Foundation Mathematics Units 3 & 4

Chapter 9 — Money management: interest, loans and credit

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Theory

Whenever money is borrowed or invested, **interest** is charged or earned. Interest is the price of using someone else's money: a bank pays you interest on your savings, and charges you interest on a loan. The amount of interest depends on three things — the **principal** P (the amount borrowed or invested), the interest **rate** r (a percentage, usually per year, written "p.a." for *per annum*), and the **time** the money is borrowed or invested for.

Simple interest.

Simple interest is calculated on the original principal only, so the same amount of interest is added in every period. It is given by the formula (on your formula sheet)

$$I = \frac{P\,r\,t}{100},$$

where P is the principal, r is the interest rate per period (as a number, e.g. 5 for 5%) and t is the number of periods. The **total amount** after the interest is added is

$$A = P + I.$$

For example, $\$2000$ invested at 4% p.a. simple interest for 3 years earns $I = \frac{2000 \times 4 \times 3}{100} = \$240$, giving a total of $\$2240$. Because the interest is the same each year ($\$80$), simple interest grows in equal steps.

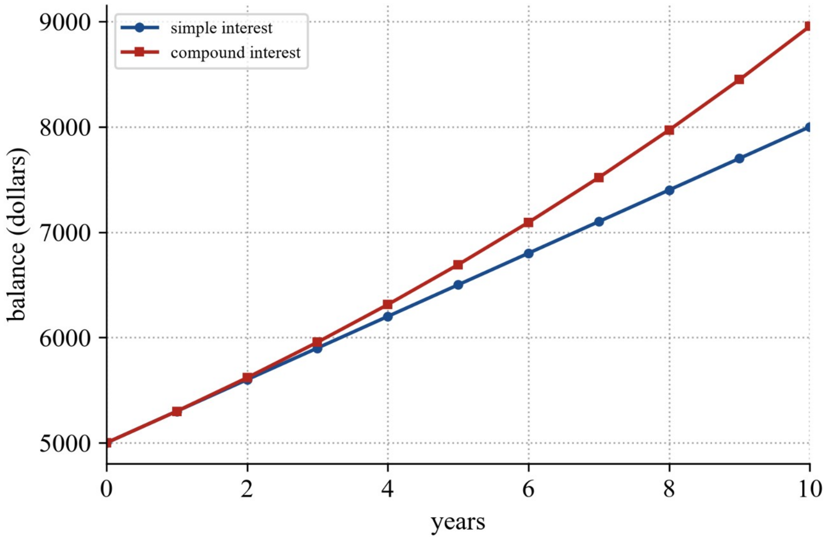
Compound interest.

With **compound interest**, each period's interest is added to the balance, so the next period earns interest on that interest as well. This makes an investment grow faster and faster. If interest is added once per period, the amount after n periods is (formula sheet)

$$A = P\,R^n, \text{ where } R = 1 + \frac{r}{100}.$$

Here R is the **growth factor** for one period. The interest earned is the amount grown minus the principal, $I = A - P$. For example, $\$2000$ at 4% p.a. compounded annually for 3 years grows to $A = 2000 \times 1.04^3 = \$2249.73$, so the interest is $\$249.73$ — a little more than the $\$240$ of simple interest, because the later years earn interest on the earlier interest.

The graph below compares $\$5000$ invested at 6% p.a. under simple interest (equal steps) and compound interest (a curve that pulls away). The longer the time, the bigger the gap.



Compounding more often than once a year.

Interest is often compounded monthly, quarterly or daily rather than yearly. To use $A = P\,R^n$, match the rate and the number of periods to the compounding:

- the rate per period is the annual rate divided by the number of periods per year (e.g. $6\,\%$ p.a. compounded monthly gives $\frac{6}{12} = 0.5\,\%$ per month);
- n is the total number of periods (e.g. 2 years compounded monthly gives $n = 2 \times 12 = 24$).

For example, $\$5000$ at $6\,\%$ p.a. compounded **monthly** for 1 year uses $R = 1.005$ and $n = 12$, giving $A = 5000 \times 1.005^{12} = \$5308.39$ — slightly more than the $\$5300$ it would reach if compounded annually, because the interest is added more often.

Loans and repayments.

When you take out a loan you repay the **principal plus interest**, usually in regular instalments. With a **reducing-balance** loan, each repayment first covers the interest charged that period and then reduces the balance, so later repayments pay off more of the loan. The table below shows the first two months of a $\$1000$ loan charged $1\,\%$ interest per month, with repayments of $\$300$ per month.

Month	Balance owing	Interest ($1\,\%$)	Repayment	New balance
1	$\$1000.00$	$\$10.00$	$\$300.00$	$\$710.00$
2	$\$710.00$	$\$7.10$	$\$300.00$	$\$417.10$

The process continues until the balance reaches zero. A quick way to find the **total cost** of a loan with fixed repayments is

$$\text{total repaid} = \text{repayment} \times \text{number of repayments}\,,$$

and the **total interest** is the total repaid minus the amount borrowed. Comparing the total cost of two loans (or the total interest earned by two investments) is how you decide which option is better.

Worked examples

Example 1 — scaffolded

Rana invests $\$2500$ in an account paying $4\,\%$ p.a. simple interest for 3 years. Find the interest earned and the total amount in the account.

Working	Comment
$P=2500$, $r=4$, $t=3$.	Identify the principal, rate and time.
$I=\frac{P\,r\,t}{100}=\frac{2500\times4\times3}{100}$.	Substitute into the simple interest formula.
$I=\frac{30000}{100}=\$300$.	Evaluate on the calculator.
$A=P+I=2500+300=\$2800$.	The total amount is principal plus interest.

Example 2 — scaffolded

Rana instead invests the $\$2500$ at 4% p.a. compounded annually for 3 years. Find the total amount and the interest earned.

Working	Comment
$P=2500$, $r=4$, $n=3$.	Interest is compounded once per year, so n is the number of years.
$R=1+\frac{r}{100}=1+\frac{4}{100}=1.04$.	Find the growth factor for one year.
$A=P\,R^n=2500\times1.04^3$.	Substitute into the compound interest formula.
$A=2500\times1.124864=\$2812.16$.	Evaluate on the calculator.
$I=A-P=2812.16-2500=\$312.16$.	The interest is the amount grown minus the principal ($\$12.16$ more than simple interest).

Example 3 — unscaffolded

A credit union pays $\$600$ simple interest on an investment of $\$5000$ over 4 years. Find the annual interest rate.

Rearranging $I=\frac{P\,r\,t}{100}$ to make r the subject gives $r=\frac{100\,I}{P\,t}$. Substituting $I=600$, $P=5000$ and $t=4$,

$$r=\frac{100\times600}{5000\times4}=\frac{60000}{20000}=3.$$

∴ the account pays 3% p.a. simple interest.

Example 4 — unscaffolded

Compare the value of a $\$5000$ investment after 4 years at 6% p.a. under (i) simple interest and (ii) interest compounded annually. How much more does compound interest earn?

For simple interest, $I=\frac{5000\times6\times4}{100}=\$1200$, so the amount is $A=5000+1200=\$6200$. For compound interest, $R=1.06$ and $n=4$, so

$$A=5000\times1.06^4=5000\times1.26247696=\$6312.38.$$

The compound interest earns $6312.38-6200=\$112.38$ more than simple interest.

∴ compound interest gives $\$6312.38$ versus $\$6200$ for simple interest — $\$112.38$ more over the four years.

Practice questions

Scaffolded

Q1. Tom invests $\$1500$ at 5% p.a. simple interest for 2 years. (a) Write down P, r and t. (b) Find the simple interest earned. (c) Find the total amount in the account.

Q2. $\$4000$ is invested at 3% p.a. compounded annually for 5 years. (a) Write down the growth factor R. (b) Find the total amount after 5 years. (c) Find the interest earned.

Q3. An investment of $\$2000$ at 5% p.a. simple interest earns $\$300$ in interest. (a) Write the simple interest formula and substitute the known values. (b) Rearrange to make t the subject. (c) Find the number of years the money was invested.

