Foundation Mathematics Units 3 & 4

Chapter 4 — Algebra, formulas and equations

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Theory

Algebra uses letters, called **pronumerals**, to stand for numbers we do not yet know or that can change. An **algebraic expression** is a combination of pronumerals and numbers joined by operations — for example $3n+5$. Algebra lets us write a general rule once and then use it for any values, which is exactly what a **formula** does.

The conventions of algebra.

Algebra is written in a compact, agreed way:

- A number written next to a pronumeral means **multiply**: $3n$ means $3\times n$, and the 3 is called the **coefficient** of n.
- ab means $a\times b$; a pronumeral times itself is written as a power: $a\times a=a^2$.
- Division is written as a fraction: $n\div 4=\frac{n}{4}$.
- A **term** is a single number, pronumeral, or product of these (e.g. $3n$, -5, $2ab$); terms are separated by $+$ or $-$ signs.

Writing an expression from words.

Turn the words into operations, one piece at a time. "$\$20$ plus $\$6$ for each ticket" becomes $20+6t$, where t is the number of tickets. Watch the order: "6 less than m" is $m-6$ (not $6-m$), and "add 4 to m, then multiply the result by 3" is $3(m+4)$.

Substituting values.

To **substitute** (or **evaluate**), replace each pronumeral with its number and calculate, following the order of operations. For example, if $C=30+9n$ then when $n=4$,

$$C=30+9\times 4=30+36=66.$$

Collecting like terms.

Like terms have exactly the same pronumeral part, such as $7m$ and $2m$, or $4ab$ and ab. Only like terms can be added or subtracted, by combining their coefficients:

$$7m+2m=9m, 5c+3d-2c+d=3c+4d.$$

Unlike terms (such as $3c$ and $4d$) cannot be combined — they stay separate.

Expanding brackets.

To **expand** a bracket, multiply every term inside it by the term outside. This is the **distributive law** (on the Foundation Mathematics formula sheet):

$$a(b+c)=ab+ac.$$

For example $4(3n-2)=12n-8$. After expanding, collect any like terms:

$$2(3x+5)+4(x+1)=6x+10+4x+4=10x+14.$$

Worked examples

Example 1 — scaffolded

A plumber charges an $\$80$ call-out fee plus $\$65$ for each hour worked. Write an expression for the cost of a job lasting h hours, then find the cost of a 3-hour job.

Working	Comment
Cost $=80+65\times h=80+65h$ dollars.	Fixed fee plus $\$65$ for each hour: 65 times h.
For $h=3$: cost $=80+65\times 3$.	Substitute $h=3$.
$=80+195=\$275$.	Multiply before adding (order of operations).

Example 2 — scaffolded

The cost of hiring a car is given by the formula $C=45d+0.25k$, where d is the number of days and k is the number of kilometres driven. Find the cost of hiring the car for 3 days and driving 200 km.

Working	Comment
$C=45d+0.25k$ with $d=3$ and $k=200$.	Identify each pronumeral's value.
$C=45\times 3+0.25\times 200$.	Substitute both values.
$=135+50=\$185$.	Do both multiplications, then add.

Example 3 — unscaffolded

Simplify $6a+4b-2a+3b+5$.

Group the like terms. The a-terms are $6a$ and $-2a$; the b-terms are $4b$ and $3b$; the number 5 has no like term.

$$6a-2a=4a,\ 4b+3b=7b.$$

$\therefore 6a+4b-2a+3b+5=4a+7b+5$.

Example 4 — unscaffolded

Expand and simplify $3(2x+4)+2(x-1)$.

Expand each bracket using the distributive law, then collect like terms:

$$3(2x+4)+2(x-1)=6x+12+2x-2.$$

The x-terms give $6x+2x=8x$, and the numbers give $12-2=10$.

$\therefore 3(2x+4)+2(x-1)=8x+10$.

Practice questions

Scaffolded

Q1. Write an algebraic expression for each description. (a) 5 more than a number n. (b) 3 times a number p. (c) A number t is reduced by 8, and the result is then doubled.

Q2. Evaluate each expression when $a=4$ and $b=3$. (a) $2a+b$. (b) ab. (c) a^2-b.

Q3. A market stall pays a $\$25$ site fee for the day plus $\$12$ for each box of produce it buys, so its costs are $C=25+12n$ dollars, where n is the number of boxes. (a) Find C when $n=5$. (b) Find C when $n=10$. (c) The stall sells all n boxes for $\$20$ a box, so its takings are $R=20n$ dollars. Find R when $n=10$.

Q4. Simplify by collecting like terms. (a) $3x+5x$. (b) $7y-2y+y$. (c) $4a+3b+2a-b$.

Q5. Expand each bracket. (a) $2(x+3)$. (b) $5(2n-1)$. (c) $3(4-p)$.

Q6. Expand and simplify. (a) $2(x+5)+3x$. (b) $3(n+2)+2(n+1)$. (c) $5(a+2)-3a$.

Unscaffolded

Q7. A taxi charges a $\$4$ flagfall plus $\$2$ for each kilometre. Write an expression for the cost of a trip of k kilometres.

Q8. Evaluate $3x-2y$ when $x=5$ and $y=2$.

Q9. Simple interest is $I=\frac{Prt}{100}$. Find I when $P=2000$, $r=4$ and $t=3$.

Q10. Simplify $8m-3n+2m+5n$.

Q11. Expand $6(2x-5)$.

Q12. Expand and simplify $4(x+2)+3(x-1)$.

Q13. A gym charges a $\$50$ joining fee plus $\$15$ per week. Write an expression for the total cost after w weeks, and find the cost after 8 weeks.

Q14. The area of a triangle is $A=\frac{1}{2}bh$. Find A when $b=12$ and $h=5$.

Q15. Simplify $5p+7-2p-3$.

Q16. Expand and simplify $2(3n+1)-4$.

Q17. A rectangle has width w metres and a length that is 3 m more than its width. (a) Write an expression for its length. (b) Write and simplify an expression for its perimeter. (c) Write an expression for its area.

Q18. A different plumber charges a $\$90$ call-out fee plus $\$70$ per hour. Write an expression for the cost of an h-hour job, and find the cost of a job lasting 4.5 hours.

Answers

Q1. (a) $n+5$. (b) $3p$. (c) $2(t-8)$. **Q2.** (a) 11. (b) 12. (c) 13. **Q3.** (a) $\$85$. (b) $\$145$. (c) $\$200$. **Q4.** (a) $8x$. (b) $6y$. (c) $6a+2b$. **Q5.** (a) $2x+6$. (b) $10n-5$. (c) $12-3p$. **Q6.** (a) $5x+10$. (b) $5n+8$. (c) $2a+10$. **Q7.** $4+2k$ dollars. **Q8.** 11. **Q9.** $\$240$. **Q10.** $10m+2n$. **Q11.** $12x-30$. **Q12.** $7x+5$. **Q13.** $50+15w$; $\$170$. **Q14.** 30. **Q15.** $3p+4$. **Q16.** $6n-2$. **Q17.** (a) $w+3$. (b) $2(w+(w+3))=4w+6$. (c) $w(w+3)$. **Q18.** $90+70h$; $\$405$.

Fully-worked solutions

Q1. (a) "5 more than n" adds 5 to n: $n+5$. (b) "3 times p" is $3p$. (c) First "t reduced by 8" is $t-8$; "the result doubled" multiplies that by 2: $2(t-8)$.

Q2. Substitute $a=4$, $b=3$. (a) $2a+b=2\times4+3=8+3=11$. (b) $ab=4\times3=12$. (c) $a^2-b=4^2-3=16-3=13$.

Q3. (a) $C=25+12\times5=25+60=\$85$. (b) $C=25+12\times10=25+120=\$145$. (c) $R=20n=20\times10=\$200$. Note that the takings have no fixed term: selling no boxes brings in nothing, whereas the $\$25$ site fee is paid whatever happens.

Q4. (a) $3x+5x=8x$. (b) $7y-2y+y=6y$. (c) $4a+2a=6a$ and $3b-b=2b$, so $4a+3b+2a-b=6a+2b$.

Q5. (a) $2(x+3)=2x+6$. (b) $5(2n-1)=10n-5$. (c) $3(4-p)=12-3p$.

Q6. (a) $2(x+5)+3x=2x+10+3x=5x+10$. (b) $3(n+2)+2(n+1)=3n+6+2n+2=5n+8$. (c) $5(a+2)-3a=5a+10-3a=2a+10$.

Q7. A $\$4$ flagfall plus $\$2$ per kilometre for k kilometres is $4+2k$ dollars.

Q8. $3x-2y=3\times5-2\times2=15-4=11$.

Q9. $I=\dfrac{Prt}{100}=\dfrac{2000\times4\times3}{100}=\dfrac{24\,000}{100}=\$240$.

Q10. The m-terms give $8m+2m=10m$; the n-terms give $-3n+5n=2n$. So $8m-3n+2m+5n=10m+2n$.

Q11. $6(2x-5)=6\times2x-6\times5=12x-30$.

Q12. $4(x+2)+3(x-1)=4x+8+3x-3=7x+5$.

Q13. A $\$50$ joining fee plus $\$15$ per week for w weeks is $50+15w$. After 8 weeks: $50+15\times8=50+120=\$170$.

Q14. $A=\dfrac{1}{2}bh=\dfrac{1}{2}\times12\times5=6\times5=30$.

Q15. The p-terms give $5p-2p=3p$; the numbers give $7-3=4$. So $5p+7-2p-3=3p+4$.

Q16. $2(3n+1)-4=6n+2-4=6n-2$.

