straight line next to an angle of  $43^\circ$ .

**Q2.** Three angles meet at the point  $O$ , as shown.



- (a) State the angle property that relates the three angles.
- (b) Find  $x$ .
- (c) State whether the angle  $x$  is acute or obtuse.

**Q3.** Two straight roads cross, making four angles. One of them is  $68^\circ$ . (a) Find the angle vertically opposite the  $68^\circ$  angle. (b) Find an angle next to the  $68^\circ$  angle (on a straight line). (c) Find the fourth angle.

**Q4.** A transversal crosses two parallel lines, forming eight angles. One of them is  $75^\circ$ . (a) State the size of a corresponding angle. (b) State the size of an alternate angle. (c) State the size of a co-interior angle.

**Q5.** A triangular garden bed has two angles measuring  $52^\circ$  and  $61^\circ$ . (a) Find the third angle  $x$ . (b) Classify the triangle as acute, right or obtuse. (c) State the angle property you used in part (a).

**Q6.** A quadrilateral tile has three angles measuring  $90^\circ$ ,  $115^\circ$  and  $78^\circ$ . (a) State the sum of the angles of a quadrilateral. (b) Find the fourth angle  $x$ . (c) Classify the angle  $x$ .

### Unscaffolded

**Q7.** Find the complement of  $27^\circ$ .

**Q8.** Find the supplement of  $134^\circ$ .

**Q9.** Four angles meet at a point. Three of them are  $130^\circ$ ,  $95^\circ$  and  $80^\circ$ . Find the fourth angle  $x$ .

**Q10.** Two straight lines cross. One angle is  $118^\circ$ . (a) Find the angle vertically opposite it. (b) Find an angle adjacent to it on a straight line.

**Q11.** A transversal crosses two parallel fences. Find the angle corresponding to an angle of  $63^\circ$ .

**Q12.** A transversal crosses two parallel lines. Find the angle alternate to an angle of  $108^\circ$ .

**Q13.** A transversal crosses two parallel lines. Find the co-interior angle to an angle of  $124^\circ$ .

**Q14.** A right-angled triangular bracket has one angle of  $90^\circ$  and another of  $38^\circ$ . Find the third angle  $x$ .

**Q15.** An isosceles triangle has an apex angle of  $40^\circ$  and two equal base angles. Find the size of each base angle.

**Q16.** A triangle has interior angles of  $47^\circ$  and  $88^\circ$ . Find the exterior angle at the third vertex.

**Q17.** A quadrilateral window frame has three angles of  $100^\circ$ ,  $90^\circ$  and  $90^\circ$ . Find the fourth angle  $x$ .

**Q18.** A transversal crosses two parallel lines  $PQ$  and  $RS$ , making a  $118^\circ$  angle. The co-interior angle to it is one angle of a triangle; a second angle of the triangle is  $71^\circ$ . Find the third angle  $x$  of the triangle, giving reasons.

---

## Answers

**Q1.** (a)  $58^\circ$ . (b)  $75^\circ$ . (c)  $137^\circ$ . **Q2.** (a) angles at a point add to  $360^\circ$ . (b)  $x = 125^\circ$ . (c) obtuse. **Q3.** (a)  $68^\circ$ . (b)  $112^\circ$ . (c)  $112^\circ$ . **Q4.** (a)  $75^\circ$ . (b)  $75^\circ$ . (c)  $105^\circ$ . **Q5.** (a)  $67^\circ$ . (b) acute. (c) the angles of a triangle add to  $180^\circ$ . **Q6.** (a)  $360^\circ$ . (b)  $77^\circ$ . (c) acute. **Q7.**  $63^\circ$ . **Q8.**  $46^\circ$ . **Q9.**  $55^\circ$ . **Q10.** (a)  $118^\circ$ . (b)  $62^\circ$ . **Q11.**  $63^\circ$ . **Q12.**  $108^\circ$ . **Q13.**  $56^\circ$ . **Q14.**  $52^\circ$ . **Q15.**  $70^\circ$  each. **Q16.**  $135^\circ$ . **Q17.**  $80^\circ$ . **Q18.**  $47^\circ$ .

---

## Fully-worked solutions

**Q1.** (a) Complement  $= 90^\circ - 32^\circ = 58^\circ$ . (b) Supplement  $= 180^\circ - 105^\circ = 75^\circ$ . (c) Angles on a straight line are supplementary:  $180^\circ - 43^\circ = 137^\circ$ .

**Q2.** (a) The three angles meet at a point, and angles at a point add to  $360^\circ$ . (b)  $90^\circ + 145^\circ + x = 360^\circ$ , so  $x = 360^\circ - 235^\circ = 125^\circ$ . (c) Since  $125^\circ$  is between  $90^\circ$  and  $180^\circ$ , it is **obtuse**.

**Q3.** (a) Vertically opposite angles are equal, so the angle opposite  $68^\circ$  is  $68^\circ$ . (b) An adjacent angle is on a straight line with  $68^\circ$ , so it is  $180^\circ - 68^\circ = 112^\circ$ . (c) The fourth angle is vertically opposite the  $112^\circ$  angle, so it is also  $112^\circ$ .

**Q4.** (a) Corresponding angles are equal, so a corresponding angle is  $75^\circ$ . (b) Alternate angles are equal, so an alternate angle is  $75^\circ$ . (c) Co-interior angles are supplementary, so a co-interior angle is  $180^\circ - 75^\circ = 105^\circ$ .

**Q5.** (a) The angles of a triangle add to  $180^\circ$ :  $x = 180^\circ - 52^\circ - 61^\circ = 67^\circ$ . (b) All three angles ( $52^\circ$ ,  $61^\circ$ ,  $67^\circ$ ) are less than  $90^\circ$ , so the triangle is **acute**. (c) The angle sum of a triangle is  $180^\circ$ .

**Q6.** (a) The angles of a quadrilateral add to  $360^\circ$ . (b)  $x = 360^\circ - 90^\circ - 115^\circ - 78^\circ = 77^\circ$ . (c)  $77^\circ$  is less than  $90^\circ$ , so it is **acute**.

**Q7.**  $90^\circ - 27^\circ = 63^\circ$ .

**Q8.**  $180^\circ - 134^\circ = 46^\circ$ .

**Q9.** Angles at a point add to  $360^\circ$ :  $x = 360^\circ - 130^\circ - 95^\circ - 80^\circ = 55^\circ$ .

**Q10.** (a) Vertically opposite angles are equal, so the opposite angle is  $118^\circ$ . (b) An adjacent angle on a straight line is  $180^\circ - 118^\circ = 62^\circ$ .

**Q11.** Corresponding angles are equal, so the corresponding angle is  $63^\circ$ .

**Q12.** Alternate angles are equal, so the alternate angle is  $108^\circ$ .

**Q13.** Co-interior angles are supplementary, so the co-interior angle is  $180^\circ - 124^\circ = 56^\circ$ .

**Q14.** The angles of a triangle add to  $180^\circ$ :  $x = 180^\circ - 90^\circ - 38^\circ = 52^\circ$ .

**Q15.** Let each base angle be  $x$ . Then  $x + x + 40^\circ = 180^\circ$ , so  $2x = 140^\circ$  and  $x = 70^\circ$ . Each base angle is  $70^\circ$ .