Q4. Priya takes out a personal loan and repays it with 24 monthly payments of $\$290$. The amount she borrowed was $\$6000$. (a) Find the total amount she repays. (b) Find the total interest she is charged. (c) Express the interest as a fraction of the amount borrowed, as a percentage (to the nearest whole per cent).

Q5. $\$10\,000$ is invested for 3 years at 4% p.a. (a) Find the total amount if simple interest is paid. (b) Find the total amount if interest is compounded annually. (c) State how much more the compound-interest option is worth.

Q6. $\$4500$ is invested at 3.6% p.a. compounded **monthly** for 3 years. (a) State the interest rate per month and the number of periods n. (b) Find the total amount after 3 years. (c) Find the interest earned.

Unscaffolded

Q7. Find the simple interest earned, and the total amount, when $\$3500$ is invested at 4.5% p.a. for 2 years.

Q8. An investment of $\$8000$ earns $\$960$ simple interest over 3 years. Find the annual interest rate.

Q9. How many years does it take for $\$1200$ to earn $\$216$ simple interest at 6% p.a.?

Q10. Find the total amount when $\$7000$ is invested at 5% p.a. compounded annually for 4 years, and state the interest earned.

Q11. $\$2500$ is invested at 3.5% p.a. compounded annually for 6 years. Find the interest earned.

Q12. $\$6000$ is invested for 5 years at 5% p.a. Find the total amount under simple interest and under annual compound interest, and state the difference.

Q13. A loan of $\$9000$ is charged 12% p.a. interest compounded **monthly**. Find the amount owing after 2 years if no repayments are made, and the interest charged.

Q14. A family can repay a $\$20\,000$ loan in one of two ways: Option A is 48 monthly payments of $\$520$; Option B is 60 monthly payments of $\$470$. Find the total cost of each option and state which is cheaper, and by how much.

Q15. How much must be invested now at 4% p.a. compounded annually to grow to $\$10\,000$ in 5 years? (Give your answer to the nearest cent.)

Q16. $\$5000$ is invested at 8% p.a. for 1 year. Find the total amount if interest is compounded (a) annually, (b) quarterly, (c) monthly, and comment on the effect of compounding more often.

Q17. A fridge costs $\$1800$ cash. On a payment plan it can be bought for 24 monthly payments of $\$85$ with no deposit. (a) Find the total paid under the payment plan. (b) Find the extra amount paid compared with the cash price. (c) Express the extra amount as a percentage of the cash price (to two decimal places).

Q18. Two banks offer a 4-year investment of $\$5000$: Bank A pays 5.5% p.a. simple interest; Bank B pays 5% p.a. compounded annually. Find the final value of each and state which is better, and by how much.

Answers

Q1. (a) $P=1500$, $r=5$, $t=2$. (b) $\$150$. (c) $\$1650$. **Q2.** (a) $R=1.03$. (b) $\$4637.10$. (c) $\$637.10$. **Q3.** (a) $300=\frac{2000\times 5\times t}{100}$. (b) $t=\frac{100\,I}{P\,r}$. (c) 3 years. **Q4.** (a) $\$6960$. (b) $\$960$. (c) $16\,\%$. **Q5.** (a) $\$11\,200$. (b) $\$11\,248.64$. (c) $\$48.64$. **Q6.** (a) $0.3\,\%$ per month, $n=36$. (b) $\$5012.40$. (c) $\$512.40$. **Q7.** $I=\$315$, $A=\$3815$. **Q8.** $4\,\%$ p.a. **Q9.** 3 years. **Q10.** $A=\$8508.54$, interest $\$1508.54$. **Q11.** $\$573.14$. **Q12.** simple $\$7500$, compound $\$7657.69$; difference $\$157.69$. **Q13.** $A=\$11\,427.61$, interest $\$2427.61$. **Q14.** Option A $\$24\,960$, Option B $\$28\,200$; Option A is cheaper by $\$3240$. **Q15.** $\$8219.27$. **Q16.** (a) $\$5400$; (b) $\$5412.16$; (c) $\$5415.00$. Compounding more often earns slightly more interest. **Q17.** (a) $\$2040$. (b) $\$240$. (c) $13.33\,\%$. **Q18.** Bank A $\$6100$, Bank B $\$6077.53$; Bank A is better by $\$22.47$.

Fully-worked solutions

Q1. (a) $P=1500$, $r=5$, $t=2$. (b) $I=\frac{P\,r\,t}{100}=\frac{1500\times 5\times 2}{100}=\frac{15000}{100}=\$150$. (c) $A=P+I=1500+150=\$1650$.

Q2. (a) $R=1+\frac{3}{100}=1.03$. (b) $A=P\,R^n=4000\times 1.03^5=4000\times 1.15927407=\$4637.10$. (c) $I=A-P=4637.10-4000=\$637.10$.

Q3. (a) With $I=300$, $P=2000$, $r=5$: $300=\frac{2000\times 5\times t}{100}=100\,t$. (b) Dividing both sides by 100, $t=\frac{100\,I}{P\,r}=\frac{300}{100}$. (c) $t=3$ years.

Q4. (a) Total repaid $=290\times 24=\$6960$. (b) Interest $=6960-6000=\$960$. (c) As a percentage of the amount borrowed, $\frac{960}{6000}\times 100=16\,\%$.

Q5. (a) Simple interest: $I=\frac{10\,000\times 4\times 3}{100}=\$1200$, so $A=10\,000+1200=\$11\,200$. (b) Compound: $R=1.04$, $n=3$, so $A=10\,000\times 1.04^3=10\,000\times 1.124864=\$11\,248.64$. (c) The compound option is worth $11\,248.64-11\,200=\$48.64$ more.

Q6. (a) The monthly rate is $\frac{3.6}{12}=0.3\,\%$, and $n=3\times 12=36$ months. (b) $R=1+\frac{0.3}{100}=1.003$, so $A=4500\times 1.003^{36}=4500\times 1.11386764=\$5012.40$. (c) $I=5012.40-4500=\$512.40$.

Q7. $I=\frac{3500\times 4.5\times 2}{100}=\frac{31\,500}{100}=\$315$, and $A=3500+315=\$3815$.

Q8. Rearranging $I=\frac{P\,r\,t}{100}$ gives $r=\frac{100\,I}{P\,t}=\frac{100\times 960}{8000\times 3}=\frac{96\,000}{24\,000}=4$. So the rate is $4\,\%$ p.a.

Q9. $t=\frac{100\,I}{P\,r}=\frac{100\times 216}{1200\times 6}=\frac{21\,600}{7200}=3$ years.

Q10. $R=1.05$, $n=4$, so $A=7000\times 1.05^4=7000\times 1.21550625=\$8508.54$, and the interest is $8508.54-7000=\$1508.54$.

Q11. $R=1.035$, $n=6$, so $A=2500\times 1.035^6=2500\times 1.22925533=\$3073.14$, and the interest is $3073.14-2500=\$573.14$.

Q12. Simple: $I=\frac{6000\times 5\times 5}{100}=\$1500$, so $A=\$7500$. Compound: $A=6000\times 1.05^5=6000\times 1.27628156=\$7657.69$. The difference is $7657.69-7500=\$157.69$.

Q13. The monthly rate is $\frac{12}{12}=1\%$, and $n=2\times12=24$. So $R=1.01$ and $A=9000\times1.01^{24}=9000\times1.26973465=\$11\,427.61$. The interest charged is $11\,427.61-9000=\$2427.61$.

Q14. Option A total $=520\times48=\$24\,960$. Option B total $=470\times60=\$28\,200$. Option A costs $28\,200-24\,960=\$3240$ less, so Option A is cheaper by $\$3240$.