Q17. (a) The length is 3 m more than the width w, so length $=w+3$. (b) Perimeter $=2(\text{length}+\text{width})=2(w+(w+3))=2(2w+3)=4w+6$. (c) Area $=\text{length}\times\text{width}=w(w+3)$.

Q18. The cost is a $\$90$ call-out plus $\$70$ per hour for h hours: $90+70h$. For a 4.5-hour job: $90+70\times4.5=90+315=\$405$.

Theory

An **equation** is a mathematical statement that two expressions are equal — it always contains an equals sign, such as $3x+7=22$. To **solve** an equation is to find the value of the unknown (here x) that makes the statement true. That value is called the **solution**.

The balance method.

An equation is like a set of balanced scales: the two sides are equal, so whatever you do to one side you must do to the **other side** as well to keep it balanced. To get the unknown by itself we undo the operations around it using **inverse operations** — addition undoes subtraction, multiplication undoes division, and vice versa.

One-step equations.

If a single operation is applied to the unknown, undo it once. For $x+8=20$, subtract 8 from both sides: $x=12$. For $5x=45$, divide both sides by 5: $x=9$.

Two-step equations.

When two operations are applied, undo them in **reverse order** — undo the + or − first, then the × or ÷. To solve $3x+7=22$:

$$3x+7=22 \Rightarrow 3x=15 \Rightarrow x=5.$$

(Subtract 7, then divide by 3.)

Unknown on both sides.

If the unknown appears on both sides, first collect the unknowns on one side (usually the side with more of them) by adding or subtracting. To solve $6x-5=2x+19$, subtract $2x$ from both sides to get $4x-5=19$, then solve as a two-step equation: $4x=24$, so $x=6$.

Equations with brackets.

Expand the bracket first (multiply through), then solve. To solve $4(x-3)=20$, expand to $4x-12=20$, giving $4x=32$ and $x=8$. (You could also divide both sides by 4 first: $x-3=5$.)

Equations with a fraction.

If the unknown is inside a fraction, **multiply** both sides by the denominator to clear it. To solve $\frac{x+1}{2}=6$, multiply both sides by 2: $x+1=12$, so $x=11$.

Writing an equation from a problem.

Many real problems are solved by turning words into an equation:

1. Choose a pronumeral for the unknown quantity and say what it stands for.
2. Write an equation that describes the situation.
3. Solve the equation.
4. Answer the question in words, with units.

Checking a solution.

Always check by **substituting** your answer back into the original equation to confirm both sides are equal. For $x=5$ in $3x+7=22$: $3(5)+7=15+7=22$. ✓

Worked examples

Example 1 — scaffolded

Solve $9x-7=5x+13$.

Working	Comment
$9x-7-5x=5x+13-5x$	Subtract $5x$ from both sides to gather the unknowns on the left.
$4x-7=13$	Simplify: $9x-5x=4x$.
$4x=20$	Add 7 to both sides.
$x=5$	Divide both sides by 4.
Check: $9(5)-7=38$ and $5(5)+13=38$. ✓	Substitute back into the original equation.

Example 2 — scaffolded

Solve $3(2x-1)=21$.

Working	Comment
$6x-3=21$	Expand the bracket: $3\times 2x=6x$, $3\times(-1)=-3$.
$6x=24$	Add 3 to both sides.
$x=4$	Divide both sides by 6.
Check: $3(2\times 4-1)=3(7)=21$. ✓	Substitute back to confirm.

Example 3 — unscaffolded

Solve $5(x+2)=45$.

Expanding the bracket gives $5x+10=45$. Subtracting 10 from both sides gives $5x=35$, and dividing both sides by 5 gives $x=7$. Checking: $5(7+2)=5\times 9=45$. ✓

$\therefore x=7$.

Example 4 — unscaffolded

A plumber charges an $\$80$ call-out fee plus $\$65$ for each hour worked. A job cost $\$340$ in total. How many hours did the plumber work?

Let h be the number of hours worked. The total cost is the call-out fee plus $\$65$ per hour, so

$$80+65h=340.$$

Subtracting 80 from both sides gives $65h=260$, and dividing both sides by 65 gives $h=4$.

$\therefore$ the plumber worked 4 hours.

Practice questions

Scaffolded

Q1. Solve each one-step equation. (a) $x+12=30$ (b) $x-7=15$ (c) $4x=52$

Q2. Solve each two-step equation. (a) $2x+5=17$ (b) $3x-8=13$ (c) $\frac{x}{4}+2=6$

Q3. Solve each equation with the unknown on both sides. (a) $4x+3=2x+13$ (b) $7x-2=4x+16$

Q4. Solve each equation by expanding the bracket first. (a) $2(x+4)=18$ (b) $5(x-1)=35$

Q5. When a number is tripled and 4 is added, the result is 25. (a) Write an equation using n for the number. (b) Solve it to find the number.

Q6. A rectangle is 3 cm longer than it is wide, and its perimeter is 26 cm. (a) Taking the width as w, write an equation for the perimeter. (b) Solve it to find the width. (c) State the length.

Unscaffolded

Q7. Solve $5x+9=44$.

Q8. Solve $8x-13=3x+22$.

Q9. Solve $3(x+5)=27$.

Q10. Solve $\frac{x}{5}-3=2$.

Q11. Solve $6x+4=4x+20$.

Q12. Solve $2(3x-1)=28$.

Q13. Solve $\frac{x-3}{4}=5$.

Q14. A taxi charges a $\$4$ flagfall plus $\$2.20$ per kilometre. A trip cost $\$26$. Write and solve an equation to find the distance travelled.

Q15. A gym charges a $\$50$ joining fee plus $\$15$ per week. A member has paid $\$200$ in total so far. Write and solve an equation to find how many weeks they have been a member.

Q16. Solve $8x-5=4x+31$.

Q17. Solve $4(2x+3)=5(x+6)$.

Q18. A water tank holds 500 L and drains at 25 L per minute. Write and solve an equation to find how many minutes until 150 L remain.

Answers

Q1. (a) 18; (b) 22; (c) 13. **Q2.** (a) 6; (b) 7; (c) 16. **Q3.** (a) 5; (b) 6. **Q4.** (a) 5; (b) 8. **Q5.** (a) $3n+4=25$; (b) $n=7$. **Q6.** (a) $2(w+w+3)=26$ (or $4w+6=26$); (b) $w=5$ cm; (c) length $=8$ cm. **Q7.** $x=7$. **Q8.** $x=7$. **Q9.** $x=4$. **Q10.** $x=25$. **Q11.** $x=8$. **Q12.** $x=5$. **Q13.** $x=23$. **Q14.** $4+2.20k=26$; $k=10$ km. **Q15.** $50+15w=200$; $w=10$ weeks. **Q16.** $x=9$. **Q17.** $x=6$. **Q18.** $500-25t=150$; $t=14$ minutes.

Fully-worked solutions

Q1. (a) Subtract 12: $x=30-12=18$. (b) Add 7: $x=15+7=22$. (c) Divide by 4: $x=52\div 4=13$.

Q2. (a) $2x=17-5=12$, so $x=6$. (b) $3x=13+8=21$, so $x=7$. (c) Subtract 2: $\frac{x}{4}=4$, then multiply by 4: $x=16$.

Q3. (a) Subtract $2x$: $2x+3=13$, so $2x=10$ and $x=5$. (b) Subtract $4x$: $3x-2=16$, so $3x=18$ and $x=6$.

Q4. (a) Expand: $2x+8=18$, so $2x=10$ and $x=5$. (b) Expand: $5x-5=35$, so $5x=40$ and $x=8$.

Q5. (a) "Tripled" is $3n$ and "add 4" gives $3n+4$, so $3n+4=25$. (b) $3n=21$, so $n=7$.

Q6. (a) The width is w and the length is $w+3$; the perimeter is $2(\text{width}+\text{length})=2(w+w+3)=26$. (b) $2(2w+3)=26$, so $4w+6=26$, giving $4w=20$ and $w=5$ cm. (c) Length $=w+3=8$ cm.

Q7. $5x=44-9=35$, so $x=7$.

Q8. Subtract $3x$: $5x-13=22$, so $5x=35$ and $x=7$.

Q9. Expand: $3x+15=27$, so $3x=12$ and $x=4$. (Or divide by 3 first: $x+5=9$.)

Q10. Add 3: $\frac{x}{5}=5$, then multiply by 5: $x=25$.

Q11. Subtract $4x$: $2x+4=20$, so $2x=16$ and $x=8$.

Q12. Expand: $6x-2=28$, so $6x=30$ and $x=5$.

Q13. Multiply both sides by 4: $x-3=20$, so $x=23$.

Q14. Let k be the distance in km. The fare is $4+2.20k=26$. Subtract 4: $2.20k=22$, so $k=22\div 2.20=10$ km.

Q15. Let w be the number of weeks. The total paid is $50+15w=200$. Subtract 50: $15w=150$, so $w=10$ weeks.

Q16. Subtract $4x$: $4x-5=31$, so $4x=36$ and $x=9$.

Q17. Expand both sides: $8x+12=5x+30$. Subtract $5x$: $3x+12=30$, so $3x=18$ and $x=6$.

Q18. Let t be the number of minutes. The water remaining is $500-25t=150$. Subtract 500: $-25t=-350$, so $t=14$ minutes.

Theory

A **formula** is a rule written with pronumerals, such as $P=4s$ for the perimeter of a square with side length s. The pronumeral on its own on the left, here P, is called the **subject** of the formula. **Transposing** (or **rearranging**) a formula means rewriting it so that a *different* pronumeral becomes the subject — for example turning $P=4s$ into $s=\frac{P}{4}$ so that we can find the side length of a square from its perimeter.