**Q16.** The exterior angle of a triangle equals the sum of the two opposite interior angles:  $47^\circ + 88^\circ = 135^\circ$ .

**Q17.** The angles of a quadrilateral add to  $360^\circ$ :  $x = 360^\circ - 100^\circ - 90^\circ - 90^\circ = 80^\circ$ .

**Q18.** The co-interior angle to  $118^\circ$  is  $180^\circ - 118^\circ = 62^\circ$  (co-interior angles are supplementary,  $PQ \parallel RS$ ). This  $62^\circ$  is one angle of the triangle, so with the second angle of  $71^\circ$ ,

$$x = 180^\circ - 62^\circ - 71^\circ = 47^\circ$$

(the angles of a triangle add to  $180^\circ$ ).



## Chapter 12 Shape, geometry and scale — 12F Maps and scale

### Theory

A map shows a large area — a suburb, a national park, a whole country — shrunk down to fit on a page. To use a map for real calculations you need two things: its **scale** (how much the real world has been shrunk) and a way to describe **direction**.

### Map scales.

A scale tells you how a distance on the map relates to the real distance on the ground. It is written in one of two ways:

- a **ratio scale** such as  $1:50\,000$  — one unit of length on the map represents 50 000 of the **same** units in reality (so 1 cm on the map is 50 000 cm on the ground);
- a **statement scale** such as “1 cm represents 5 km” — said in words, often easier to use directly.

### Turning a ratio into a “1 cm represents ...” statement.

With a ratio scale, convert the real distance into sensible units. For  $1:100\,000$ ,

$$1 \text{ cm on the map} = 100\,000 \text{ cm} = 1000 \text{ m} = 1 \text{ km}.$$

So  $1:100\,000$  is the same as “1 cm represents 1 km”. In general, for a scale  $1:n$ , one centimetre represents  $\frac{n}{100\,000}$  km (because  $100\,000 \text{ cm} = 1 \text{ km}$ ).

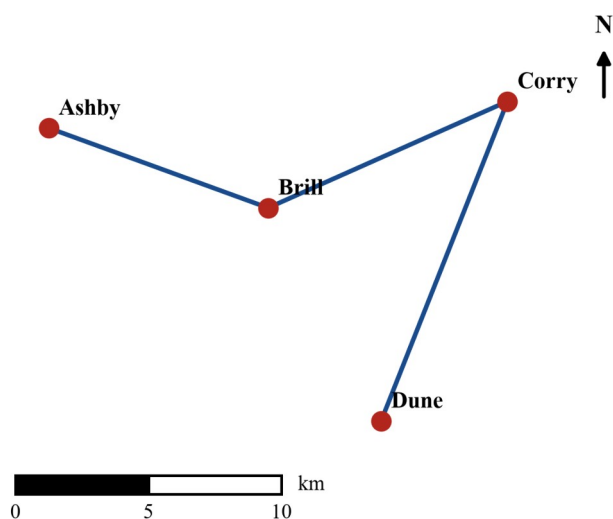
### Converting between map and real distances.

- **Map** → **real**: multiply the map distance by what 1 cm (or 1 unit) represents.
- **Real** → **map**: divide the real distance by what 1 cm represents.

Keep the units consistent, and state the units in every answer.

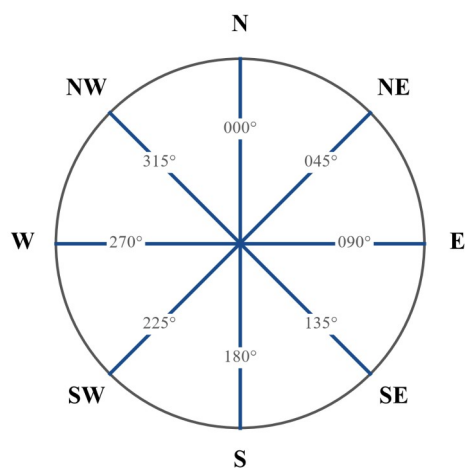
### Scale bars.

Many maps also show a **scale bar** — a small ruled bar marked with real distances (for example 0, 5, 10 km). You read off how much one centimetre represents, or lay the bar against a distance to measure it directly.

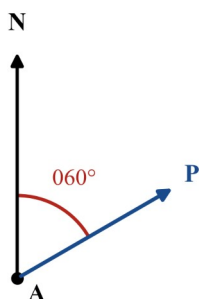


### Direction: compass points and bearings.

Directions are given from the **compass**: the four main points N, E, S, W and the in-between points NE, SE, SW, NW. For a more precise direction we use a **three-figure (compass) bearing** — the angle, measured **clockwise from north**, written with **three digits** and a degree symbol.



North is  $000^\circ$ , east is  $090^\circ$ , south is  $180^\circ$  and west is  $270^\circ$ ; the in-between directions are  $045^\circ$  (NE),  $135^\circ$  (SE),  $225^\circ$  (SW) and  $315^\circ$  (NW). A bearing is always written with three digits, so a bearing of sixty degrees is written  $060^\circ$ .



**Back bearings.** The bearing of B from A and the bearing of A from B differ by  $180^\circ$ : to reverse a bearing, add  $180^\circ$  (or subtract  $180^\circ$  if the result would exceed  $360^\circ$ ). For example, if B is due east of A ( $090^\circ$ ), then A is due west of B ( $090^\circ + 180^\circ = 270^\circ$ ).

## Worked examples

### Example 1 — scaffolded

A road map has a scale of  $1:50\,000$ . Two towns are  $8\text{ cm}$  apart on the map. Find the real distance between them, in kilometres.

Working

$1\text{ cm}$  represents  $50\,000\text{ cm} = 500\text{ m} = 0.5\text{ km}$ .

Real distance  $= 8 \times 0.5\text{ km}$ .

$= 4\text{ km}$ .

Comment

Convert the ratio to a “ $1\text{ cm}$  represents ...” statement ( $50\,000\text{ cm} = 0.5\text{ km}$ ).

Map  $\rightarrow$  real: multiply the map distance by what  $1\text{ cm}$  represents.

State the answer with units.

### Example 2 — scaffolded

A map uses the scale “1 cm represents 25 km”. A highway is 340 km long. How long is it on the map?

Working

1 cm represents 25 km.

Map length =  $340 \div 25$ .

= 13.6 cm.

Comment

The statement scale can be used directly.

Real  $\rightarrow$  map: divide the real distance by what 1 cm represents.

The highway is 13.6 cm long on the map.

### Example 3 — unscaffolded

A hiking map has a scale of 1 : 25 000. A walking track measures 14 cm along its path on the map. Find the real length of the track, and the time to walk it at 4 km/h.

For 1 : 25 000, 1 cm represents 25 000 cm = 250 m = 0.25 km. The real length is

$$14 \times 0.25 = 3.5 \text{ km}.$$

Walking at 4 km/h, the time is  $\frac{3.5}{4} = 0.875 \text{ h} = 0.875 \times 60 = 52.5 \text{ minutes}$ .