Q15. Rearranging $A=PR^n$ gives $P=\frac{A}{R^n}$. With $A=10\,000$, $R=1.04$ and $n=5$,

$$P=\frac{10\,000}{1.04^5}=\frac{10\,000}{1.21665290}=\$8219.27.$$

Q16. (a) Annually: $A=5000\times1.08^1=\$5400$. (b) Quarterly: rate $\frac{8}{4}=2\%$, $n=4$, so $A=5000\times1.02^4=5000\times1.08243216=\$5412.16$. (c) Monthly: rate $\frac{8}{12}\%$, $n=12$, so $A=5000\times\left(1+\frac{8}{1200}\right)^{12}=\$5415.00$. The more often interest is compounded, the more interest is earned (though the increases become small).

Q17. (a) Total paid $=85\times24=\$2040$. (b) Extra $=2040-1800=\$240$. (c) As a percentage of the cash price, $\frac{240}{1800}\times100=13.33\%$ (to 2 d.p.).

Q18. Bank A (simple): $I=\frac{5000\times5.5\times4}{100}=\$1100$, so $A=5000+1100=\$6100$. Bank B (compound): $A=5000\times1.05^4=5000\times1.21550625=\$6077.53$. Bank A gives $6100-6077.53=\$22.47$ more, so Bank A is the better option.

Theory

When you cannot (or choose not to) pay the full price of something straight away, you use **credit** — you get the goods now and pay over time. Credit almost always costs more than paying up front, and different kinds of credit cost very different amounts. This section compares the main options by working out the **total cost** of each.

The main credit options.

- A **personal loan** lends you a fixed amount, repaid in equal instalments over a fixed term (for example $205 a month for 4 years). You know exactly what each repayment is and when the loan ends.
- A **credit card** lets you borrow up to a limit. If you pay the whole balance within the **interest-free period** you pay no interest, but any balance carried past it is charged a **high** interest rate (often around 18% to 22% per year).
- **Buy now, pay later** (BNPL) splits a purchase into instalments (often fortnightly) with no interest, but usually charges **account or late fees**.
- **Store credit** is finance offered by a shop, sometimes interest-free for a set period, then charged interest on whatever is still owing.

Total cost of a loan with fixed repayments.

For any plan with equal repayments, the **total amount repaid** is

$$\text{total repaid} = \text{repayment} \times \text{number of repayments},$$

and the **cost of the credit** (interest and fees) is the total repaid minus the price:

$$\text{cost of credit} = \text{total repaid} - \text{amount borrowed}.$$

If there is an establishment fee or monthly fee, add it to the total repaid.

Interest on a credit-card balance.

Interest charged on a balance carried on a card is

$$\text{interest} = \text{balance} \times \frac{\text{annual rate}}{100} \times \frac{\text{months}}{12}.$$

For example, a $3200 balance at 18.5% per year is charged $3200 \times \frac{18.5}{100} \times \frac{1}{12} \approx \$49.33$ in a single month. Only paying the **minimum** repayment is expensive, because most of a small repayment goes on interest and the balance barely falls.

Comparing options.

To compare two options for the **same purchase**, work out the total cost of each (including all fees) and find the difference. A lower monthly repayment is **not** automatically cheaper — a smaller repayment usually means a **longer** term and more interest overall.

Headline rate vs comparison rate.

The advertised (headline) interest rate ignores fees. The **comparison rate** builds the fees into a single rate so that two loans can be compared fairly, which is why the comparison rate is always **higher** than the headline rate when a loan has fees.

Credit versus debit.

A **debit** card (or paying from your savings) spends your **own** money, so there is no interest and no debt — you can only spend what you actually have. A **credit** card spends **borrowed** money that must be repaid, and interest is charged if the balance is not cleared in time. Using debit for everyday spending avoids interest completely; credit is borrowing.

Renting versus buying with a mortgage.

A **mortgage** is a large loan used to buy a home, repaid over many years. To compare **renting** a home with **buying** it, work out the total cost of each over the same period: rent = rent per period × number of periods; mortgage cost = repayment × number of repayments. Buying also builds **ownership** in the home, while rent only buys the right to live there.

Debt consolidation.

Debt consolidation combines several separate debts — each with its own repayment — into a **single** loan with one repayment. It saves money when the new loan's total repaid is less than the sum of the old debts' totals. Compare the **total repaid before** (the old debts added together) with the **total repaid after** (the single new loan).

Worked examples

Example 1 — scaffolded

Priya borrows $8000 for a car. She repays the loan at $205 per month for 4 years. Find (a) the total amount she repays and (b) the total interest she pays.

Working	Comment
Number of repayments $=4\times 12=48$.	4 years at one repayment per month.
Total repaid $=\$205\times 48=\$9840$.	Total = repayment × number of repayments.
Total interest $=\$9840-\$8000=\$1840$.	Cost of credit = total repaid − amount borrowed.

Example 2 — scaffolded

Sam has a credit-card balance of $3200 at an interest rate of $18.5\,\%$ per year. He makes no new purchases. Find the interest charged (a) in one month and (b) over six months if the balance is unchanged.

Working	Comment
One month: interest $=3200\times\frac{18.5}{100}\times\frac{1}{12}\approx\$49.33$.	Rate per year × one-twelfth of a year.
Six months: interest $=3200\times\frac{18.5}{100}\times\frac{6}{12}=\$296.00$.	Six-twelfths $=1/2$ of a year.
Carrying the balance is costly — nearly $300 in half a year.	Interpret the result in context.

Example 3 — unscaffolded

A $2400 fridge can be bought two ways: store finance at $110 per month for 24 months, or a personal loan repaid at $108 per month for 24 months. Which option costs less, and by how much?

Store finance costs $\$110\times 24=\$2640$, so its interest is $\$2640-\$2400=\$240$. The personal loan costs $\$108\times 24=\$2592$, so its interest is $\$2592-\$2400=\$192$. The difference is $\$2640-\$2592=\$48$.

∴ the personal loan is cheaper by $48 over the two years.

Example 4 — unscaffolded

A $12 000 car can be financed two ways. Loan A: $360 per month for 36 months plus a $200 establishment fee. Loan B: $340 per month for 42 months with no fee. Loan B has the smaller monthly repayment — is it the cheaper loan?

Loan A costs $\$360 \times 36 + \$200 = \$12\,960 + \$200 = \$13\,160$, so its cost of credit is $\$13\,160 - \$12\,000 = \$1160$. Loan B costs $\$340 \times 42 = \$14\,280$, so its cost of credit is $\$14\,280 - \$12\,000 = \$2280$. Even with the fee, Loan A costs $\$14\,280 - \$13\,160 = \$1120$ less.

∴ despite its higher monthly repayment, Loan A is cheaper by \$1120, because its term is shorter and it charges interest for less time.

Practice questions

Scaffolded

Q1. A \$5000 personal loan is repaid at \$160 per month for 36 months. (a) Find the total amount repaid. (b) Find the total interest paid. (c) State one advantage of a personal loan over a credit card for this purchase.

Q2. A credit-card balance of \$1500 is charged interest at $20\,\%$ per year. (a) Find the interest charged in one month if the balance is unchanged. (b) Find the interest charged over three months if the balance is unchanged. (c) Explain why paying only the minimum each month is expensive.

Q3. A \$2000 television can be paid off two ways: Option A at \$95 per month for 24 months, or Option B at \$185 per month for 12 months. (a) Find the total cost of Option A. (b) Find the total cost of Option B. (c) State which option is cheaper and by how much.

Q4. An \$800 purchase is made with a buy-now-pay-later plan: four fortnightly payments of \$200, plus an \$8 monthly account fee for the two months of the plan. (a) Find the total paid under the plan. (b) State how much more this is than paying \$800 up front.

Q5. A \$6000 loan can be repaid as Option A: \$300 per month for 24 months, or Option B: \$200 per month for 39 months. (a) Find the total interest for Option A. (b) Find the total interest for Option B. (c) State which option costs less overall, and one reason someone might still choose the other.

Q6. A \$10 000 personal loan is repaid at \$250 per month for 48 months, plus a \$500 establishment fee. (a) Find the total amount repaid, including the fee. (b) Find the total cost of credit.

Unscaffolded

Q7. A \$15 000 car loan is repaid at \$375 per month for 48 months. Find the total amount repaid and the total interest.