The golden rule.

Transposing uses exactly the same idea as solving an equation: **whatever you do to one side, you must do to the other**, and you **undo** the operations attached to the variable you want, using **inverse operations** (+ undoes −, × undoes ÷, a square root undoes a square). Work in the reverse order to the order of operations.

Making a variable the subject — one step at a time.

To make h the subject of $W=rh$ (weekly pay = hourly rate × hours worked), the h is multiplied by r, so divide both sides by r:

$$W=rh \Rightarrow \frac{W}{r}=h, \text{that is } h=\frac{W}{r}.$$

When more than one operation is attached, undo them one at a time. An electricity bill is $B=f+cu$, where f is the fixed supply charge, c is the cost of one unit of power and u is the number of units used. To make u the subject:

$$B=f+cu \Rightarrow B-f=cu \Rightarrow \frac{B-f}{c}=u, \text{so } u=\frac{B-f}{c}.$$

(First subtract f from both sides, then divide both sides by c.)

Formulas with squares and square roots.

A square is undone by a square root, and a square root by squaring. The surface area of a cube with edge length s is $A=6s^2$, so to make s the subject, divide by 6 and then take the square root (an edge length is positive):

$$A=6s^2 \Rightarrow \frac{A}{6}=s^2 \Rightarrow s=\sqrt{\frac{A}{6}}.$$

Going the other way, squaring undoes a square root: from $s=\sqrt{A}$, the side length of a square of area A, squaring both sides gives $A=s^2$.

Two ways to find an unknown value.

If you only need a single value, you have a choice:

- **Substitute first, then solve** — put the known numbers in straight away and solve the resulting equation.
- **Transpose first, then substitute** — rearrange the formula for the variable you want, then put the numbers in.

Transposing first is usually quicker when you must find the **same** unknown for several different sets of numbers (for example, finding the interest rate for many different loans), because you rearrange only once.

A note on brackets.

When the variable you want is inside a bracket, deal with everything *outside* the bracket first. The area of a trapezium is $A=\frac{1}{2}(a+b)h$, where a and b are the parallel sides and h is the distance between them. To make b the subject:

$$A=\frac{1}{2}(a+b)h \Rightarrow 2A=(a+b)h \Rightarrow \frac{2A}{h}=a+b \Rightarrow b=\frac{2A}{h}-a.$$

Worked examples

Example 1 — scaffolded

The formula $v=u+at$ gives the final speed v of an object with starting speed u, acceleration a and time t. Make a the subject, and then find a when $v=25$, $u=5$ and $t=4$ (in SI units).

Working	Comment
$v=u+at$. Subtract u from both sides: $v-u=at$.	Undo the "$+u$" first (it is furthest from a).
Divide both sides by t: $\frac{v-u}{t}=a$.	Undo the "$\times t$".
So $a=\frac{v-u}{t}$.	a is now the subject.
$a=\frac{25-5}{4}=\frac{20}{4}=5$.	Substitute $v=25$, $u=5$, $t=4$.

Example 2 — scaffolded

The simple interest formula is $I=\frac{Prt}{100}$, where P is the principal, r the rate (% per year) and t the time in years. Make r the subject, and find the rate when $I=\$180$, $P=\$1500$ and $t=3$ years.

Working	Comment
$I=\frac{Prt}{100}$. Multiply both sides by 100: $100I=Prt$.	Clear the fraction first.
Divide both sides by Pt: $\frac{100I}{Pt}=r$.	Undo the "$\times Pt$".
So $r=\frac{100I}{Pt}$.	r is now the subject.
$r=\frac{100\times 180}{1500\times 3}=\frac{18\,000}{4500}=4$.	The interest rate is 4% per year.

Example 3 — unscaffolded

The area of a circle is $A=\pi r^2$. A circular garden bed has area 78.5 m². Make r the subject and find the radius, correct to one decimal place.

Dividing both sides by π gives $\frac{A}{\pi}=r^2$, and taking the square root of both sides gives $r=\sqrt{\frac{A}{\pi}}$ (the radius is positive). Substituting $A=78.5$,

$$r=\sqrt{\frac{78.5}{\pi}}=\sqrt{24.98\ldots}=4.998\ldots\approx 5.0\,\text{m}.$$

$\therefore$ the radius of the garden bed is about 5.0 m.

Example 4 — unscaffolded

The formula $C=\frac{5(F-32)}{9}$ converts a temperature from degrees Fahrenheit (F) to degrees Celsius (C). Make F the subject, and convert 25 °C to °F.

Multiplying both sides by 9 gives $9C=5(F-32)$; dividing by 5 gives $\frac{9C}{5}=F-32$; and adding 32 gives

$$F=\frac{9C}{5}+32.$$

Substituting $C=25$,

$$F=\frac{9\times 25}{5}+32=45+32=77.$$

∴ 25 °C is equal to 77 °F.

Practice questions

Scaffolded

Q1. Make x the subject of each formula. (a) $y=x+7$. (b) $y=x-3$. (c) $y=5x$.

Q2. Make a the subject of each formula. (a) $P=a+b+c$. (b) $M=\frac{a+b+c}{3}$. (c) $F=ma$.

Q3. The area of a rectangle is $A=lw$. (a) Make l the subject. (b) Make w the subject. (c) Find w when $A=48$ and $l=6$.

Q4. The speed formula is $s=\frac{d}{t}$. (a) Make d the subject. (b) Make t the subject. (c) Find t when $d=240$ km and $s=60$ km/h.

Q5. The circumference of a circle is $C=2\pi r$. (a) Make r the subject. (b) Find r when $C=44$ cm, correct to one decimal place. (c) Find r when $C=31.4$ cm, correct to one decimal place.

Q6. The simple interest formula is $I=\frac{Prt}{100}$. (a) Make P the subject. (b) Make t the subject. (c) Find P when $I=\$200$, $r=5$ and $t=2$.

Unscaffolded

Q7. The surface area of a sphere is $A=4\pi r^2$. A spherical water tank has a surface area of 200 m². Make r the subject and find the radius, correct to one decimal place.

Q8. Using $v=u+at$, make t the subject and find t when $v=30$, $u=6$ and $a=4$.

Q9. An object dropped from rest falls h metres in t seconds, where $t=\sqrt{\frac{h}{4.9}}$. Make h the subject and find h when $t=2$ s.

Q10. The perimeter of a rectangle is $P=2(l+w)$. Make w the subject and find w when $P=26$ m and $l=8$ m.

Q11. Pythagoras' theorem is $c^2=a^2+b^2$. Make a the subject and find a when $c=13$ and $b=5$.

Q12. The volume of a box is $V=lwh$. Make h the subject and find h when $V=60$ cm³, $l=5$ cm and $w=3$ cm.

Q13. A tradesperson's charge is $C=40+15n$ dollars for a job of n hours. Make n the subject and find n when $C=\$190$.

Q14. Density is $\rho=\frac{m}{V}$. Make m the subject and find m when $\rho=2.5$ g/cm³ and $V=40$ cm³.

Q15. The area of a triangle is $A=\frac{1}{2}bh$. Make h the subject and find h when $A=24$ cm² and $b=6$ cm.

Q16. A straight line has equation $y=mx+c$. Make x the subject and find x when $y=17$, $m=3$ and $c=2$.

Q17. A car's fuel use, in litres per 100 km, is $F=\frac{100L}{d}$, where L litres of fuel are used over a distance of d km. Make L the subject and find L when $F=8.5$ and $d=240$ km.

Q18. The volume of a cylinder is $V=\pi r^2 h$. Make h the subject and find h when $V=500$ cm³ and $r=5$ cm, correct to one decimal place.

Answers

Q1. (a) $x=y-7$. (b) $x=y+3$. (c) $x=\frac{y}{5}$. **Q2.** (a) $a=P-b-c$. (b) $a=3M-b-c$. (c) $a=\frac{F}{m}$. **Q3.** (a) $l=\frac{A}{w}$. (b) $w=\frac{A}{l}$. (c) $w=8$. **Q4.** (a) $d=st$. (b) $t=\frac{d}{s}$. (c) $t=4$ h. **Q5.** (a) $r=\frac{C}{2\pi}$. (b) $r\approx 7.0$ cm. (c) $r\approx 5.0$ cm. **Q6.** (a) $P=\frac{100I}{rt}$. (b) $t=\frac{100I}{Pr}$. (c) $P=\$2000$. **Q7.** $r=\sqrt{\frac{A}{4\pi}}\approx 4.0$ m. **Q8.** $t=\frac{v-u}{a}=6$. **Q9.** $h=4.9t^2=19.6$ m. **Q10.** $w=\frac{P}{2}-l=5$ m. **Q11.** $a=\sqrt{c^2-b^2}=12$. **Q12.** $h=\frac{V}{lw}=4$ cm. **Q13.** $n=\frac{C-40}{15}=10$ hours. **Q14.** $m=\rho V=100$ g. **Q15.** $h=\frac{2A}{b}=8$ cm. **Q16.** $x=\frac{y-c}{m}=5$. **Q17.** $L=\frac{Fd}{100}=20.4$ L. **Q18.** $h=\frac{V}{\pi r^2}\approx 6.4$ cm.

Fully-worked solutions

Q1. (a) Subtract 7: $x=y-7$. (b) Add 3: $x=y+3$. (c) Divide by 5: $x=\frac{y}{5}$.

Q2. (a) Subtract b and c: $a=P-b-c$. (b) Multiply both sides by 3: $3M=a+b+c$, then subtract b and c: $a=3M-b-c$. (c) Divide by m: $a=\frac{F}{m}$.