$\therefore$  the track is 3.5 km long and takes about 52.5 minutes to walk.

### Example 4 — unscaffolded

A lighthouse L is directly **north-east** of a boat B. (a) Write the direction of L from B as a three-figure bearing. (b) Find the bearing of B from L.

North-east is halfway between north ( $000^\circ$ ) and east ( $090^\circ$ ), so the bearing of L from B is  $045^\circ$ . To find the bearing of B from L, add  $180^\circ$ :

$$045^\circ + 180^\circ = 225^\circ.$$

$\therefore$  L is on a bearing of  $045^\circ$  from B, and B is on a bearing of  $225^\circ$  from L.

## Practice questions

### Scaffolded

**Q1.** A map has a scale of 1 : 100 000. (a) What real distance (in km) does 1 cm on the map represent? (b) Two places are 6 cm apart on the map. Find the real distance between them. (c) A lake is 15 km long. How long is it on the map?

**Q2.** A map uses the scale “1 cm represents 20 km”. (a) A river is 4.5 cm long on the map. Find its real length. (b) A road is 250 km long. How long is it on the map? (c) In your own words, explain what the scale “1 cm represents 20 km” means.

**Q3.** A street map has a scale of 1 : 25 000. (a) What distance does 1 cm represent, in metres and in kilometres? (b) A park is 8 cm across on the map. Find its real width in kilometres. (c) A bike path is 2 km long. How long is it on the map?

**Q4.** Write each compass direction as a three-figure bearing. (a) north-east (b) south (c) west

**Q5.** A map’s scale bar shows that 2 cm represents 10 km. (a) What distance does 1 cm represent? (b) A coastline measures 7 cm on the map. Find its real length. (c) A real distance of 20 km — how long is it on the map?

**Q6.** On a map with scale 1 : 50 000, a walker’s route runs 6 cm to a hut, then 10 cm to a lookout. (a) Find the real length of each leg in kilometres. (b) Find the total real distance walked. (c) At a steady 5 km/h, how long does the walk take?

## Un scaffolded

- Q7.** A map has a scale of 1 : 200 000. Two towns are 7.5 cm apart on the map. Find the real distance between them.
- Q8.** A map uses the scale “1 cm represents 8 km”. A railway line is 100 km long. Find its length on the map.
- Q9.** A map’s scale bar shows that 4 cm represents 20 km. A river measures 13 cm along its course on the map. Find the real length of the river.
- Q10.** A map has a scale of 1 : 500 000. What real distance, in kilometres, does 1 cm on the map represent?
- Q11.** On a map with scale 1 : 100 000, a cycling route has three straight legs of 4 cm, 9 cm and 5 cm. Find the total real distance of the route.
- Q12.** A control tower T is due **east** of an airport A, and a hangar H is to the **south-west** of A. Write the bearing of (a) T from A and (b) H from A.
- Q13.** A plane leaves an airport flying to the **north-west**. Write its direction as a three-figure bearing.
- Q14.** A road is 6 cm long on a map with scale 1 : 250 000. (a) Find the real length of the road. (b) At an average speed of 80 km/h, how many minutes does it take to drive?
- Q15.** A fence on a farm is 1.5 km long. On a plan with scale 1 : 25 000, how long is the fence?
- Q16.** Two survey pegs are 5.5 cm apart on a plan with scale 1 : 40 000. Find the real distance between them, in metres and in kilometres.
- Q17.** A buoy B is on a bearing of  $090^\circ$  from a jetty A (that is, due east). Find the bearing of the jetty A from the buoy B.
- Q18.** A bushwalker’s map has a scale of 1 : 50 000. From her camp she walks a route that is 8 cm north, then 6 cm east, on the map. (a) Find the real distance of each leg, in kilometres. (b) Find the total distance she actually walked. (c) Using Pythagoras’ theorem, find her straight-line (“as the crow flies”) distance from the camp at the end.
-

## Answers

**Q1.** (a) 1 km. (b) 6 km. (c) 15 cm. **Q2.** (a) 90 km. (b) 12.5 cm. (c) every 1 cm measured on the map stands for 20 km on the ground. **Q3.** (a) 250 m = 0.25 km. (b) 2 km. (c) 8 cm. **Q4.** (a)  $045^\circ$ . (b)  $180^\circ$ . (c)  $270^\circ$ . **Q5.** (a) 5 km. (b) 35 km. (c) 4 cm. **Q6.** (a) 3 km and 5 km. (b) 8 km. (c)  $1.6 \text{ h} = 1 \text{ h } 36 \text{ min}$ . **Q7.** 15 km. **Q8.** 12.5 cm. **Q9.** 65 km. **Q10.** 5 km. **Q11.** 18 km. **Q12.** (a)  $090^\circ$ . (b)  $225^\circ$ . **Q13.**  $315^\circ$ . **Q14.** (a) 15 km. (b) 11.25 min. **Q15.** 6 cm. **Q16.** 2200 m = 2.2 km. **Q17.**  $270^\circ$  (due west). **Q18.** (a) 4 km north and 3 km east. (b) 7 km. (c) 5 km.

---

## Fully-worked solutions

**Q1.** For 1 : 100 000, 1 cm represents 100 000 cm = 1000 m = 1 km. (a) 1 km. (b)  $6 \times 1 = 6 \text{ km}$ . (c)  $15 \div 1 = 15 \text{ cm}$ .

**Q2.** The scale is “1 cm represents 20 km”. (a)  $4.5 \times 20 = 90 \text{ km}$ . (b)  $250 \div 20 = 12.5 \text{ cm}$ . (c) It means that each 1 cm of length measured on the map corresponds to 20 km of real distance on the ground.

**Q3.** For 1 : 25 000, 1 cm represents 25 000 cm. (a) 25 000 cm = 250 m = 0.25 km. (b)  $8 \times 0.25 = 2 \text{ km}$ . (c)  $2 \div 0.25 = 8 \text{ cm}$ .

**Q4.** Bearings are measured clockwise from north, with three digits. (a) North-east is halfway between  $000^\circ$  and  $090^\circ$  :  $045^\circ$ . (b) South is  $180^\circ$ . (c) West is  $270^\circ$ .

**Q5.** If 2 cm represents 10 km, then 1 cm represents  $10 \div 2 = 5 \text{ km}$ . (a) 5 km. (b)  $7 \times 5 = 35 \text{ km}$ . (c)  $20 \div 5 = 4 \text{ cm}$ .

**Q6.** For 1 : 50 000, 1 cm represents 0.5 km. (a)  $6 \times 0.5 = 3 \text{ km}$  and  $10 \times 0.5 = 5 \text{ km}$ . (b)  $3 + 5 = 8 \text{ km}$ . (c) Time =  $\frac{8}{5} = 1.6 \text{ h}$ . Since  $0.6 \text{ h} = 0.6 \times 60 = 36 \text{ min}$ , this is 1 h 36 min.