Q8. A credit-card balance of \$4200 is charged $21\,\%$ per year. Find the interest charged (a) in one month and (b) over a whole year, if the balance is unchanged.

Q9. A \$1600 laptop can be bought at \$75 per month for 24 months, or at \$145 per month for 12 months. Find the total cost of each option and state which is cheaper and by how much.

Q10. A \$9000 loan can be repaid at \$450 per month for 24 months, or at \$300 per month for 39 months. Find the total cost of each and state how much is saved by choosing the cheaper one.

Q11. An \$8000 loan is offered two ways: Loan A at \$380 per month for 24 months with no fees, or Loan B at \$360 per month for 24 months plus a \$350 establishment fee. Which loan costs less, and by how much?

Q12. A \$3000 purchase can be financed three ways: Option A \$135 per month for 24 months; Option B \$175 per month for 18 months; Option C \$290 per month for 12 months. Which option is the cheapest?

Q13. A \$1200 purchase is made with a buy-now-pay-later plan that charges a \$9.95 monthly fee for the four months of the plan (the instalments total \$1200). How much more than the cash price of \$1200 does the plan cost?

Q14. A \$2500 lounge is bought on store credit that is interest-free for 6 months. If \$1000 is still owing after 6 months and is carried for a further 3 months at $22\,\%$ per year, find the interest charged on that \$1000.

Q15. A credit card has a \$2000 balance at $19\,\%$ per year, and the minimum monthly repayment is \$40. (a) Find the interest added in the first month. (b) Hence find how much of the \$40 repayment actually reduces the balance, and explain why the balance falls so slowly.

Q16. A personal loan is advertised at a headline rate but also charges a \$400 establishment fee and an \$8 monthly fee over its 60-month term. (a) Find the total of the fees over the loan. (b) Explain why the loan's comparison rate is higher than its headline rate.

Q17. A \$20 000 loan can be repaid at \$500 per month for 48 months, or at \$400 per month for 66 months. Find the total interest for each, and state how much extra the longer loan costs.

Q18. A \$1500 appliance can be paid for by: Option A, a credit card paid in full within the interest-free period; Option B, a personal loan at \$70 per month for 24 months; or Option C, a buy-now-pay-later plan with \$40 in fees. Find the total cost of each and state, with a reason, which option is best.

Q19. A \$1200 purchase can be paid for with a **debit** card (from savings), or with a **credit** card whose \$1200 balance is then carried for 3 months at $20\,\%$ per year before being cleared. (a) State the interest cost of paying by debit card. (b) Find the interest charged on the credit-card balance over the 3 months. (c) State which method avoids interest, and why.

Q20. A couple can **rent** a house for \$520 per week, or **buy** it with a mortgage repaid at \$2400 per month. Compare the two over one year (52 weeks $= 12$ months). (a) Find the total rent for the year. (b) Find the total mortgage repayments for the year. (c) State which costs less in cash over the year and by how much, and give one non-cash advantage of buying.

Q21. A person has three debts, each repaid over 24 months: a personal loan at \$190 per month, a store-card debt at \$110 per month, and a credit-card debt at \$95 per month. A **consolidation** loan would instead repay the combined debt at \$330 per month for 24 months. (a) Find the total repaid under the three separate debts. (b) Find the total repaid under the consolidation loan. (c) State how much the consolidation loan saves.

Answers

Q1. (a) \$5760. (b) \$760. (c) the repayment and end date are fixed and known, and the interest rate is usually lower than a credit card's. **Q2.** (a) \$25. (b) \$75. (c) most of a small repayment goes on interest, so the balance barely falls and interest keeps being charged. **Q3.** (a) \$2280. (b) \$2220. (c) Option B, by \$60. **Q4.** (a) \$816. (b) \$16 more. **Q5.** (a) \$1200. (b) \$1800. (c) Option A costs \$600 less; someone might choose B for the smaller monthly repayment. **Q6.** (a) \$12 500. (b) \$2500. **Q7.** total \$18 000, interest \$3000. **Q8.** (a) \$73.50. (b) \$882. **Q9.** \$75/month option \$1800; \$145/month option \$1740; the \$145/month option is cheaper by \$60. **Q10.** \$450/month option \$10 800; \$300/month option \$11 700; the \$450/month option saves \$900. **Q11.** Loan A \$9120; Loan B \$8990; Loan B is cheaper by \$130. **Q12.** A \$3240, B \$3150, C \$3480; Option B is the cheapest. **Q13.** \$39.80 more. **Q14.** \$55. **Q15.** (a) \$31.67. (b) only \$8.33 reduces the balance; almost all the repayment is swallowed by interest, so the debt falls very slowly. **Q16.** (a) \$880. (b) the comparison rate includes the fees, which the headline rate ignores, so it is higher. **Q17.** \$500/month loan interest \$4000; \$400/month loan interest \$6400; the longer loan costs \$2400 more. **Q18.** A \$1500, B \$1680, C \$1540; Option A is best (no interest or fees) provided it is paid off within the interest-free period. **Q19.** (a) \$0. (b) \$60. (c) the debit card, because it spends your own money instead of borrowing. **Q20.** (a) \$27 040. (b) \$28 800. (c) renting costs \$1760 less in cash over the year; buying builds ownership (equity) in the home. **Q21.** (a) \$9480. (b) \$7920. (c) \$1560.

Fully-worked solutions

Q1. (a) Total repaid $= \$160 \times 36 = \$5760$. (b) Interest $= \$5760 - \$5000 = \$760$. (c) A personal loan has fixed repayments and a fixed end date, and usually a lower interest rate than a credit card.

Q2. (a) One month: $1500 \times \frac{20}{100} \times \frac{1}{12} = \$25$. (b) Three months: $1500 \times \frac{20}{100} \times \frac{3}{12} = \$75$. (c) A minimum repayment is small, and much of it only covers that month's interest, so the balance falls very slowly and interest keeps being charged on it.

Q3. (a) $\$95 \times 24 = \$2280$. (b) $\$185 \times 12 = \$2220$. (c) Option B costs $\$2280 - \$2220 = \$60$ less.

Q4. (a) The instalments total $4 \times \$200 = \$800$, plus fees $\$8 \times 2 = \$16$, giving $\$816$. (b) That is $\$816 - \$800 = \$16$ more than paying up front.

Q5. (a) Option A: $\$300 \times 24 = \$7200$, interest $\$7200 - \$6000 = \$1200$. (b) Option B: $\$200 \times 39 = \$7800$, interest $\$7800 - \$6000 = \$1800$. (c) Option A costs $\$1800 - \$1200 = \$600$ less overall; someone might still choose Option B because its monthly repayment (\$200) is easier to afford than \$300.

Q6. (a) Total $= \$250 \times 48 + \$500 = \$12\,000 + \$500 = \$12\,500$. (b) Cost of credit $= \$12\,500 - \$10\,000 = \$2500$.

Q7. Total repaid $= \$375 \times 48 = \$18\,000$; interest $= \$18\,000 - \$15\,000 = \$3000$.

Q8. (a) $4200 \times \frac{21}{100} \times \frac{1}{12} = \$73.50$. (b) $4200 \times \frac{21}{100} = \$882$ over a full year.

Q9. The \$75/month plan costs $\$75 \times 24 = \$1800$; the \$145/month plan costs $\$145 \times 12 = \$1740$. The \$145/month plan is cheaper by $\$1800 - \$1740 = \$60$.

Q10. The \$450/month plan costs $\$450 \times 24 = \$10\,800$; the \$300/month plan costs $\$300 \times 39 = \$11\,700$. Choosing the \$450/month plan saves $\$11\,700 - \$10\,800 = \$900$.

Q11. Loan A costs $\$380 \times 24 = \$9120$. Loan B costs $\$360 \times 24 + \$350 = \$8640 + \$350 = \$8990$. Loan B is cheaper by $\$9120 - \$8990 = \$130$ — the lower repayment more than makes up for the fee here.

Q12. Option A: $\$135 \times 24 = \$3240$. Option B: $\$175 \times 18 = \$3150$. Option C: $\$290 \times 12 = \$3480$. The cheapest is **Option B** at \$3150.