Q3. (a) Divide both sides of $A=lw$ by w: $l=\frac{A}{w}$. (b) Divide by l: $w=\frac{A}{l}$. (c) $w=\frac{48}{6}=8$.

Q4. (a) Multiply both sides of $s=\frac{d}{t}$ by t: $d=st$. (b) From $d=st$, divide by s: $t=\frac{d}{s}$. (c) $t=\frac{240}{60}=4$ h.

Q5. (a) Divide both sides of $C=2\pi r$ by 2π: $r=\frac{C}{2\pi}$. (b) $r=\frac{44}{2\pi}=\frac{44}{6.283\ldots}=7.003\ldots\approx 7.0$ cm. (c) $r=\frac{31.4}{2\pi}=4.997\ldots\approx 5.0$ cm.

Q6. (a) Multiply by 100 then divide by rt: $P=\frac{100I}{rt}$. (b) Multiply by 100 then divide by Pr: $t=\frac{100I}{Pr}$. (c) $P=\frac{100\times 200}{5\times 2}=\frac{20\,000}{10}=\$2000$.

Q7. Divide by 4π and take the square root: $r=\sqrt{\frac{A}{4\pi}}$. Then $r=\sqrt{\frac{200}{4\pi}}=\sqrt{15.91\ldots}=3.989\ldots\approx 4.0$ m.

Q8. Subtract u then divide by a: $t=\frac{v-u}{a}=\frac{30-6}{4}=\frac{24}{4}=6$.

Q9. Square both sides of $t=\sqrt{\frac{h}{4.9}}$: $t^2=\frac{h}{4.9}$, then multiply both sides by 4.9: $h=4.9t^2=4.9\times 2^2=4.9\times 4=19.6$ m.

Q10. Divide $P=2(l+w)$ by 2: $\frac{P}{2}=l+w$, then subtract l: $w=\frac{P}{2}-l=\frac{26}{2}-8=13-8=5$ m.

Q11. Subtract b^2 then take the square root: $a=\sqrt{c^2-b^2}=\sqrt{13^2-5^2}=\sqrt{169-25}=\sqrt{144}=12$.

Q12. Divide $V=lwh$ by lw: $h=\frac{V}{lw}=\frac{60}{5\times 3}=\frac{60}{15}=4$ cm.

Q13. Subtract 40 then divide by 15: $n=\frac{C-40}{15}=\frac{190-40}{15}=\frac{150}{15}=10$ hours.

Q14. Multiply both sides of $\rho=\frac{m}{V}$ by V: $m=\rho V=2.5\times 40=100$ g.

Q15. Multiply by 2 then divide by b: $h=\frac{2A}{b}=\frac{2\times 24}{6}=\frac{48}{6}=8$ cm.

Q16. Subtract c then divide by m: $x=\frac{y-c}{m}=\frac{17-2}{3}=\frac{15}{3}=5$.

Q17. Multiply both sides of $F=\frac{100L}{d}$ by d: $Fd=100L$, then divide by 100: $L=\frac{Fd}{100}=\frac{8.5\times 240}{100}=\frac{2040}{100}=20.4$ L.

Q18. Divide $V=\pi r^2 h$ by πr^2: $h=\frac{V}{\pi r^2}=\frac{500}{\pi\times 25}=\frac{500}{78.54\ldots}=6.366\ldots\approx 6.4$ cm.

Theory

A **linear relation** is a rule connecting two quantities whose graph is a **straight line**. Many everyday situations are linear — a taxi fare that starts with a flagfall and grows at a fixed rate per kilometre, or a wage made of a base amount plus a fixed hourly rate. Being able to draw and read a straight-line graph lets you see the whole relationship at a glance.

Making a table of values.

To graph a rule such as $y=2x+3$, choose a few values of x, work out the matching y, and plot the points. For $x=-1,0,1,2$ the rule gives $y=1,3,5,7$:

x	-1	0	1	2
y	1	3	5	7

Plotting these points and ruling a line through them gives the graph. Two points are enough to draw a line, but a third is a useful check.

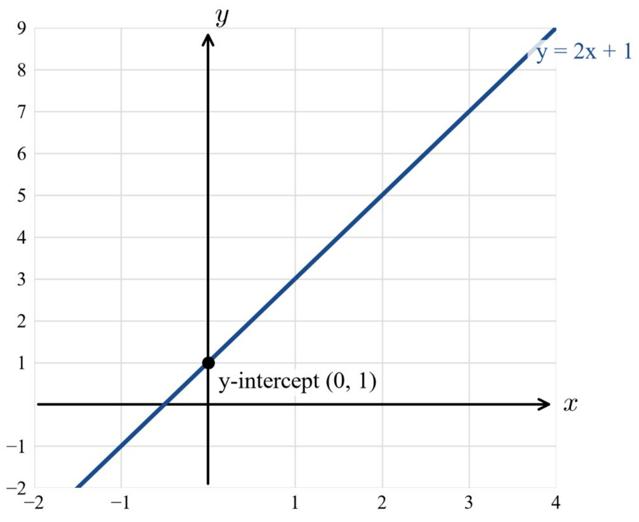
Gradient and y-intercept: $y=mx+c$.

Every straight line can be written as

$$y=mx+c,$$

where m is the **gradient** (how steep the line is) and c is the **y-intercept** (where the line crosses the y-axis, at $x=0$).

- The **gradient** m is the "rise over run" — how much y changes for each increase of 1 in x. A positive gradient rises to the right; a negative gradient falls to the right.
- The **y-intercept** c is the value of y when $x=0$ — the line's starting height.



Reading gradient and intercept in context.

In an applied rule the gradient and intercept have real meanings. For a plumber who charges a $\$40$ call-out fee plus $\$25$ per hour, the cost is

$$C=40+25h,$$

so the **y-intercept** 40 is the fixed call-out fee (the cost before any hours are worked) and the **gradient** 25 is the hourly rate (the cost rises by $\$25$ for each extra hour).

Reading values off a graph.

Once a line is drawn you can read off values without calculating: go up from an x-value to the line, then across to the y-axis (or the reverse). This is how you compare two options — for example, which of two plans is cheaper at a given level of use.

Finding the rule from a graph.

To find the rule of a drawn line, read the **y-intercept** c straight off the graph, then find the **gradient** m from two clear points using

$$m=\frac{\text{rise}}{\text{run}}=\frac{\text{change in } y}{\text{change in } x}.$$

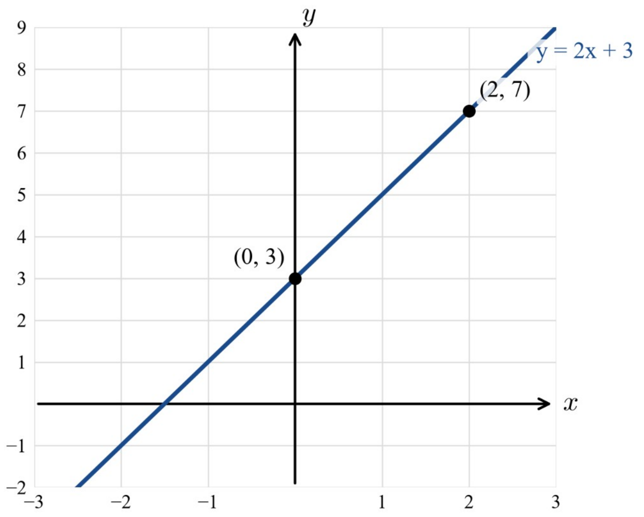
Then write $y=mx+c$.

Worked examples

Example 1 — scaffolded

Complete a table of values for $y=2x+3$ using $x=-2,-1,0,1,2$, plot the graph, and state the gradient and the y-intercept.

Working	Comment
$x=-2: y=2(-2)+3=-1$; $x=-1: y=1$; $x=0: y=3$; $x=1: y=5$; $x=2: y=7$.	Substitute each x into the rule.
Plot $(-2,-1),(-1,1),(0,3),(1,5),(2,7)$ and rule a line through them.	The points lie on a straight line.
The rule is $y=2x+3$, so the gradient is $m=2$.	The number in front of x.
The y-intercept is $c=3$ (the line crosses the y-axis at $(0,3)$).	The constant term; the value of y when $x=0$.

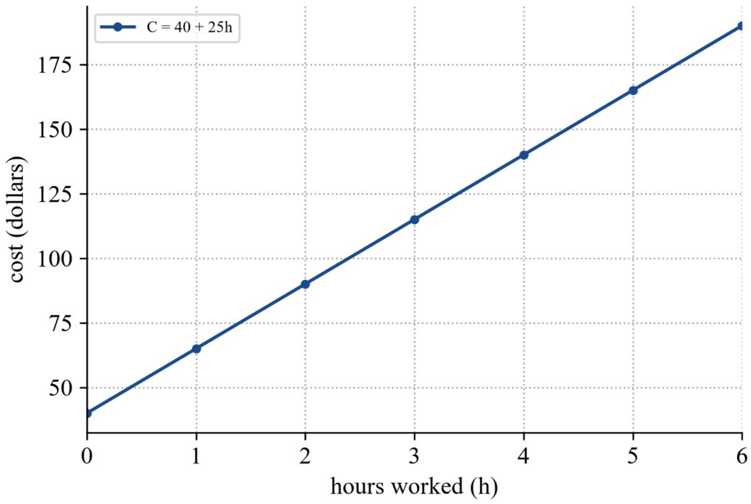


Example 2 — scaffolded

A plumber charges a $\$40$ call-out fee plus $\$25$ per hour. The cost is $C=40+25h$, where h is the number of hours. Interpret the gradient and the y-intercept, and find the cost of a 3-hour job.