**Q7.** For 1 : 200 000, 1 cm represents 200 000 cm = 2 km. Real distance =  $7.5 \times 2 = 15 \text{ km}$ .

**Q8.** Real  $\rightarrow$  map:  $100 \div 8 = 12.5 \text{ cm}$ .

**Q9.** The scale bar gives 4 cm = 20 km, so 1 cm =  $20 \div 4 = 5 \text{ km}$ . The river is  $13 \times 5 = 65 \text{ km}$ .

**Q10.** For 1 : 500 000, 1 cm represents 500 000 cm = 5000 m = 5 km.

**Q11.** For 1 : 100 000, 1 cm represents 1 km. Total map distance =  $4 + 9 + 5 = 18 \text{ cm}$ , so the real distance is  $18 \times 1 = 18 \text{ km}$ .

**Q12.** (a) Due east is  $090^\circ$ . (b) South-west is halfway between south ( $180^\circ$ ) and west ( $270^\circ$ ), so  $225^\circ$ .

**Q13.** North-west is halfway between west ( $270^\circ$ ) and north ( $360^\circ$ ), i.e.  $315^\circ$ .

**Q14.** For 1 : 250 000, 1 cm represents 250 000 cm = 2.5 km. (a)  $6 \times 2.5 = 15 \text{ km}$ . (b) Time =  $\frac{15}{80} \text{ h} = \frac{15}{80} \times 60 = 11.25 \text{ min}$ .

**Q15.** For 1 : 25 000, 1 cm represents 0.25 km. Map length =  $1.5 \div 0.25 = 6 \text{ cm}$ .

**Q16.** For 1 : 40 000, 1 cm represents 40 000 cm = 400 m. Real distance =  $5.5 \times 400 = 2200 \text{ m} = 2.2 \text{ km}$ .

**Q17.** The bearing of A from B is the reverse of the bearing of B from A. Add  $180^\circ$ :  $090^\circ + 180^\circ = 270^\circ$ , which is due west.

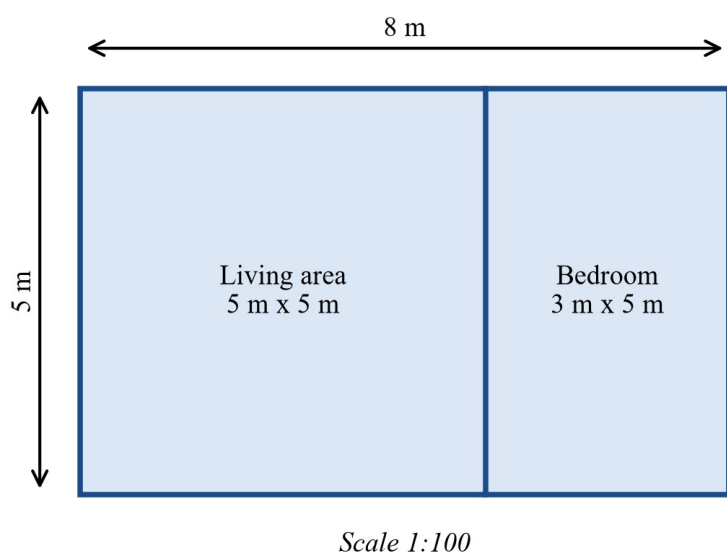
**Q18.** For 1 : 50 000, 1 cm represents 0.5 km. (a) North leg:  $8 \times 0.5 = 4 \text{ km}$ . East leg:  $6 \times 0.5 = 3 \text{ km}$ . (b) Total distance walked =  $4 + 3 = 7 \text{ km}$ . (c) The two legs are at right angles, so by Pythagoras' theorem the straight-line distance is  $\sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5 \text{ km}$ .

### Theory

Builders, landscapers and designers work from **scale plans** — accurate drawings in which every length is shrunk by the same factor so that a whole house or block fits on a page. To use a plan you must be able to convert between a length **on the plan** and the matching **real** length, and to work out real areas and quantities of materials.

#### What a scale means.

A scale can be written as a **ratio**, such as **1 : 100**, or as a **statement**, such as **1 cm represents 1 m**. A scale of 1 : 100 means that *1 unit on the plan represents 100 of the same units in real life* — so 1 cm on the plan is 100 cm = 1 m in reality. The two forms describe the same plan.



#### Converting between plan and real lengths.

For a scale of 1 :  $n$ :

- **real length** = **plan length**  $\times n$  (the real thing is bigger, so multiply);
- **plan length** = **real length**  $\div n$  (the drawing is smaller, so divide).

Keep both lengths in the **same unit** while you multiply or divide, then convert to a sensible unit at the end. For example, on a 1 : 50 plan a wall drawn 8 cm long is  $8 \times 50 = 400$  cm = 4 m in reality.

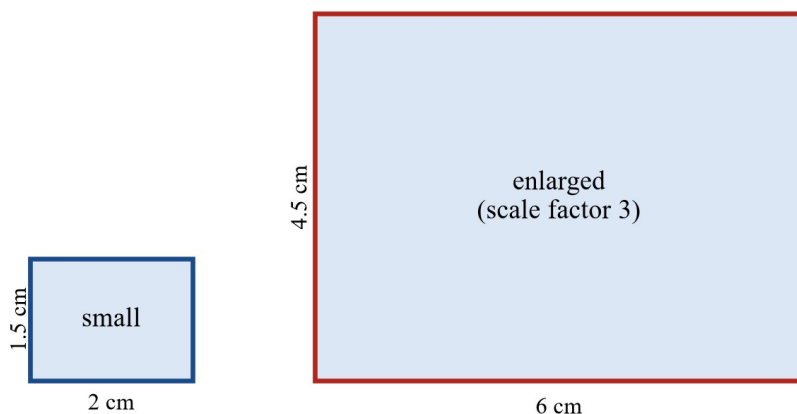
#### Real areas from a plan.

To find a real **area**, first convert the plan lengths to real lengths, then multiply. Do **not** just multiply the plan area by the scale — because area has two dimensions, a 1 :  $n$  scale makes a real area  $n^2$  times the plan area. It is safest to convert each length first, then find the area, and to state the answer in  $\text{m}^2$ .

#### Enlargement, reduction and scale factor.

Redrawing a shape larger or smaller multiplies every length by a **scale factor**  $k$ . An **enlargement** has  $k > 1$ ; a **reduction** has  $k < 1$  (for example  $k = 1/2$  halves each length). The scale factor affects lengths and areas differently:

- every **length** is multiplied by  $k$ ;
- every **area** is multiplied by  $k^2$ .



So enlarging a shape by a scale factor of 3 makes it 3 times as *wide* but  $3^2 = 9$  times as large in *area*.

### Similar figures.

Two figures are **similar** if one is an enlargement or reduction of the other — they have the same shape, and their matching (corresponding) sides are all in the same ratio, the scale factor. To find a missing length in a pair of similar figures, first find the scale factor  $k$  from a pair of known matching sides, then multiply (or divide) the other side by  $k$ .

## Worked examples

### Example 1 — scaffolded

A floor plan is drawn to a scale of 1 : 50. A rectangular bedroom measures 8 cm by 6 cm on the plan. Find the real length, the real width and the real floor area of the bedroom.

Working

Comment

Real length =  $8 \times 50 = 400$  cm = 4 m.