Q13. The fees total $\$9.95 \times 4 = \$39.80$, and the instalments total the cash price, so the plan costs $\$39.80$ more than paying \$1200 in cash.

Q14. After the interest-free period, the \$1000 owing is charged $1000\times\frac{22}{100}\times\frac{3}{12}=\$55$.

Q15. (a) First-month interest $=2000\times\frac{19}{100}\times\frac{1}{12}\approx\$31.67$. (b) Of the \$40 repayment, only $\$40-\$31.67=\$8.33$ reduces the balance. Because almost all of each repayment is taken up by interest, the balance falls very slowly and the debt takes a long time to clear.

Q16. (a) Fees $=\$400+\$8\times60=\$400+\$480=\$880$. (b) The comparison rate folds these fees into the rate, whereas the headline rate ignores them, so the comparison rate is higher.

Q17. The \$500/month loan costs $\$500\times48=\$24\,000$, so interest is $\$24\,000-\$20\,000=\$4000$. The \$400/month loan costs $\$400\times66=\$26\,400$, so interest is $\$26\,400-\$20\,000=\$6400$. The longer loan costs $\$6400-\$4000=\$2400$ more.

Q18. Option A: paid within the interest-free period, so it costs the price, \$1500. Option B: $\$70\times24=\$1680$. Option C: $\$1500+\$40=\$1540$. Option A is the cheapest at \$1500 and is best **provided** the balance is cleared within the interest-free period; otherwise credit-card interest would make it dearer.

Q19. (a) A debit card spends your own money, so there is no borrowing and no interest: \$0. (b) Interest $=1200\times\frac{20}{100}\times\frac{3}{12}=\$60$. (c) The debit card avoids interest, because it uses money you already have rather than borrowing.

Q20. (a) Yearly rent $=\$520\times52=\$27\,040$. (b) Yearly mortgage repayments $=\$2400\times12=\$28\,800$. (c) Renting costs $\$28\,800-\$27\,040=\$1760$ less in cash over the year; however, mortgage repayments build ownership in the home, whereas rent does not.

Q21. (a) The three debts together repay $(\$190+\$110+\$95)\times24=\$395\times24=\$9480$. (b) The consolidation loan repays $\$330\times24=\$7920$. (c) Consolidating saves $\$9480-\$7920=\$1560$.

Chapter 9 Money management: interest, loans and credit — 9E Student loan schemes

Theory

Most Australian university students pay their tuition using the government's **HECS-HELP** student loan scheme. It works very differently from the loans in the previous sections, so its arithmetic is worth understanding before you take one on.

How a HECS-HELP loan works.

- The government pays your course fees; the amount becomes a **debt** you owe.
- The debt charges **no interest**. Instead, once a year (on 1 June) the balance is **indexed** — increased by a rate the government sets as the **lower** of the Consumer Price Index (a measure of price rises) and the Wage Price Index (a measure of wage growth), so the debt never grows faster than wages.
- You make **no repayments at all** until your income passes a set **threshold**. After that, repayments are **compulsory** and are collected automatically through the tax system.
- You may also make **voluntary** repayments at any time, which reduce the balance directly.
- **VET Student Loans**, for vocational courses, work the same way — the same indexation and the same repayment thresholds and rates.

Indexation.

Indexation is just a percentage increase applied to the whole balance. If a balance B is indexed at a rate of $r\,\%$, the new balance is

$$B_{\text{new}} = B \times \left(1 + \frac{r}{100}\right).$$

For example, a \$20 000 debt indexed at $3\,\%$ becomes $20\,000 \times 1.03 = \$\,20\,600$ — it has grown by \$600 even though no interest was charged. This is why a HECS-HELP debt that is not being repaid slowly **grows** over time.

Compulsory repayments.

Once your **repayment income** passes the threshold, you must make a compulsory repayment each year. Since 1 July 2025 the repayment has been worked out on a **marginal** basis: only the part of your income **above the threshold** is counted, not your whole income. For the 2026–27 income year the threshold is \$69 528, and the rates are:

Repayment income	Compulsory repayment
\$0 – \$69 528	Nil (no repayment)
\$69 529 – \$129 717	15c for each \$1 over \$69 528
\$129 718 – \$186 050	\$9028 plus 17c for each \$1 over \$129 717
\$186 051 and above	10% of the whole repayment income

The thresholds are raised each year in line with average earnings, so the exact figures change — always use the table you are given. The last row is a cap that applies only to very high incomes.

To find a compulsory repayment, locate the income in the table, subtract the threshold to find the income **above** it, then apply the rate in that row. For an income of \$80 000:

$$80\,000 - 69\,528 = \$\,10\,472,\ 15\,\% \times 10\,472 = \$\,1570.80.$$

Because only the income above the threshold is counted, the repayment starts at nothing at the threshold and then rises steadily: each extra \$1 of income adds only 15c to the repayment, so a \$1000 pay rise adds just \$150. There is no sudden jump at a band boundary.

Voluntary repayments and the order of events.

Each year the balance is **indexed first**, then any compulsory repayment is subtracted. A **voluntary** repayment made *before* the indexation date reduces the balance that gets indexed, so it saves a little extra. For a \$30 000 debt indexed at $4\,\%$ with a \$2500 compulsory repayment, the balance becomes $30\,000 \times 1.04 - 2500 = 31\,200 - 2500 = \$\,28\,700$.

Estimating the time to repay.

Ignoring indexation, a rough estimate of the years to clear a debt is

$$\text{years} \approx \frac{\text{balance}}{\text{annual repayment}}.$$

In reality indexation makes it take **longer**, because each year's indexation adds to the balance before the repayment comes off.

Worked examples

Example 1 — scaffolded

A graduate has a HECS-HELP debt of \$24 000. It is indexed at $3.2\,\%$. Find the indexation increase and the new balance.

Working	Comment
Indexation increase $= 3.2\,\% \times 24\,000 = 0.032 \times 24\,000 = \$\,768$.	A percentage increase of the balance.
New balance $= 24\,000 + 768 = \$\,24\,768$.	Add the increase to the original balance.
Or directly: $24\,000 \times 1.032 = \$\,24\,768$.	Multiplying by 1.032 does both steps at once.

Example 2 — scaffolded

Using the repayment table in the Theory, find the compulsory annual repayment for a graduate whose repayment income is \$86 000.

Working	Comment
\$86 000 lies in the \$69 529 – \$129 717 band.	Locate the income in the table.
Income above the threshold $= 86\,000 - 69\,528 = \$\,16\,472$.	Only this part is counted.
Repayment $= 15\,\% \times 16\,472 = 0.15 \times 16\,472 = \$\,2470.80$.	Apply the 15c-in-the-dollar rate.

Example 3 — unscaffolded

A student owes \$30 000. This year the debt is indexed at $4\,\%$, and then a compulsory repayment of \$2500 is made. Find the balance at the end of the year.

The balance is indexed first:

$$30\,000 \times 1.04 = \$\,31\,200\,.$$

The compulsory repayment is then subtracted:

$$31\,200 - 2500 = \$\,28\,700\,.$$

∴ the balance at the end of the year is \$28 700 — despite the \$2500 repayment, the debt has fallen by only $30\,000 - 28\,700 = \$\,1300$, because indexation added \$1200 first.

Example 4 — unscaffolded

Using the repayment table, compare the compulsory repayment of a graduate earning \$89 000 with one earning \$90 000, and comment.

Both incomes lie in the $69 529 – $129 717 band, so each graduate repays 15c for every $1 above $69 528.

For $89 000:

$$15\% \times (89\,000 - 69\,528) = 15\% \times 19\,472 = \$2920.80.$$

For $90 000:

$$15\% \times (90\,000 - 69\,528) = 15\% \times 20\,472 = \$3070.80.$$

The second graduate earns $1000 more and repays $3070.80 - 2920.80 = \$150$ more.

∴ because only the income above the threshold is counted, an extra $1000 of income adds just $15\% \times 1000 = \$150$ to the repayment — the repayment rises smoothly, with no jump.