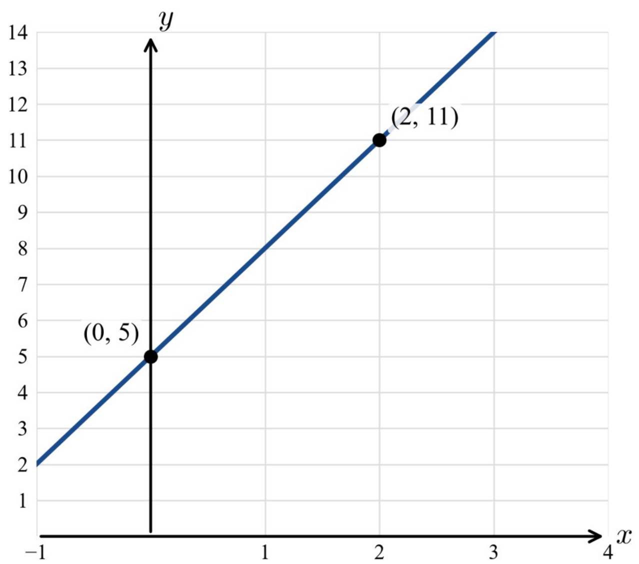
Working	Comment
The y-intercept is 40: the fixed $\$40$ call-out fee, charged before any hours.	The value of C when $h=0$.
The gradient is 25: the cost rises by $\$25$ for each extra hour worked.	The rate of change of cost with hours.

Working	Comment
For $h=3$: $C=40+25\times3=40+75=\$115$.	Substitute $h=3$ into the rule.



Example 3 — unscaffolded

The straight line below passes through $(0,5)$ and $(2,11)$. Find its rule.



The line crosses the y-axis at $(0,5)$, so the y-intercept is $c=5$. Using the two points, the gradient is

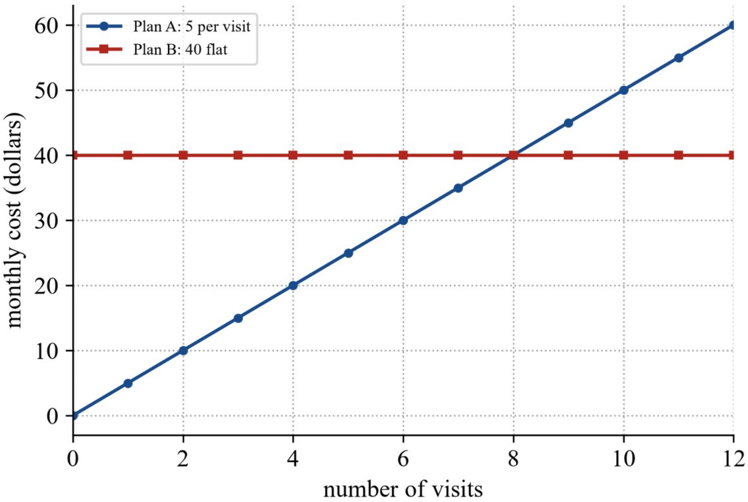
$$m=\frac{\text{rise}}{\text{run}}=\frac{11-5}{2-0}=\frac{6}{2}=3.$$

So the rule is $y=3x+5$.

$\therefore$ the line has rule $y=3x+5$.

Example 4 — unscaffolded

A gym offers two membership options. **Plan A** charges $\$5$ per visit (cost $=5x$ for x visits). **Plan B** is a flat $\$40$ per month for unlimited visits. Use the graph to decide which plan is cheaper for 6 visits and for 10 visits a month.



Plan A is the sloping line through the origin; Plan B is the horizontal line at $\$40$. For 6 visits, Plan A costs $5 \times 6 = \$30$, which is below Plan B's $\$40$. For 10 visits, Plan A costs $5 \times 10 = \$50$, which is above $\$40$. The lines cross at 8 visits.

$\therefore$ Plan A is cheaper for 6 visits ($\$30$ vs $\$40$); Plan B is cheaper for 10 visits ($\$40$ vs $\$50$). They cost the same at 8 visits.

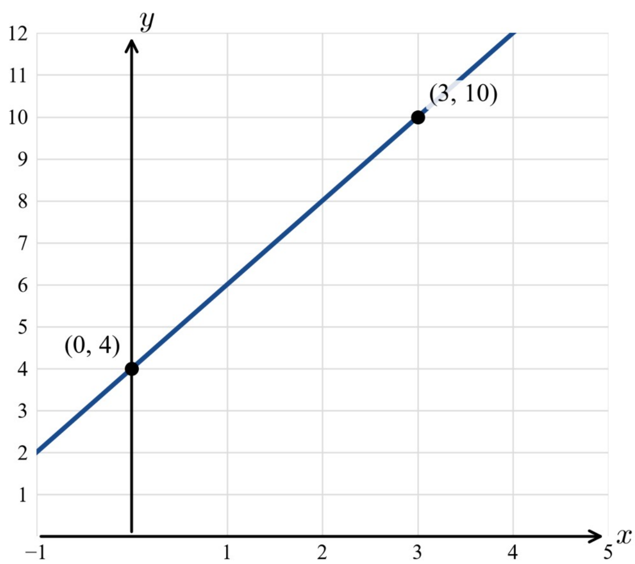
Practice questions

Scaffolded

Q1. Consider the rule $y = 3x - 2$. (a) Complete a table of values for $x = -1, 0, 1, 2, 3$. (b) Plot the points and rule the line. (c) State the gradient and the y-intercept.

Q2. A mechanic charges a $\$15$ booking fee plus $\$8$ per part fitted, so the cost is $C = 15 + 8t$ where t is the number of parts. (a) What does the y-intercept 15 represent? (b) What does the gradient 8 represent? (c) Find the cost of a job using 5 parts.

Q3. The straight line below passes through $(0, 4)$ and $(3, 10)$.



(a) Write down the y-intercept.
(b) Find the gradient.
(c) State the rule of the line.

Q4. A taxi charges a $\$4$ flagfall plus $\$2.50$ per kilometre, so the fare is $F=4+2.5d$ where d is the distance in kilometres. (a) What is the flagfall? (b) What is the charge per kilometre? (c) Find the fare for a 6 km trip.

Q5. Consider the rule $y=-2x+6$. (a) Complete a table of values for $x=0,1,2,3,4$. (b) State the gradient and the y-intercept. (c) At what value of x does the line cross the x-axis (where $y=0$)?

Q6. A full water tank drains so that its volume is $V=200-20t$ litres after t minutes. (a) What is the starting volume? (b) What does the gradient -20 tell you? (c) After how many minutes is the tank empty?

Unscaffolded

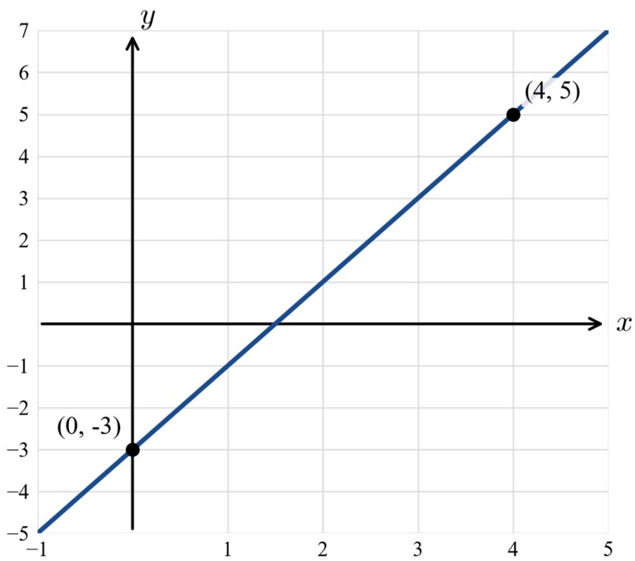
Q7. State the gradient and the y-intercept of $y=4x-1$.

Q8. Complete a table of values for $y=-3x+5$ using $x=0,1,2,3$.

Q9. A line has a y-intercept of 2 and a gradient of 3. Write down its rule.

Q10. A carpet cleaner charges $C=25+12h$ dollars for h hours. Find the cost of a 4-hour job and explain what the 25 represents.

Q11. The line below passes through $(0,-3)$ and $(4,5)$. Find its rule.



Q12. A swimming pool offers two options: pay-per-swim at $\$6$ a swim, or a season pass at a flat $\$54$ for unlimited swims. State which option is cheaper for 7 swims and which is cheaper for 12 swims.

Q13. A salesperson is paid $\$200$ per week plus 10% commission on sales, so the weekly pay is $P=200+0.1s$ where s is the value of sales in dollars. Find the pay in a week with $\$3000$ of sales.

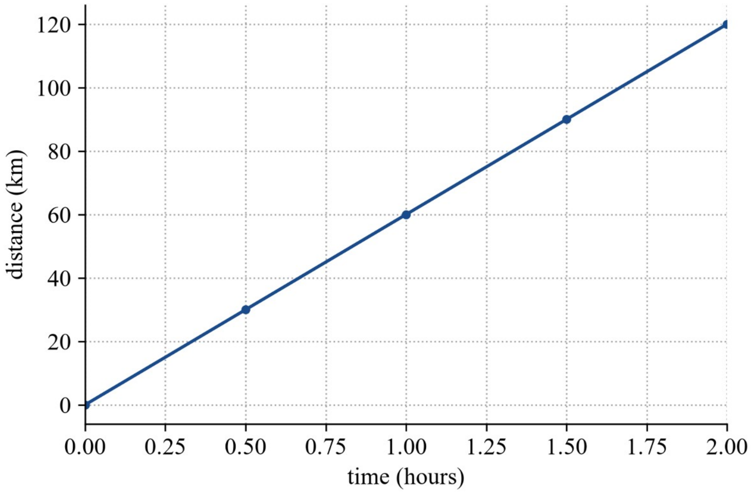
Q14. A line has rule $y=1/2x+1$. State its gradient and y-intercept, and find y when $x=6$.