Real length = plan length  $\times n$ , with  $n = 50$ .

Real width =  $6 \times 50 = 300$  cm = 3 m.

Same conversion for the width.

Real area =  $4 \times 3 = 12$  m<sup>2</sup>.

Convert lengths first, then multiply — area in m<sup>2</sup>.

### Example 2 — scaffolded

A house plan uses the scale 1 cm represents 0.5 m (that is, 1 : 50). (a) A wall is 7 m long in reality. How long is it on the plan? (b) A room measures 9 cm on the plan. How long is it in reality?

Working

Comment

(a) Plan length =  $7 \div 0.5 = 14$  cm.

Plan length = real length  $\div$  (metres per cm).

(b) Real length =  $9 \times 0.5 = 4.5$  m.

Real length = plan length  $\times$  (metres per cm).

### Example 3 — unscaffolded

A company logo is 5 cm wide and has an area of 12 cm<sup>2</sup>. It is enlarged by a scale factor of 3 for a shopfront sign. Find the width and the area of the enlarged logo.

Every length is multiplied by the scale factor, so the enlarged width is

$$5 \times 3 = 15 \text{ cm.}$$

Area is multiplied by the *square* of the scale factor, so the enlarged area is

$$12 \times 3^2 = 12 \times 9 = 108 \text{ cm}^2.$$

$\therefore$  the enlarged logo is 15 cm wide with an area of 108 cm<sup>2</sup>.

### Example 4 — unscaffolded

A living room is drawn on a 1 : 100 plan as a rectangle 5 cm by 4 cm. Carpet costs \$ 45 per square metre. Find the cost of carpeting the room.

Converting each plan length to a real length,

$$5 \times 100 = 500 \text{ cm} = 5 \text{ m}, 4 \times 100 = 400 \text{ cm} = 4 \text{ m}.$$

The real floor area is  $5 \times 4 = 20 \text{ m}^2$ , so the carpet cost is

$$20 \times \$ 45 = \$ 900.$$

$\therefore$  carpeting the room costs \$ 900.

---

### Practice questions

#### Scaffolded

**Q1.** A floor plan is drawn to a scale of 1 : 50. A kitchen measures 10 cm by 7 cm on the plan. (a) Find the real length. (b) Find the real width. (c) Find the real floor area.

**Q2.** A plan uses the scale 1 cm represents 2 m. (a) A fence is 16 m long. How long is it on the plan? (b) A path is 5 cm long on the plan. How long is it in reality? (c) A driveway is 12 cm long on the plan. How long is it in reality?

**Q3.** A shape is redrawn using a scale factor of 4. (a) A side that was 3 cm becomes how long? (b) An area that was 6  $\text{cm}^2$  becomes how large? (c) A different shape is reduced by a scale factor of  $1/2$ . A 20 cm length becomes how long?

**Q4.** A block of land is drawn on a 1 : 200 plan as a rectangle 6 cm by 4.5 cm. (a) Find the real length and width. (b) Find the real area of the block.

**Q5.** A small rectangle 4 cm by 3 cm is enlarged so that the 4 cm side becomes 10 cm. (a) Find the scale factor. (b) Find the length of the other side after the enlargement.

**Q6.** A lounge room is drawn on a 1 : 100 plan as a rectangle 5.5 cm by 4 cm. Floorboards cost \$ 38 per square metre. (a) Find the real length and width. (b) Find the real floor area. (c) Find the cost of the floorboards.

#### Unscaffolded

**Q7.** On a 1 : 50 plan a study measures 9 cm by 8 cm. Find its real floor area.

**Q8.** A plan uses the scale 1 cm represents 0.25 m. A wall is 6 m long in reality. How long is it on the plan?

**Q9.** A garden is drawn on a 1 : 150 plan as a rectangle 8 cm by 5 cm. Find its real dimensions and its real perimeter.

**Q10.** A model car is built to a scale of 1 : 20. The real car is 4.6 m long. Find the length of the model, in centimetres.

**Q11.** A rectangle has an area of 24  $\text{cm}^2$ . It is enlarged by a scale factor of 3. Find the area of the enlarged rectangle.

**Q12.** A bedroom is drawn on a 1 : 100 plan as a rectangle 4.5 cm by 3 cm. Tiles cost \$ 55 per square metre. Find the cost of tiling the bedroom floor.

**Q13.** Two triangles are similar. The first has sides 5 cm, 12 cm and 13 cm. In the second, the side matching the 5 cm side is 15 cm. Find the other two sides of the second triangle.

**Q14.** A wardrobe is 3 m wide in reality. On a 1 : 50 plan, how wide is it drawn?

**Q15.** On a 1 : 200 plan a rectangular house floor measures 7 cm by 6 cm. Find the real floor area.

**Q16.** A photo 10 cm wide is enlarged so that it is 25 cm wide. The original photo has an area of 60  $\text{cm}^2$ . (a) Find the scale factor of the enlargement. (b) Find the area of the enlarged photo.

**Q17.** A plan drawn to a scale of 1:50 shows an L-shaped room: a 10 cm by 6 cm rectangle with a 4 cm by 3 cm rectangle removed from one corner. Find the real floor area of the room.

**Q18.** A rectangular paddock is drawn on a 1:500 plan as 5 cm by 4 cm. Fencing costs \$25 per metre. (a) Find the real length and width of the paddock. (b) Find the real perimeter. (c) Find the cost of fencing the paddock.

---

## Answers

**Q1.** (a) 5 m. (b) 3.5 m. (c) 17.5 m<sup>2</sup>. **Q2.** (a) 8 cm. (b) 10 m. (c) 24 m. **Q3.** (a) 12 cm. (b) 96 cm<sup>2</sup>. (c) 10 cm. **Q4.** (a) 12 m by 9 m. (b) 108 m<sup>2</sup>. **Q5.** (a) 2.5. (b) 7.5 cm. **Q6.** (a) 5.5 m by 4 m. (b) 22 m<sup>2</sup>. (c) \$836. **Q7.** 18 m<sup>2</sup>. **Q8.** 24 cm. **Q9.** 12 m by 7.5 m; perimeter 39 m. **Q10.** 23 cm. **Q11.** 216 cm<sup>2</sup>. **Q12.** \$742.50. **Q13.** 36 cm and 39 cm. **Q14.** 6 cm. **Q15.** 168 m<sup>2</sup>. **Q16.** (a) 2.5. (b) 375 cm<sup>2</sup>. **Q17.** 12 m<sup>2</sup>. **Q18.** (a) 25 m by 20 m. (b) 90 m. (c) \$2250.

---

## Fully-worked solutions

**Q1.** With  $n=50$ : (a) real length =  $10 \times 50 = 500$  cm = 5 m. (b) real width =  $7 \times 50 = 350$  cm = 3.5 m. (c) real area =  $5 \times 3.5 = 17.5$  m<sup>2</sup>.

**Q2.** Here 1 cm = 2 m. (a) plan length =  $16 \div 2 = 8$  cm. (b) real length =  $5 \times 2 = 10$  m. (c) real length =  $12 \times 2 = 24$  m.