Practice questions

Scaffolded

Q1. A HECS-HELP debt of $18 500 is indexed at 2.8%. (a) Find the indexation increase. (b) Find the new balance. (c) Explain how the debt can grow even though it charges no interest.

Q2. Use the repayment table in the Theory for a graduate with a repayment income of $72 000. (a) State which income band the graduate is in. (b) Find how much of the income is above the threshold. (c) Find the compulsory annual repayment.

Q3. A student owes $26 000. The debt is indexed at 3.5%, and then a compulsory repayment of $1800 is made. (a) Find the balance after indexation. (b) Find the balance after the repayment. (c) State by how much the debt has fallen over the year.

Q4. Using the repayment table, two graduates earn $78 000 and $95 000. (a) Find the compulsory repayment of the graduate earning $78 000. (b) Find the compulsory repayment of the graduate earning $95 000. (c) Find how much more the second graduate repays.

Q5. A graduate owes $40 000, about to be indexed at 4%. (a) Find the balance if it is simply indexed. (b) Find the balance if the graduate first makes a $5000 voluntary repayment and the reduced balance is then indexed. (c) Find how much the voluntary repayment saves compared with paying nothing.

Q6. A graduate owes $21 000 and makes compulsory repayments of $3000 each year. (a) Ignoring indexation, estimate the number of years to clear the debt. (b) Explain why, in reality, it will take longer than this.

Unscaffolded

Q7. A HECS-HELP debt of $33 000 is indexed at 3.9%. Find the new balance and the indexation increase.

Q8. Using the repayment table in the Theory, find the compulsory annual repayment for a graduate whose repayment income is $88 000.

Q9. A debt of $15 000 is indexed at 2.5%. Find the new balance.

Q10. Using the repayment table, find the compulsory repayment for a graduate earning $62 000 and for one earning $102 000.

Q11. A student owes $45 000. The debt is indexed at 4.2% and then a compulsory repayment of $3200 is made. Find the balance at the end of the year.

Q12. Using the repayment table, compare the compulsory repayment for a graduate earning $74 000 with one earning $76 000. State how much extra the $76 000 graduate repays, and explain why that extra is so small compared with the $2000 of extra income.

Q13. A graduate owes $50 000, about to be indexed at 4%. Find how much is saved over the year if the graduate first makes a $10 000 voluntary repayment (so only the reduced balance is indexed), compared with making no voluntary repayment.

Q14. Using the repayment table in the Theory, find the compulsory annual repayment for a graduate with a repayment income of $140 000.

Q15. A debt of $20 000 is left unpaid and is indexed at 3% at the end of each year for two years. Find the balance after the two years.

Q16. A graduate has a repayment income of $100 000. Find their compulsory repayment, and find what percentage of their whole repayment income it is, correct to one decimal place.

Q17. A graduate's compulsory repayment is $3000, and their repayment income lies in the $69 529 – $129 717 band. Find the graduate's repayment income.

Q18. A debt of $24 000 is indexed at 3% at the start of each year, and a compulsory repayment of $4000 is made at the end of each year. Find the balance after two years.

Answers

Q1. (a) \$518. (b) \$19 018. (c) indexation increases the balance each year, so an unpaid debt still grows even though no interest is charged. **Q2.** (a) \$69 529 – \$129 717. (b) \$2472. (c) \$370.80. **Q3.** (a) \$26 910. (b) \$25 110. (c) \$890. **Q4.** (a) \$1270.80. (b) \$3820.80. (c) \$2550. **Q5.** (a) \$41 600. (b) \$36 400. (c) \$5200. **Q6.** (a) 7 years. (b) indexation adds to the balance each year before the repayment, so more must be repaid overall. **Q7.** new balance \$34 287; increase \$1287. **Q8.** \$2770.80. **Q9.** \$15 375. **Q10.** nil (\$0) and \$4870.80. **Q11.** \$43 690. **Q12.** \$74 000 → \$670.80; \$76 000 → \$970.80; \$300 more, because only the income above the \$69 528 threshold is counted, so each extra \$1 adds just 15c. **Q13.** \$10 400. **Q14.** \$10 776.11. **Q15.** \$21 218. **Q16.** \$4570.80, which is 4.6 % of the whole income. **Q17.** \$89 528. **Q18.** \$17 341.60.

Fully-worked solutions

Q1. (a) Increase $=2.8\% \times 18\,500=0.028 \times 18\,500=\$518$. (b) New balance $=18\,500+518=\$19\,018$ (or $18\,500 \times 1.028=\$19\,018$). (c) Each year the balance is indexed — increased by a set percentage — so even with no interest, a debt that is not being paid down still grows.

Q2. (a) \$72 000 falls in the \$69 529 – \$129 717 band. (b) Income above the threshold $=72\,000-69\,528=\$2472$. (c) Repayment $=15\% \times 2472=0.15 \times 2472=\$370.80$.

Q3. (a) Indexed balance $=26\,000 \times 1.035=\$26\,910$. (b) After the repayment: $26\,910-1800=\$25\,110$. (c) The debt has fallen by $26\,000-25\,110=\$890$ (the \$1800 repayment was partly offset by the \$910 of indexation).

Q4. (a) \$78 000 is above the threshold by $78\,000-69\,528=\$8472$, so the repayment is $15\% \times 8472=\$1270.80$. (b) \$95 000 is above the threshold by $95\,000-69\,528=\$25\,472$, so the repayment is $15\% \times 25\,472=\$3820.80$. (c) The difference is $3820.80-1270.80=\$2550$ (which is 15% of the \$17 000 difference in income).

Q5. (a) Indexed directly: $40\,000 \times 1.04=\$41\,600$. (b) After a \$5000 voluntary repayment the balance is \$35 000, which is then indexed: $35\,000 \times 1.04=\$36\,400$. (c) The saving is $41\,600-36\,400=\$5200$ (the \$5000 repaid, plus \$200 of indexation that is no longer charged on it).

Q6. (a) $\frac{21\,000}{3000}=7$ years. (b) In reality the balance is indexed each year before the repayment is taken, so the total amount repaid is more than \$21 000 and it takes longer than 7 years.

Q7. New balance $=33\,000 \times 1.039=\$34\,287$; the increase is $34\,287-33\,000=\$1287$.

Q8. \$88 000 is in the \$69 529 – \$129 717 band. Income above the threshold $=88\,000-69\,528=\$18\,472$, so the repayment is $15\% \times 18\,472=\$2770.80$.

Q9. New balance $=15\,000 \times 1.025=\$15\,375$.

Q10. \$62 000 is below the \$69 528 threshold, so there is no compulsory repayment (\$0). \$102 000 is above the threshold by $102\,000-69\,528=\$32\,472$, so the repayment is $15\% \times 32\,472=\$4870.80$.

Q11. Indexed balance $=45\,000 \times 1.042=\$46\,890$. After the repayment: $46\,890-3200=\$43\,690$.

Q12. \$74 000 is above the threshold by $74\,000-69\,528=\$4472$, so the repayment is $15\% \times 4472=\$670.80$. \$76 000 is above the threshold by $76\,000-69\,528=\$6472$, so the repayment is $15\% \times 6472=\$970.80$. The \$76 000 graduate repays $970.80-670.80=\$300$ more. The extra is small because only the income **above** the threshold is counted, so the extra \$2000 is charged at 15c in the dollar: $15\% \times 2000=\$300$.

Q13. With no voluntary repayment: $50\,000 \times 1.04=\$52\,000$. With a \$10 000 voluntary repayment first, the \$40 000 balance is indexed: $40\,000 \times 1.04=\$41\,600$. The saving over the year is $52\,000-41\,600=\$10\,400$.

Q14. \$140 000 is in the \$129 718 – \$186 050 band, so the repayment is \$9028 plus 17% of the income over \$129 717:

$$140\,000 - 129\,717 = \$10\,283, 17\% \times 10\,283 = \$1748.11,$$

so the repayment is $9028 + 1748.11 = \$10\,776.11$.