Q15. A phone plan costs $C=20+0.05m$ dollars, where m is the number of call minutes. Find the cost of a month with 400 minutes of calls.

Q16. A line has a y-intercept of 8 and a gradient of -2. Write down its rule and find where it crosses the x-axis.

Q17. Two couriers price a delivery by distance d (km): Rapid charges $\$12+\$3d$ and Metro charges $\$6+\$4d$. (a) Find the cost of each for a 4 km delivery. (b) Find the cost of each for an 8 km delivery. (c) State which courier is cheaper for the short trip and which for the long trip.

Q18. The distance–time graph below shows a car's journey.



The car travels 120 km in 2 hours at a steady speed. Use the gradient of the line to find the car's speed in km/h.

Answers

Q1. (a) $y=-5,-2,1,4,7$. (c) gradient 3, y-intercept -2. **Q2.** (a) the fixed $\$15$ booking fee. (b) the cost rises $\$8$ per part. (c) $\$55$. **Q3.** (a) 4. (b) 2. (c) $y=2x+4$. **Q4.** (a) $\$4$. (b) $\$2.50$. (c) $\$19$. **Q5.** (a) $y=6,4,2,0,-2$. (b) gradient -2, y-intercept 6. (c) $x=3$. **Q6.** (a) 200 litres. (b) the tank loses 20 litres each minute. (c) 10 minutes. **Q7.** gradient 4, y-intercept -1. **Q8.** $y=5,2,-1,-4$. **Q9.** $y=3x+2$. **Q10.** $\$73$; the $\$25$ is the fixed call-out/booking fee. **Q11.** $y=2x-3$. **Q12.** 7 swims: pay-per-swim ($\$42$ vs $\$54$). 12 swims: the season pass ($\$54$ vs $\$72$). **Q13.** $\$500$. **Q14.** gradient $1/2$, y-intercept 1; $y=4$. **Q15.** $\$40$. **Q16.** $y=-2x+8$; crosses the x-axis at $x=4$. **Q17.** (a) Rapid $\$24$, Metro $\$22$. (b) Rapid $\$36$, Metro $\$38$. (c) Metro is cheaper for 4 km; Rapid is cheaper for 8 km. **Q18.** 60 km/h.

Fully-worked solutions

Q1. (a) Substituting into $y=3x-2$: $x=-1\Rightarrow-5$; $x=0\Rightarrow-2$; $x=1\Rightarrow1$; $x=2\Rightarrow4$; $x=3\Rightarrow7$. (b) Plot $(-1,-5),(0,-2),(1,1),(2,4),(3,7)$ and rule a line through them. (c) Comparing with $y=mx+c$, the gradient is $m=3$ and the y-intercept is $c=-2$.

Q2. (a) The y-intercept 15 is the cost when $t=0$ parts — the fixed $\$15$ booking fee. (b) The gradient 8 is the increase in cost for each extra part: $\$8$ per part. (c) $C=15+8\times5=15+40=\$55$.

Q3. (a) The line crosses the y-axis at $(0,4)$, so the y-intercept is 4. (b) Gradient $=\dfrac{10-4}{3-0}=\dfrac{6}{3}=2$. (c) With $m=2$ and $c=4$, the rule is $y=2x+4$.

Q4. (a) The flagfall is the fixed part, $\$4$. (b) The charge per kilometre is the gradient, $\$2.50$. (c) $F=4+2.5\times6=4+15=\$19$.

Q5. (a) Substituting into $y=-2x+6$: $x=0\Rightarrow6$; $x=1\Rightarrow4$; $x=2\Rightarrow2$; $x=3\Rightarrow0$; $x=4\Rightarrow-2$. (b) Gradient $m=-2$ (the line falls to the right), y-intercept $c=6$. (c) The line crosses the x-axis where $y=0$: $0=-2x+6$, so $2x=6$ and $x=3$.

Q6. (a) The starting volume is the value when $t=0$: $V=200$ litres. (b) The gradient -20 means the volume falls by 20 litres each minute. (c) Empty when $V=0$: $0=200-20t$, so $20t=200$ and $t=10$ minutes.

Q7. Comparing $y=4x-1$ with $y=mx+c$: gradient 4, y-intercept -1.

Q8. Substituting into $y=-3x+5$: $x=0\Rightarrow5$; $x=1\Rightarrow2$; $x=2\Rightarrow-1$; $x=3\Rightarrow-4$.

Q9. With gradient $m=3$ and y-intercept $c=2$, the rule is $y=3x+2$.

Q10. $C=25+12\times4=25+48=\$73$. The $\$25$ is the fixed call-out fee, charged before any hours are worked.

Q11. The y-intercept is -3 (the line crosses the y-axis at $(0,-3)$). The gradient is $\dfrac{5-(-3)}{4-0}=\dfrac{8}{4}=2$. So the rule is $y=2x-3$.

Q12. For 7 swims: pay-per-swim costs $6\times7=\$42$, against the $\$54$ season pass, so pay-per-swim is cheaper. For 12 swims: pay-per-swim costs $6\times12=\$72$, against the $\$54$ season pass, so the season pass is cheaper.

Q13. $P=200+0.1\times3000=200+300=\$500$.

Q14. Comparing $y=1/2x+1$ with $y=mx+c$: gradient $1/2$, y-intercept 1. At $x=6$: $y=1/2\times6+1=3+1=4$.

Q15. $C=20+0.05\times400=20+20=\$40$.

Q16. With $m=-2$ and $c=8$, the rule is $y=-2x+8$. It crosses the x-axis where $y=0$: $0=-2x+8$, so $2x=8$ and $x=4$.

Q17. (a) At $d = 4$: Rapid $= 12 + 3 \times 4 = \$24$; Metro $= 6 + 4 \times 4 = \$22$. (b) At $d = 8$: Rapid $= 12 + 3 \times 8 = \$36$; Metro $= 6 + 4 \times 8 = \$38$. (c) Metro is cheaper for the 4 km trip ($\$22 < \$24$); Rapid is cheaper for the 8 km trip ($\$36 < \$38$). (Their costs are equal at 6 km.)

Q18. The gradient of a distance–time graph is the speed. Here the line rises 120 km over a run of 2 hours, so the speed is $\frac{120}{2} = 60$ km/h.

Theory

Some problems involve **two** unknown quantities linked by **two** conditions. Each condition can be written as a linear equation, and together they form a pair of **simultaneous equations**. The **solution** is the pair of values that makes **both** equations true at the same time. There are three standard ways to find it.

The graphical method.

Each linear equation is a straight line. Because the solution satisfies both equations, it is the point where the two lines **cross**. So you can solve a pair of simultaneous equations by drawing both lines on the same axes and reading off the coordinates of the **point of intersection**.

The substitution method.

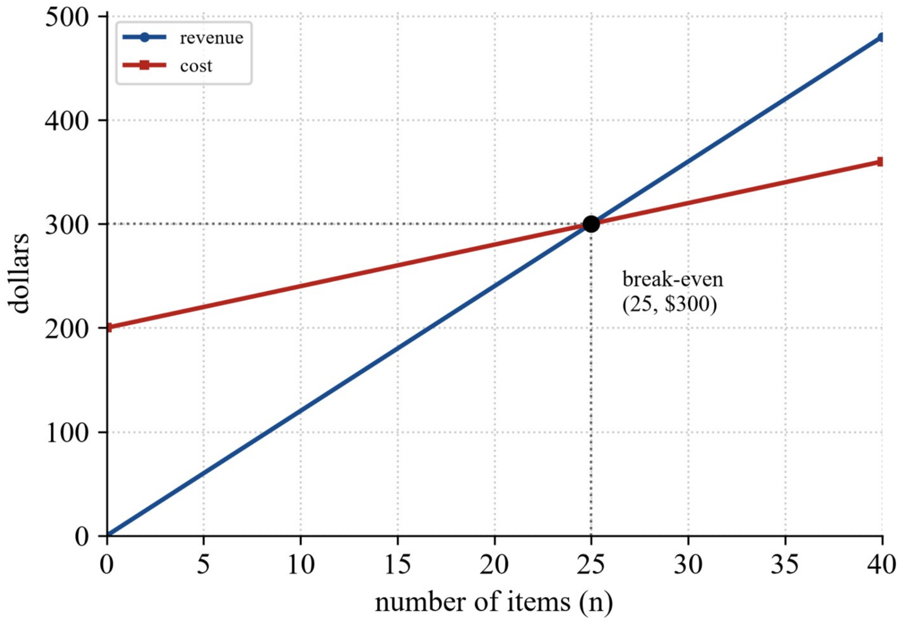
If one equation already gives one variable in terms of the other (for example $y=2x$), **substitute** that expression into the other equation. This leaves a single equation in one variable, which you solve as usual; then back-substitute to find the second variable.

The elimination method.

If both equations are written as $ax+by=c$, you can **add** or **subtract** them to remove ("eliminate") one variable. **Subtract** when that variable has the same coefficient in both equations; **add** when the two coefficients are the same size but opposite in sign. For example, adding $2x+y=12$ and $x-y=3$ removes y because $+y$ and $-y$ cancel, leaving $3x=15$; and subtracting $x+3y=9$ from $4x+3y=18$ removes y because both equations have $+3y$, leaving $3x=9$.

The break-even point.