**Q3.** (a) Length  $\times k = 3 \times 4 = 12$  cm. (b) Area  $\times k^2 = 6 \times 4^2 = 6 \times 16 = 96$  cm<sup>2</sup>. (c) With  $k = 1/2$ ,  $20 \times 1/2 = 10$  cm.

**Q4.** With  $n=200$ : (a) real length =  $6 \times 200 = 1200$  cm = 12 m; real width =  $4.5 \times 200 = 900$  cm = 9 m. (b) real area =  $12 \times 9 = 108$  m<sup>2</sup>.

**Q5.** (a) The scale factor is  $k = \frac{10}{4} = 2.5$ . (b) The other side becomes  $3 \times 2.5 = 7.5$  cm.

**Q6.** With  $n=100$ : (a) real length =  $5.5 \times 100 = 550$  cm = 5.5 m; real width =  $4 \times 100 = 400$  cm = 4 m. (b) real area =  $5.5 \times 4 = 22$  m<sup>2</sup>. (c) cost =  $22 \times \$38 = \$836$ .

**Q7.** Real length =  $9 \times 50 = 450$  cm = 4.5 m; real width =  $8 \times 50 = 400$  cm = 4 m. Real area =  $4.5 \times 4 = 18$  m<sup>2</sup>.

**Q8.** Here 1 cm = 0.25 m, so plan length =  $6 \div 0.25 = 24$  cm.

**Q9.** With  $n=150$ : real length =  $8 \times 150 = 1200$  cm = 12 m; real width =  $5 \times 150 = 750$  cm = 7.5 m. Perimeter =  $2 \times (12 + 7.5) = 39$  m.

**Q10.** With a 1:20 scale, model length = real length  $\div 20 = 4.6 \div 20 = 0.23$  m = 23 cm.

**Q11.** Area is multiplied by  $k^2$ :  $24 \times 3^2 = 24 \times 9 = 216$  cm<sup>2</sup>.

**Q12.** With  $n=100$ : real length =  $4.5 \times 100 = 450$  cm = 4.5 m; real width =  $3 \times 100 = 300$  cm = 3 m. Real area =  $4.5 \times 3 = 13.5$  m<sup>2</sup>, so cost =  $13.5 \times \$55 = \$742.50$ .

**Q13.** The scale factor is  $k = \frac{15}{5} = 3$ . The other sides are  $12 \times 3 = 36$  cm and  $13 \times 3 = 39$  cm.

**Q14.** 3 m = 300 cm. On a 1:50 plan, plan width =  $300 \div 50 = 6$  cm.

**Q15.** With  $n=200$ : real length =  $7 \times 200 = 1400$  cm = 14 m; real width =  $6 \times 200 = 1200$  cm = 12 m. Real area =  $14 \times 12 = 168$  m<sup>2</sup>.

**Q16.** (a) Scale factor  $k = \frac{25}{10} = 2.5$ . (b) Area is multiplied by  $k^2$ :  $60 \times 2.5^2 = 60 \times 6.25 = 375$  cm<sup>2</sup>.

**Q17.** Convert the plan lengths ( $n=50$ ): the large rectangle is  $10 \times 50 = 500$  cm = 5 m by  $6 \times 50 = 300$  cm = 3 m, with area  $5 \times 3 = 15$  m<sup>2</sup>. The removed corner is  $4 \times 50 = 200$  cm = 2 m by  $3 \times 50 = 150$  cm = 1.5 m, with area  $2 \times 1.5 = 3$  m<sup>2</sup>. The real floor area is  $15 - 3 = 12$  m<sup>2</sup>.

**Q18.** With  $n=500$ : (a) real length =  $5 \times 500 = 2500$  cm = 25 m; real width =  $4 \times 500 = 2000$  cm = 20 m. (b) perimeter =  $2 \times (25 + 20) = 90$  m. (c) cost =  $90 \times \$25 = \$2250$ .

### Theory

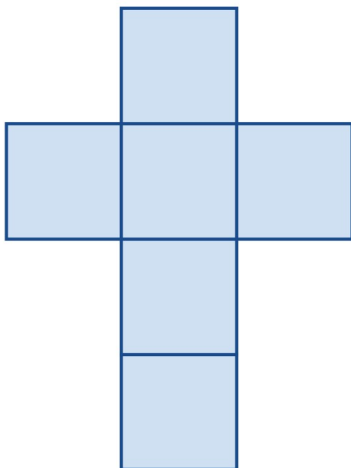
Every 3D object has a 2D “skin”. Being able to move between the solid and its flat representations lets you design packaging, read construction drawings, and work out how much material a surface needs.

#### Nets.

A **net** is a flat pattern of joined 2D shapes that folds up to make a 3D solid. Each face of the solid appears once in the net. For example:

- a **cube** folds from **6 squares**;
- a **rectangular prism** folds from **6 rectangles** (three matching pairs);
- a **triangular prism** folds from **2 triangles and 3 rectangles**;
- a **square pyramid** folds from **1 square and 4 triangles**;
- a **cylinder** folds from **2 circles and 1 rectangle** (the rectangle wraps around, so its length equals the circle’s circumference);
- a **cone** folds from **1 circle and 1 sector**.

Net of a cube (6 squares)



Not every arrangement of the right shapes folds up correctly. Six squares only make a cube if they are arranged so that no two would overlap when folded — a straight strip of six squares in a row, for instance, will **not** fold into a cube.

#### Surface area from a net.

Because a net shows every face laid flat, the **surface area** of a solid is simply the **sum of the areas of all the faces in its net**:

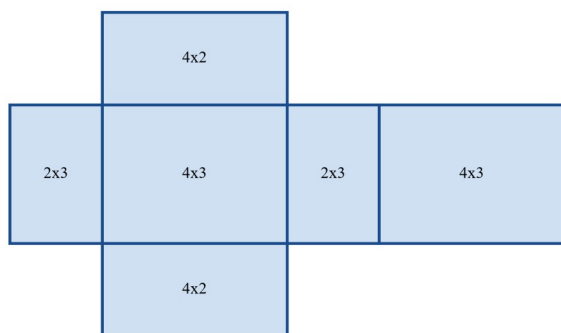
$$\text{surface area} = \text{sum of the areas of all faces.}$$

This is often the easiest way to find a surface area — draw or picture the net, work out each face, and add them up. (Area formulas are on the Foundation Mathematics formula sheet.)