Q15. Indexing twice: $20\,000 \times 1.03 = 20\,600$, then $20\,600 \times 1.03 = \$21\,218$. (Equivalently $20\,000 \times 1.03^2 = \$21\,218$.)

Q16. Income above the threshold $= 100\,000 - 69\,528 = \$30\,472$, so the repayment is $15\% \times 30\,472 = \$4570.80$. As a percentage of the whole income,

$$\frac{4570.80}{100\,000} \times 100\% = 4.5708\% \approx 4.6\%.$$

This is well below the 15% marginal rate, because only the income above the threshold is charged.

Q17. The repayment is 15% of the income above the threshold, so the income above the threshold is $\frac{3000}{0.15} = \$20\,000$. The repayment income is therefore $69\,528 + 20\,000 = \$89\,528$.

Q18. Year 1: indexed $24\,000 \times 1.03 = 24\,720$, then $24\,720 - 4000 = \$20\,720$. Year 2: indexed $20\,720 \times 1.03 = 21\,341.60$, then $21\,341.60 - 4000 = \$17\,341.60$. ∴ after two years the balance is \$17 341.60.

Chapter 9 Money management: interest, loans and credit — 9F Wage growth and the cost of living

Theory

Wages rise over time, but so do prices. Whether you are actually **better off** depends on how your pay rise compares with the rising **cost of living**. This section looks at pay rises as percentage increases, at how inflation is measured, and at the difference between a **nominal** gain (more dollars) and a **real** gain (more buying power).

Pay rises as a percentage increase.

A pay rise is almost always quoted as a **percentage** of the current wage. To apply a rise of $r\,\%$ to a wage, multiply by $\left(1+\frac{r}{100}\right)$:

$$\text{new wage} = \text{old wage} \times \left(1+\frac{r}{100}\right).$$

For example, a $\$50\,000$ salary with a $3\,\%$ rise becomes $50\,000 \times 1.03 = \$51\,500$, a dollar increase of $\$1\,500$.

Because the rise is a **percentage** of the wage, the same percentage gives a bigger dollar increase on a bigger salary. When comparing two pay rises, always work out the **dollar** increase, not just the percentage — a larger percentage on a smaller wage can be worth fewer dollars.

Inflation and the Consumer Price Index.

Inflation is the general rise in prices over time. In Australia it is measured by the **Consumer Price Index (CPI)** — the cost of a fixed “basket” of everyday goods and services, written as an index number. A base period is set to 100; if the index later reads 107, prices have risen $7\,\%$ since the base period. The **annual inflation rate** is the percentage change in the CPI over a year:

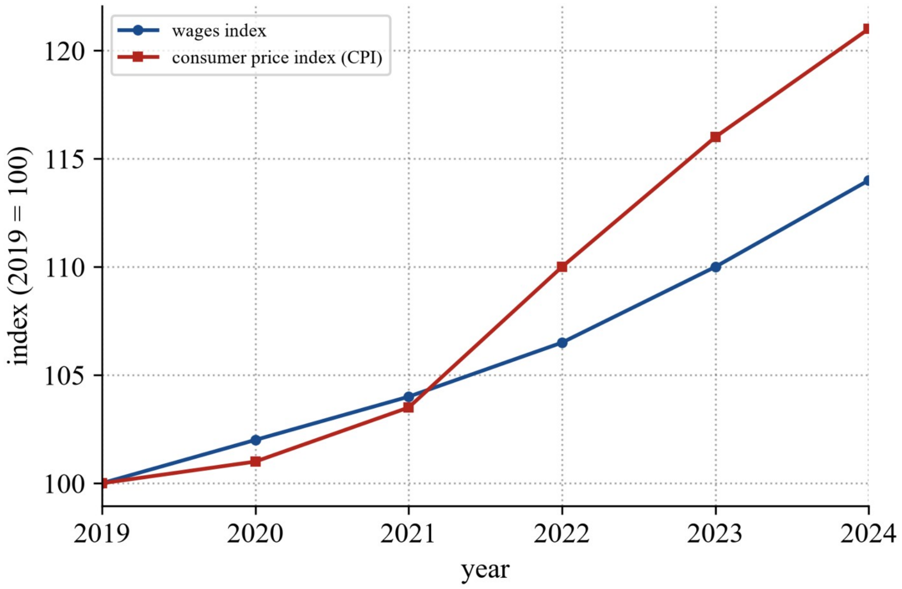
$$\text{inflation rate} = \frac{\text{new CPI} - \text{old CPI}}{\text{old CPI}} \times 100\,\%.$$

Nominal versus real change.

A **nominal** change is the change in the number of dollars. A **real** change measures the change in **buying power** — what those dollars can actually purchase. A pay rise only makes you better off if it **beats inflation**. A good working estimate at this level is

$$\text{real increase} \approx \text{wage rise} - \text{inflation rate}.$$

So a $4\,\%$ pay rise in a year of $2.5\,\%$ inflation is a real gain of about $1.5\,\%$ (your buying power grew); a $2\,\%$ pay rise in a year of $3\,\%$ inflation is a real **loss** of about $1\,\%$ (you can buy less than before), even though you earn more dollars.



In the graph above, wages and prices both start at an index of 100. Wages climb steadily, but from 2022 the CPI rises **faster**, so buying power falls even while wages keep going up — the nominal wage rises but the real wage does not.

Comparing prices across years (adjusting for inflation).

To compare a past price or wage fairly with today's, scale it by the ratio of the CPI values:

$$\text{value in today's dollars} = \text{past value} \times \frac{\text{CPI today}}{\text{CPI in the past year}}.$$

If a loaf of bread cost $\$2.00$ when the CPI was 100 and the CPI is now 130, then in today's dollars that loaf effectively cost $2.00 \times \frac{130}{100} = \$2.60$ — a useful way to see whether something has become more or less expensive in real terms.

Always state the **units** (dollars, dollars per hour, dollars per week) and, when a question asks about being "better off", say whether you mean a **nominal** or a **real** change.

Worked examples

Example 1 — scaffolded

An employee currently earns $\$58\,000$ per year and receives a $3.5\,\%$ pay rise. Find the new salary and the dollar amount of the increase.

Working	Comment
Multiplier $= 1 + \frac{3.5}{100} = 1.035$.	A rise of $r\,\%$ multiplies by $1 + r/100$.
New salary $= 58\,000 \times 1.035 = \$60\,030$.	Multiply the old salary by 1.035.
Increase $= 60\,030 - 58\,000 = \$2\,030$.	The dollar rise is new − old.
Check: $3.5\,\%$ of $58\,000 = 0.035 \times 58\,000 = \$2\,030$.	The increase should equal $r\,\%$ of the old salary.

Example 2 — scaffolded

In one year a worker receives a $2.8\,\%$ pay rise, while inflation over the same year is $3.5\,\%$. Decide whether the worker's buying power has risen or fallen, and estimate the real change.

Working	Comment
Nominal change $= +2.8\,\%$ (more dollars).	The pay rise is the nominal change.

Working	Comment
Real change ≈ wage rise − inflation $= 2.8\% - 3.5\% = -0.7\%$.	Compare the rise with inflation.
The real change is negative.	A negative real change means less buying power.
So the worker's buying power has **fallen** by about 0.7%, even though the wage rose.	More dollars, but they buy less.

Example 3 — unscaffolded

A litre of milk cost $\$2.40$ in 2010, when the CPI was 95.0. By 2024 the CPI had risen to 135.0. Express the 2010 price in 2024 dollars, and comment on what this tells you.

Scaling the past price by the ratio of the CPI values,

$$\text{value in 2024 dollars} = 2.40 \times \frac{135.0}{95.0} = 2.40 \times 1.4211 = \$3.41 .$$

So the $\$2.40$ that a litre of milk cost in 2010 is equivalent to $\$3.41$ in 2024 dollars. If milk actually costs less than $\$3.41$ in 2024, it has become **cheaper** in real terms; if it costs more, it has become dearer.

∴ in today's dollars, the 2010 milk price was about $\$3.41$.