A common application is a business deciding how many items to make. The **cost** of making n items is a fixed set-up cost plus a cost per item, and the **revenue** is the selling price times n. The **break-even point** is where **revenue equals cost** — the business neither makes a profit nor a loss. Below the break-even point the business runs at a **loss** (cost is above revenue); above it, the business makes a **profit**.



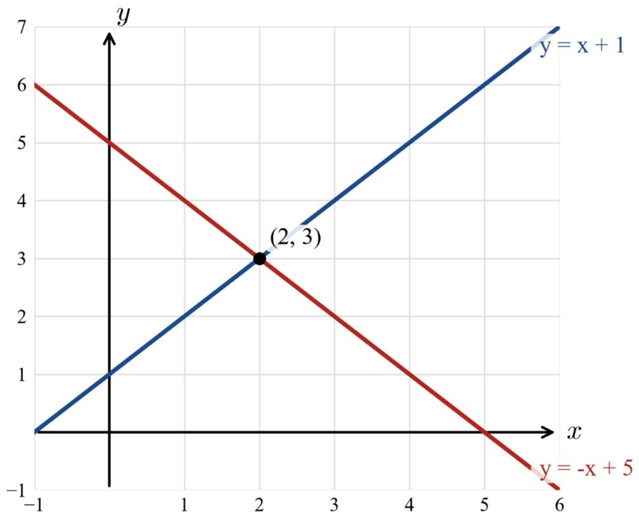
In the graph above the cost is $C=200+4n$ dollars and the revenue is $R=12n$ dollars. The lines cross at $n=25$, so the business breaks even when it sells 25 items, where the revenue and the cost are both $\$300$. Selling more than 25 items makes a profit.

Worked examples

Example 1 — scaffolded

Solve the simultaneous equations $y = x + 1$ and $y = -x + 5$ graphically.

Working	Comment
Both equations are already in the form $y = mx + c$.	Ready to plot: $y = x + 1$ has gradient 1, intercept 1; $y = -x + 5$ has gradient -1, intercept 5.
Plot both lines on the same axes.	See the graph below.
The lines cross at the point $(2, 3)$.	Read the coordinates of the intersection.
Check: in $y = x + 1$, $3 = 2 + 1$ ✓; in $y = -x + 5$, $3 = -2 + 5$ ✓.	The solution must satisfy **both** equations.



So the solution is $x = 2$, $y = 3$.

Example 2 — scaffolded

Solve $y = 2x$ and $x + y = 9$ using substitution.

Working	Comment
The first equation gives $y = 2x$.	y is already the subject — substitute it into the other equation.
Substitute into $x + y = 9$: $x + 2x = 9$.	Replace y with $2x$.
$3x = 9$, so $x = 3$.	Solve the single-variable equation.
$y = 2x = 2 \times 3 = 6$.	Back-substitute to find y.
Check: $x + y = 3 + 6 = 9$ ✓.	Confirm in the second equation.

Example 3 — unscaffolded

Solve $2x + y = 12$ and $x - y = 3$ using elimination.

The variable y has coefficient $+1$ in the first equation and -1 in the second, so **adding** the two equations eliminates it:

$$(2x + y) + (x - y) = 12 + 3 \quad 3x = 15 \quad x = 5.$$

Substituting $x = 5$ into $x - y = 3$ gives $5 - y = 3$, so $y = 2$. Checking in the first equation, $2(5) + 2 = 12$ ✓.

∴ the solution is $x = 5$, $y = 2$.

Example 4 — unscaffolded

A stall selling hand-made candles has a fixed daily set-up cost of $\$200$ and each candle costs $\$4$ in materials. The candles sell for $\$12$ each. Find the break-even number of candles, and state what happens if 30 candles are sold.

The cost of making n candles is $C=200+4n$ dollars, and the revenue is $R=12n$ dollars. Break-even is where $R=C$:

$$12n=200+4n \quad 8n=200 \quad n=25.$$

At $n=25$, $R=12\times 25=\$300$ and $C=200+4\times 25=\$300$, which agree. If 30 candles are sold, revenue is $12\times 30=\$360$ and cost is $200+4\times 30=\$320$, giving a profit of $\$40$.

∴ the stall breaks even at 25 candles, and selling 30 makes a $\$40$ profit.

Practice questions

Scaffolded

Q1. Solve $y=2x-1$ and $y=x+2$ graphically. (a) State the gradient and y-intercept of each line. (b) Plot both lines and mark the point of intersection. (c) Write down the solution and check it in both equations.

Q2. Solve $y=3x$ and $2x+y=20$ by substitution. (a) Substitute $y=3x$ into the second equation. (b) Solve for x. (c) Find y and check your solution.

Q3. Solve $x+y=10$ and $x-y=4$ by elimination. (a) Add the two equations to eliminate y. (b) Solve for x. (c) Find y.

Q4. A market gardener's cost to grow n trays of seedlings is $C=150+5n$ dollars, and the revenue from selling them is $R=20n$ dollars. (a) Write the equation for break-even. (b) Solve it to find the break-even number of trays. (c) Find the revenue at the break-even point.

Q5. Solve $3x+2y=16$ and $x+2y=8$ by elimination. (a) Subtract one equation from the other to eliminate y. (b) Solve for x. (c) Find y.

Q6. Two phone plans charge, for m minutes: Plan A $=30+0.10m$ dollars and Plan B $=20+0.20m$ dollars. (a) Write an equation for when the two plans cost the same. (b) Solve it to find the number of minutes. (c) Find the cost at that number of minutes.

Unscaffolded

Q7. Solve $y=x+3$ and $y=9-x$ graphically.

Q8. Solve $y=4x$ and $3x+y=21$ by substitution.

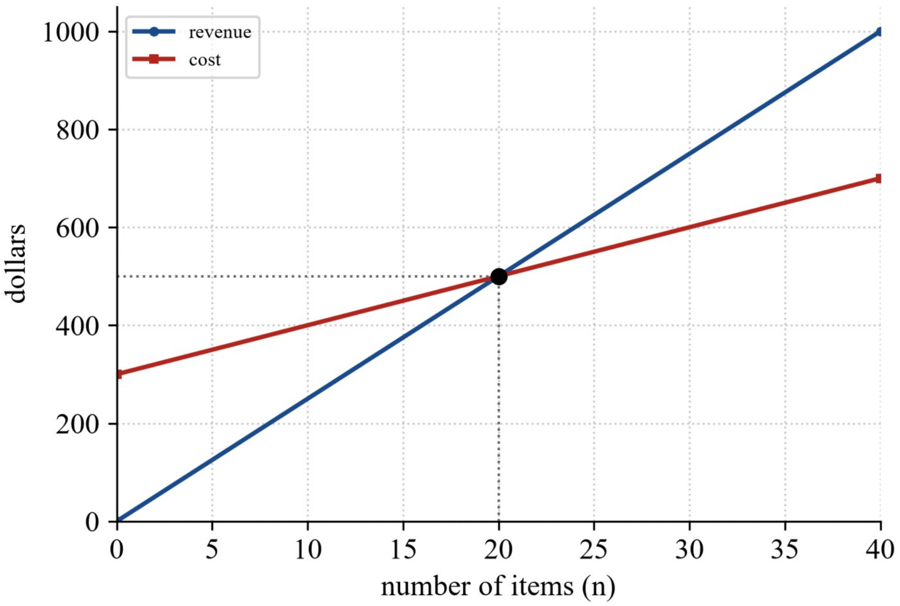
Q9. Solve $2x+y=11$ and $x+y=7$ by elimination.

Q10. Solve $3x+y=17$ and $x-y=3$ by elimination.

Q11. Solve $x=y+2$ and $2x+3y=24$ by substitution.

Q12. A small print shop has a fixed monthly cost of $\$500$ and each printed hoodie costs $\$8$ to make. The hoodies sell for $\$18$ each. Find the break-even number of hoodies and the revenue at that point.

Q13. The graph below shows the cost and revenue for a food stall selling rolls.



(a) Use the graph to state the break-even number of rolls and the revenue there.

(b) The cost is $C = 300 + 10n$ and the revenue is $R = 25n$. Confirm the break-even point algebraically.

Q14. A business has cost $C = 400 + 6n$ dollars and revenue $R = 14n$ dollars for n items. (a) Find the break-even number of items. (b) Find the profit if 70 items are sold.

Q15. Gym A charges $\$40$ joining plus $\$5$ per visit; Gym B charges $\$70$ joining plus $\$2$ per visit. After how many visits do the two gyms cost the same, and what is that cost?

Q16. Solve $5x + 2y = 24$ and $3x - 2y = 8$ by elimination.

Q17. At a cafe, 2 coffees and 3 teas cost $\$17$, while 1 coffee and 2 teas cost $\$10$. Set up and solve a pair of simultaneous equations to find the price of a coffee and the price of a tea.

Q18. A food truck has a fixed daily cost of $\$240$ and each meal costs $\$3$ to make; meals sell for $\$9$. (a) Find the break-even number of meals. (b) Find the profit if 60 meals are sold.