For a **rectangular prism** with length  $l$ , width  $w$  and height  $h$  there are three pairs of matching faces, so

$$\text{surface area} = 2(lw + lh + wh).$$

Net of a rectangular prism (4 x 2 x 3)

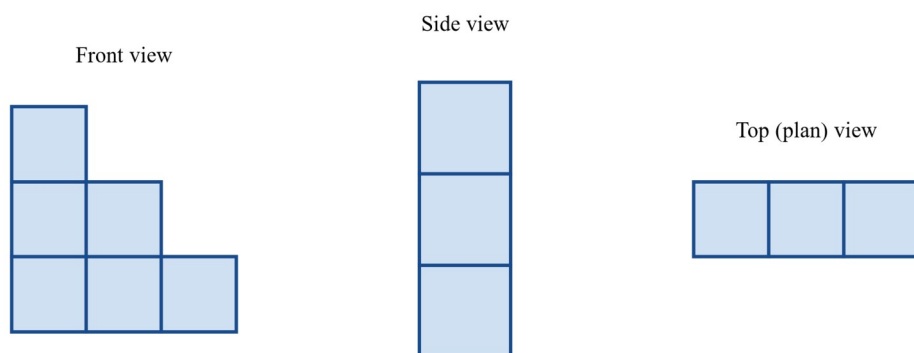


## Plans and elevations.

A solid can also be described by three 2D views:

- the **front view** (front elevation) — what you see looking straight at the front;
- the **side view** (side elevation) — looking from the side;
- the **top view** or **plan** — looking straight down from above.

These views are drawn to scale as flat shapes, with no perspective. Engineers and builders use them because three simple 2D drawings pin down a 3D object exactly. The three views below describe a “staircase” made of unit cubes (columns 3, 2 and 1 cubes high, one cube deep):



You can work **backwards** too: given the front, side and top views, you can build or picture the solid they describe.

## Worked examples

### Example 1 — scaffolded

A closed box (rectangular prism) is 8 cm long, 5 cm wide and 3 cm high. Use its net to find the surface area.

#### Working

The box has three pairs of faces: two  $8 \times 5$ , two  $8 \times 3$  and two  $5 \times 3$ .

Two  $8 \times 5$  faces:  $2 \times 40 = 80 \text{ cm}^2$ .

Two  $8 \times 3$  faces:  $2 \times 24 = 48 \text{ cm}^2$ .

Two  $5 \times 3$  faces:  $2 \times 15 = 30 \text{ cm}^2$ .

Surface area =  $80 + 48 + 30 = 158 \text{ cm}^2$ .

#### Comment

Every rectangular prism has three matching pairs.

Top and bottom.

Front and back.

The two ends.

Add the areas of all six faces.

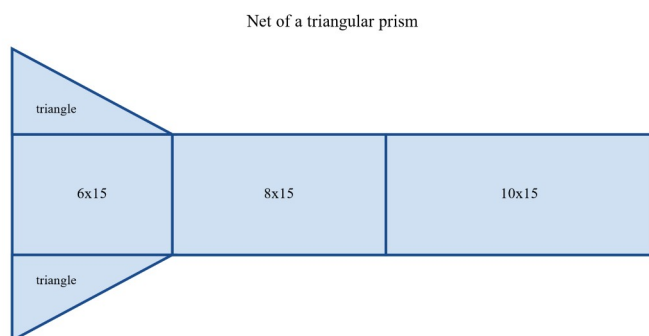
### Example 2 — scaffolded

A net is made of **2 triangles and 3 rectangles**. Name the solid it folds into and state how many faces it has.

| Working                                                    | Comment                                      |
|------------------------------------------------------------|----------------------------------------------|
| Two triangles are the two matching ends.                   | The identical parallel faces of a prism.     |
| Three rectangles join the matching sides of the triangles. | One rectangle per side of the triangle.      |
| So the solid is a <b>triangular prism</b> .                | Triangular cross-section, rectangular sides. |
| It has $2 + 3 = 5$ faces.                                  | Count every shape in the net.                |

### Example 3 — unscaffolded

A triangular prism has a right-angled triangular cross-section with sides 6 cm, 8 cm and 10 cm, and a length of 15 cm. Find its surface area.



The net has two triangular ends and three rectangles. Each triangle has area  $\frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$ , so the two ends contribute  $2 \times 24 = 48 \text{ cm}^2$ . The three rectangles are each 15 cm long and as wide as a side of the triangle, so their total area is

$$15 \times (6 + 8 + 10) = 15 \times 24 = 360 \text{ cm}^2.$$

Adding the ends and the rectangles, the surface area is  $48 + 360 = 408 \text{ cm}^2$ .

$\therefore$  the surface area of the prism is  $408 \text{ cm}^2$ .

### Example 4 — unscaffolded

An object is made of unit cubes stacked as three columns, 3, 2 and 1 cubes high, all one cube deep. Describe its front, side and top (plan) views.

Looking from the **front**, the columns appear at their full heights side by side, giving a staircase shape — three squares in the bottom row, two above the left of those, and one above the far left. Looking from the **side** (the object is one cube deep), all three columns line up behind one another, so you see a single column three squares high. Looking from the **top**, you see the footprint: a row of three squares.

$\therefore$  front view = a 3-step staircase; side view = a  $1 \times 3$  column; top view = a  $1 \times 3$  row (see the diagram in the Theory).

## Practice questions

### Scaffolded

**Q1.** A cube has edge length 4 cm. (a) How many faces does it have? (b) What is the area of one face? (c) Find the surface area.

**Q2.** A closed box is 10 cm by 6 cm by 4 cm. (a) Find the total area of the two  $10 \times 6$  faces. (b) Find the total area of the two  $10 \times 4$  faces and the two  $6 \times 4$  faces. (c) Find the surface area of the box.

**Q3.** Name the solid that each net folds into. (a) six squares (b) two triangles and three rectangles (c) one square and four triangles

- Q4.** An object is made of unit cubes: a row of three cubes on the ground, with one extra cube stacked on top of the left-hand cube. (a) Draw or describe the front view. (b) Draw or describe the top (plan) view. (c) How many cubes are used altogether?
- Q5.** A cylinder has radius 3 cm and height 10 cm. Its net is two circles and a rectangle. (a) Find the area of one circular end (leave your answer in terms of  $\pi$ ). (b) The rectangle's length equals the circumference of the circle. Find the area of the rectangle in terms of  $\pi$ . (c) Find the total surface area in terms of  $\pi$ , and as a decimal to one decimal place.
- Q6.** Six squares are arranged in a straight strip, all in one row. (a) Will this net fold into a cube? (b) Explain your answer.

### **Unscaffolded**

- Q7.** A net has two identical pentagons and five rectangles. Name the solid it folds into and state how many faces it has.
- Q8.** A student draws six squares in a  $2 \times 3$  block (two rows of three). Explain why this arrangement will **not** fold into a cube.
- Q9.** State the number of faces, edges and vertices of a rectangular prism, and hence state how many rectangles its net is made from.
- Q10.** Find the surface area of a closed box measuring 12 cm by 8 cm by 5 cm.
- Q11.** Find the surface area of a cube with edge length 7 cm.
- Q12.** A triangular prism has a right-angled triangular cross-section with sides 3 cm, 4 cm and 5 cm, and a length of 10 cm. Find its surface area.
- Q13.** A cylinder has radius 5 cm and height 12 cm. Find its surface area in terms of  $\pi$ , and as a decimal correct to two decimal places.
- Q14.** A square pyramid has a base of side 8 cm and a slant height of 6 cm. Find its surface area.
- Q15.** An object is a rectangular block 3 cubes wide, 2 cubes deep and 1 cube high. Describe its front, side and top views.
- Q16.** The three views of an object are: front = two squares side by side; side = one square; top = two squares side by side. Describe the solid.
- Q17.** A company ships a product in a closed cardboard box measuring 25 cm by 20 cm by 15 cm. How much cardboard (in  $\text{cm}^2$ ) is needed to make one box, ignoring overlaps?
- Q18.** An **open** rectangular tray (no lid) is 10 cm long, 6 cm wide and 4 cm high. Find the area of cardboard needed to make it.
-