Example 4 — unscaffolded

Worker A earns $\$90\,000$ and is offered a 2.5% rise. Worker B earns $\$60\,000$ and is offered a 4% rise. Which worker gains more **dollars**?

Worker A's increase is $90\,000 \times \frac{2.5}{100} = \$2\,250$, giving a new salary of $\$92\,250$. Worker B's increase is $60\,000 \times \frac{4}{100} = \$2\,400$, giving a new salary of $\$62\,400$. Although Worker A earns much more, Worker B's higher percentage on a smaller salary produces the larger **dollar** rise ($\$2\,400$ versus $\$2\,250$).

∴ Worker B gains more dollars, showing that a percentage rise and a dollar rise are not the same thing.

Practice questions

Scaffolded

Q1. A teacher earns $\$72\,000$ per year and receives a 3% pay rise. (a) Find the new salary. (b) Find the dollar amount of the increase.

Q2. A casual worker is paid $\$34.80$ per hour and receives a 4.5% rise. (a) Find the new hourly rate, correct to the nearest cent. (b) Find the increase per hour, correct to the nearest cent.

Q3. A weekly grocery basket costs $\$240$. Over one year, grocery prices rise by 3.2%. (a) Find the new cost of the basket. (b) Find the increase in cost.

Q4. A worker receives a 5% pay rise in a year when inflation is 3%. (a) State whether this is a real gain or a real loss in buying power. (b) Estimate the real increase as a percentage.

Q5. A cup of coffee cost $\$3.50$ in 2018, when the CPI was 100. In 2024 the CPI is 120. (a) By what percentage have prices risen since 2018? (b) Express the 2018 coffee price in 2024 dollars.

Q6. Worker P earns $\$48\,000$ and receives a 3% rise. Worker Q earns $\$75\,000$ and receives a 2% rise. (a) Find each worker's dollar increase. (b) State which worker receives the larger dollar increase.

Unscaffolded

Q7. A nurse earning $\$64\,000$ receives a $3.75\,\%$ pay rise. Find the new salary.

Q8. An employee earning $\$80\,000$ receives a $3\,\%$ rise one year and a further $2.5\,\%$ rise the next year. Find the salary after both rises.

Q9. A worker receives a $2.5\,\%$ pay rise in a year when inflation is $3.8\,\%$. State whether the worker’s buying power has risen or fallen, and estimate the real change.

Q10. The price of petrol rose from $\$1.60$ per litre to $\$2.16$ per litre. Find the percentage increase in the price.

Q11. A weekly rent of $\$420$ is increased by $5.5\,\%$. Find the new weekly rent.

Q12. A worker earned $\$38\,000$ in 2012, when the CPI was 98. Express this wage in 2024 dollars, given the 2024 CPI is 136 (answer to the nearest dollar).

Q13. The CPI rose from 112 to 119 over one year. Find the annual inflation rate, correct to two decimal places.

Q14. A retail employee’s weekly wage of $\$846.20$ is increased by $5.75\,\%$. Find the new weekly wage, correct to the nearest cent.

Q15. A basket of goods costs $\$150$ in one city and $\$174$ in another. By what percentage is the basket more expensive in the second city?

Q16. A worker receives a $4.2\,\%$ pay rise in a year when inflation is also $4.2\,\%$. Explain what happens to the worker’s buying power.

Q17. Prices are rising at $3.5\,\%$ per year. Goods that cost $\$100$ today will cost how much in one year’s time?

Q18. A worker earning $\$56\,000$ receives a $3.5\,\%$ pay rise in a year when inflation is $2.9\,\%$. (a) Find the new salary. (b) State whether the worker’s buying power has risen or fallen, and estimate the real change.

Answers

Q1. (a) $\$74\,160$. (b) $\$2\,160$. **Q2.** (a) $\$36.37$. (b) $\$1.57$. **Q3.** (a) $\$247.68$. (b) $\$7.68$. **Q4.** (a) a real gain. (b) about $+2\%$. **Q5.** (a) 20%. (b) $\$4.20$. **Q6.** (a) P $\$1\,440$, Q $\$1\,500$. (b) Worker Q. **Q7.** $\$66\,400$. **Q8.** $\$84\,460$. **Q9.** buying power has fallen; real change $\approx -1.3\%$. **Q10.** 35%. **Q11.** $\$443.10$. **Q12.** $\$52\,735$ (to the nearest dollar). **Q13.** 6.25%. **Q14.** $\$894.86$. **Q15.** 16%. **Q16.** buying power is unchanged (real change $\approx 0\%$). **Q17.** $\$103.50$. **Q18.** (a) $\$57\,960$. (b) risen; real change $\approx +0.6\%$.

Fully-worked solutions

Q1. (a) New salary $= 72\,000 \times \left(1 + \frac{3}{100}\right) = 72\,000 \times 1.03 = \$74\,160$. (b) Increase $= 74\,160 - 72\,000 = \$2\,160$ (equal to 3% of $\$72\,000$).

Q2. (a) New rate $= 34.80 \times 1.045 = 36.366 = \$36.37$ (to the nearest cent). (b) Increase $= 36.366 - 34.80 = 1.566 = \$1.57$ per hour.

Q3. (a) New cost $= 240 \times 1.032 = \$247.68$. (b) Increase $= 247.68 - 240 = \$7.68$.

Q4. (a) The pay rise of 5% is greater than inflation of 3%, so buying power grows — a **real gain**. (b) Real increase $\approx 5\% - 3\% = +2\%$.

Q5. (a) The CPI rose from 100 to 120, an increase of $\frac{120 - 100}{100} \times 100\% = 20\%$. (b) In 2024 dollars the coffee cost $3.50 \times \frac{120}{100} = \$4.20$.

Q6. (a) Worker P: $48\,000 \times \frac{3}{100} = \$1\,440$. Worker Q: $75\,000 \times \frac{2}{100} = \$1\,500$. (b) Worker Q receives the larger dollar increase ($\$1\,500 > \$1\,440$), even though Q's percentage rise is smaller, because it applies to a larger salary.

Q7. New salary $= 64\,000 \times \left(1 + \frac{3.75}{100}\right) = 64\,000 \times 1.0375 = \$66\,400$.

Q8. After the first rise: $80\,000 \times 1.03 = \$82\,400$. After the second: $82\,400 \times 1.025 = \$84\,460$. (Equivalently $80\,000 \times 1.03 \times 1.025$.)

Q9. Nominal change $= +2.5\%$. Real change $\approx 2.5\% - 3.8\% = -1.3\%$. Because this is negative, the worker's buying power has **fallen** by about 1.3%, despite the pay rise.

Q10. Percentage increase $= \frac{2.16 - 1.60}{1.60} \times 100\% = \frac{0.56}{1.60} \times 100\% = 35\%$.

Q11. New rent $= 420 \times 1.055 = \$443.10$ per week.

Q12. Value in 2024 dollars $= 38\,000 \times \frac{136}{98} = \$52\,734.69$, which is $\$52\,735$ to the nearest dollar.

Q13. Inflation rate $= \frac{119 - 112}{112} \times 100\% = \frac{7}{112} \times 100\% = 6.25\%$.

Q14. New weekly wage $= 846.20 \times 1.0575 = 894.8565 = \$894.86$ per week (to the nearest cent).

Q15. Percentage more expensive $= \frac{174 - 150}{150} \times 100\% = \frac{24}{150} \times 100\% = 16\%$.

Q16. The pay rise ($4.2\,\%$) exactly equals inflation ($4.2\,\%$), so the extra dollars are exactly cancelled by higher prices. The real change is about $0\,\%$: buying power is **unchanged**.

Q17. Cost next year $= 100 \times \left(1 + \dfrac{3.5}{100}\right) = 100 \times 1.035 = \$103.50$.

Q18. (a) New salary $= 56\,000 \times 1.035 = \$57\,960$. (b) Real change $\approx 3.5\,\% - 2.9\,\% = +0.6\,\%$. Because this is positive, the worker's buying power has **risen** slightly — the pay rise just beat inflation.