## Answers

**Q1.** (a) 6; (b)  $16 \text{ cm}^2$ ; (c)  $96 \text{ cm}^2$ . **Q2.** (a)  $120 \text{ cm}^2$ ; (b)  $80 \text{ cm}^2$  and  $48 \text{ cm}^2$ ; (c)  $248 \text{ cm}^2$ . **Q3.** (a) cube; (b) triangular prism; (c) square pyramid. **Q4.** (a) an L-shape: three squares in a row with one square above the left end; (b) a row of three squares; (c) 4 cubes. **Q5.** (a)  $9\pi \text{ cm}^2$ ; (b)  $60\pi \text{ cm}^2$ ; (c)  $78\pi \approx 245.0 \text{ cm}^2$ . **Q6.** (a) no; (b) when folded, two of the squares would overlap and two opposite faces would be missing, so a straight strip of six cannot close into a cube. **Q7.** a pentagonal prism; 7 faces. **Q8.** a cube net needs the six squares arranged so none overlap when folded; a solid  $2 \times 3$  block folds so that faces overlap and it cannot close into a cube. **Q9.** faces 6, edges 12, vertices 8; the net is made from 6 rectangles (one per face). **Q10.**  $392 \text{ cm}^2$ . **Q11.**  $294 \text{ cm}^2$ . **Q12.**  $132 \text{ cm}^2$ . **Q13.**  $170\pi \approx 534.07 \text{ cm}^2$ . **Q14.**  $160 \text{ cm}^2$ . **Q15.** front = a  $3 \times 1$  row of squares; side = a  $2 \times 1$  row of squares; top = a  $3 \times 2$  rectangle of squares. **Q16.** a rectangular prism (box) 2 units wide, 1 unit deep and 1 unit high. **Q17.**  $2350 \text{ cm}^2$ . **Q18.**  $188 \text{ cm}^2$ .

---

## Fully-worked solutions

**Q1.** A cube has (a) 6 faces. (b) Each face is a square of side 4 cm, area  $4 \times 4 = 16 \text{ cm}^2$ . (c) Surface area =  $6 \times 16 = 96 \text{ cm}^2$ .

**Q2.** (a) Two  $10 \times 6$  faces:  $2 \times 60 = 120 \text{ cm}^2$ . (b) Two  $10 \times 4$  faces:  $2 \times 40 = 80 \text{ cm}^2$ ; two  $6 \times 4$  faces:  $2 \times 24 = 48 \text{ cm}^2$ . (c) Surface area =  $120 + 80 + 48 = 248 \text{ cm}^2$ .

**Q3.** (a) Six squares fold into a **cube**. (b) Two triangles (the ends) with three rectangles (the sides) fold into a **triangular prism**. (c) One square (the base) with four triangles (the sloping faces) folds into a **square pyramid**.

**Q4.** (a) The front view is an **L-shape**: three squares in a row with one square sitting on top of the left-hand square. (b) From above you see the footprint, a **row of three squares**. (c) Three cubes on the ground plus one on top = 4 cubes.

**Q5.** (a) One circular end has area  $\pi r^2 = \pi \times 3^2 = 9\pi \text{ cm}^2$ . (b) The rectangle's length is the circumference  $2\pi r = 6\pi \text{ cm}$  and its height is 10 cm, so its area is  $6\pi \times 10 = 60\pi \text{ cm}^2$ . (c) Surface area =  $2 \times 9\pi + 60\pi = 78\pi \text{ cm}^2 \approx 245.0 \text{ cm}^2$ .

**Q6.** (a) No. (b) A cube net must fold so that the six squares wrap into six faces without overlapping. A straight strip of six squares wraps around on itself — some squares land on top of others, and there are not squares in the right positions to close the top and bottom — so it cannot form a cube.

**Q7.** Two identical pentagons are the parallel ends and five rectangles join their sides, so the solid is a **pentagonal prism**, with  $2 + 5 = 7$  faces.

**Q8.** A valid cube net has the six squares spread out (for example in a cross) so that folding brings each square up to a different face. In a solid  $2 \times 3$  block, folding makes squares overlap and leaves faces uncovered, so it will not close into a cube.

**Q9.** A rectangular prism has 6 faces, 12 edges and 8 vertices. Each face of the solid appears once in its net and every face of a rectangular prism is a rectangle, so the net is made from 6 rectangles.

**Q10.** Surface area =  $2(lw + lh + wh) = 2(12 \times 8 + 12 \times 5 + 8 \times 5) = 2(96 + 60 + 40) = 2 \times 196 = 392 \text{ cm}^2$ .

**Q11.** Surface area =  $6 \times 7^2 = 6 \times 49 = 294 \text{ cm}^2$ .

**Q12.** Each triangular end has area  $\frac{1}{2} \times 3 \times 4 = 6 \text{ cm}^2$ , giving  $2 \times 6 = 12 \text{ cm}^2$ . The three rectangles total  $10 \times (3 + 4 + 5) = 10 \times 12 = 120 \text{ cm}^2$ . Surface area =  $12 + 120 = 132 \text{ cm}^2$ .

**Q13.** Surface area =  $2\pi r(r + h) = 2\pi \times 5 \times (5 + 12) = 2\pi \times 5 \times 17 = 170\pi \text{ cm}^2 \approx 534.07 \text{ cm}^2$ .

**Q14.** The base is  $8 \times 8 = 64 \text{ cm}^2$ . Each of the four triangular faces has area  $\frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$ , giving  $4 \times 24 = 96 \text{ cm}^2$ . Surface area =  $64 + 96 = 160 \text{ cm}^2$ .

**Q15.** The block is 3 wide, 2 deep and 1 high. Front view (width  $\times$  height) = a  $3 \times 1$  row of squares; side view (depth  $\times$  height) = a  $2 \times 1$  row of squares; top view (width  $\times$  depth) = a  $3 \times 2$  rectangle of squares.

**Q16.** A front of two squares side by side (width 2, height 1), a side of one square (depth 1, height 1) and a top of two squares side by side (width 2, depth 1) describe a **rectangular prism** 2 units wide, 1 unit deep and 1 unit high — a  $2 \times 1 \times 1$  box.

**Q17.** The box is a closed rectangular prism, so surface area  
 $= 2(25 \times 20 + 25 \times 15 + 20 \times 15) = 2(500 + 375 + 300) = 2 \times 1175 = 2350 \text{ cm}^2$  of cardboard.

**Q18.** An open tray has a base and four sides but **no lid**. Base =  $10 \times 6 = 60 \text{ cm}^2$ ; the two long sides =  $2 \times (10 \times 4) = 80 \text{ cm}^2$ ; the two ends =  $2 \times (6 \times 4) = 48 \text{ cm}^2$ . Total =  $60 + 80 + 48 = 188 \text{ cm}^2$ .