Answers

Q1. (a) $y=2x-1$: gradient 2, intercept -1; $y=x+2$: gradient 1, intercept 2. (b)/(c) $(3,5)$, i.e. $x=3$, $y=5$. **Q2.** $x=4$, $y=12$. **Q3.** $x=7$, $y=3$. **Q4.** (a) $20n=150+5n$. (b) 10 trays. (c) $\$200$. **Q5.** $x=4$, $y=2$. **Q6.** (a) $30+0.10m=20+0.20m$. (b) 100 minutes. (c) $\$40$. **Q7.** $x=3$, $y=6$. **Q8.** $x=3$, $y=12$. **Q9.** $x=4$, $y=3$. **Q10.** $x=5$, $y=2$. **Q11.** $x=6$, $y=4$. **Q12.** break-even 50 hoodies; revenue $\$900$. **Q13.** (a) 20 rolls, $\$500$. (b) $25n=300+10n \Rightarrow 15n=300 \Rightarrow n=20$, $R=\$500$. **Q14.** (a) 50 items. (b) profit $\$160$. **Q15.** 10 visits; $\$90$. **Q16.** $x=4$, $y=2$. **Q17.** coffee $\$4$, tea $\$3$. **Q18.** (a) 40 meals. (b) profit $\$120$.

Fully-worked solutions

Q1. (a) In $y=2x-1$ the gradient is 2 and the y-intercept is -1; in $y=x+2$ the gradient is 1 and the y-intercept is 2. (b)/(c) Plotting both lines, they cross at $(3,5)$. Check: $y=2x-1=2(3)-1=5$ ✓ and $y=x+2=3+2=5$ ✓. So $x=3$, $y=5$.

Q2. Substitute $y=3x$ into $2x+y=20$: $2x+3x=20$, so $5x=20$ and $x=4$. Then $y=3x=12$. Check: $2(4)+12=20$ ✓.

Q3. Adding $x+y=10$ and $x-y=4$ eliminates y: $2x=14$, so $x=7$. Substituting into $x+y=10$ gives $7+y=10$, so $y=3$.

Q4. (a) Break-even is $R=C$, i.e. $20n=150+5n$. (b) $20n-5n=150$, so $15n=150$ and $n=10$ trays. (c) Revenue at break-even $=20\times10=\$200$ (and cost $=150+5\times10=\$200$, which agrees).

Q5. Subtracting $x+2y=8$ from $3x+2y=16$ eliminates y: $2x=8$, so $x=4$. Substituting into $x+2y=8$ gives $4+2y=8$, so $2y=4$ and $y=2$.

Q6. (a) The plans cost the same when $30+0.10m=20+0.20m$. (b) $30-20=0.20m-0.10m$, so $10=0.10m$ and $m=100$ minutes. (c) Cost $=30+0.10\times100=\$40$ (and Plan B $=20+0.20\times100=\$40$).

Q7. Setting $x+3=9-x$ gives $2x=6$, so $x=3$ and $y=3+3=6$. The lines cross at $(3,6)$.

Q8. Substitute $y=4x$ into $3x+y=21$: $3x+4x=21$, so $7x=21$ and $x=3$. Then $y=4\times3=12$.

Q9. Subtracting $x+y=7$ from $2x+y=11$ eliminates y: $x=4$. Substituting gives $4+y=7$, so $y=3$.

Q10. Adding $3x+y=17$ and $x-y=3$ eliminates y: $4x=20$, so $x=5$. Substituting into $x-y=3$ gives $5-y=3$, so $y=2$.

Q11. Substitute $x=y+2$ into $2x+3y=24$: $2(y+2)+3y=24$, so $2y+4+3y=24$, giving $5y=20$ and $y=4$. Then $x=y+2=6$.

Q12. Cost $C=500+8n$ and revenue $R=18n$. Break-even: $18n=500+8n$, so $10n=500$ and $n=50$ hoodies. Revenue $=18\times50=\$900$.

Q13. (a) The two lines cross at $n=20$, where both cost and revenue are $\$500$. (b) Setting $25n=300+10n$ gives $15n=300$, so $n=20$, and $R=25\times20=\$500$ — matching the graph.

Q14. (a) Break-even: $14n=400+6n$, so $8n=400$ and $n=50$ items. (b) At $n=70$: revenue $=14\times70=\$980$ and cost $=400+6\times70=\$820$, so the profit is $980-820=\$160$.

Q15. The gyms cost the same when $40+5v=70+2v$. Then $5v-2v=70-40$, so $3v=30$ and $v=10$ visits. The cost is $40+5\times10=\$90$ (and $70+2\times10=\$90$).

Q16. Adding $5x+2y=24$ and $3x-2y=8$ eliminates y: $8x=32$, so $x=4$. Substituting into $5x+2y=24$ gives $20+2y=24$, so $2y=4$ and $y=2$.

Q17. Let c be the price of a coffee and t the price of a tea. Then $2c+3t=17$ and $c+2t=10$. From the second equation $c=10-2t$. Substituting into the first: $2(10-2t)+3t=17$, so $20-4t+3t=17$, giving $20-t=17$ and $t=3$. Then $c=10-2\times3=4$. So a coffee costs $\$4$ and a tea costs $\$3$.

Q18. (a) Cost $C=240+3n$ and revenue $R=9n$. Break-even: $9n=240+3n$, so $6n=240$ and $n=40$ meals. (b) At $n=60$: revenue $=9\times60=\$540$ and cost $=240+3\times60=\$420$, so the profit is $540-420=\$120$.

Topic 1 Test A — Multiple-choice

Topic 1 (Algebra, number and structure). Materials: one scientific calculator; one bound reference that may be annotated. Section A: 15 multiple-choice questions (15 marks). Approximate time: 25 minutes. Unless otherwise indicated, diagrams are not drawn to scale.

Section A — Multiple-choice questions

Choose the response that is correct for the question.

Question 1

Which of the following numbers is irrational?

A. 0.75

B. $\frac{3}{8}$

C. $\sqrt{7}$

D. $\sqrt{16}$

Question 2

The value of $3+2\times4^2$ is

A. 19

B. 35

C. 51

D. 80

Question 3

Written in scientific notation, $45\,000\,000$ is

A. 4.5×10^7

B. 4.5×10^8

C. 4.5×10^6

D. 45×10^6

Question 4

A wall is estimated to be 48 m long, but its actual length is 50 m. The percentage error of the estimate is

A. $96\,\%$

B. $4\,\%$

C. $2\,\%$

D. $5\,\%$

Question 5

An amount of $\$600$ is shared between two people in the ratio $2:3$. The larger share is

A. $\$360$

B. $\$240$

C. $\$300$

D. $\$400$

Question 6

The variable y varies directly with x, and $y=20$ when $x=4$. When $x=7$, the value of y is

A. 140

B. 23

C. 28

D. 35

Question 7

The variable y varies inversely with x, and $y=12$ when $x=2$. When $x=8$, the value of y is

A. 6

B. 48

C. 3

D. 1.5

Question 8

A price of $\$80$ is increased by 15%. The new price is

A. $\$68$

B. $\$95$

C. $\$12$

D. $\$92$

Question 9

A jacket marked at $\$250$ is discounted by 20%. The sale price is

A. $\$210$

B. $\$230$

C. $\$200$

D. $\$50$

Question 10

If $P = 5n - 40$, then the value of P when $n = 12$ is

A. -20

B. 20

C. 100

D. 60

Question 11

The solution of the equation $3x - 7 = 11$ is

A. $x = 6$

B. $x = 1.33$

C. $x = 4$

D. $x = 18$

Question 12

The area of a triangle is $A = \frac{1}{2}bh$. Making h the subject of the formula gives

A. $h = \frac{A - b}{2}$

B. $h = \frac{2A}{b}$

C. $h = \frac{A}{2b}$

D. $h = 2Ab$

Question 13

The gradient of the line $y = 2x + 3$ is

A. 2

B. 3

C. -2

D. $\frac{2}{3}$

Question 14

A business has cost $C=200+4n$ dollars and revenue $R=12n$ dollars for n items. The business breaks even when n equals

A. 20

B. 200

C. 16

D. 25

Question 15

12% of a number is 60. The number is

A. 5

B. 720

C. 500

D. 7.2

End of Section A

Topic 1 Test A — Solutions

Question	Answer	Working
1	**C**	$\sqrt{16}=4$ and $3/8=0.375$ are rational; $\sqrt{7}$ is irrational.
2	**B**	Indices first: $4^2=16$, then $3+2\times 16=3+32=35$.
3	**A**	The point moves 7 places left: 4.5×10^7.
4	**B**	$\frac{\lvert 48-50\rvert}{50}\times 100\,\%=\frac{2}{50}\times 100\,\%=4$.
5	**A**	$600\div 5=120$; larger share $=3\times 120=\$360$.
6	**D**	$k=20\div 4=5$, so $y=5\times 7=35$.
7	**C**	$k=12\times 2=24$, so $y=24\div 8=3$.
8	**D**	$80\times 1.15=\$92$.
9	**C**	$250\times 0.80=\$200$.
10	**B**	$P=5\times 12-40=60-40=20$.
11	**A**	$3x=18$, so $x=6$.
12	**B**	$2A=bh$, so $h=\frac{2A}{b}$.
13	**A**	In $y=mx+c$, the gradient is $m=2$.
14	**D**	$12n=200+4n\Rightarrow 8n=200\Rightarrow n=25$.
15	**C**	$60\div 0.12=500$.

Topic 1 Test B — Short answer

Topic 1 (Algebra, number and structure). Materials: one scientific calculator; one bound reference. Section B: 6 questions (30 marks). Approximate time: 40 minutes. Answer all questions in the spaces provided. In all questions where a numerical answer is required, you should only round when instructed to do so. Where more than one mark is available, appropriate working must be shown.

Section B

Question 1 (5 marks)

a. Write $3\,200\,000$ in scientific notation. 1 mark

b. Evaluate $\sqrt{144}+2^3$. 1 mark

c. A phone is advertised as having 128 GB of storage, but only 119 GB is usable. Find the percentage error of the advertised figure compared with the usable storage, correct to one decimal place. 2 marks

d. State whether each of $\sqrt{9}$ and $\sqrt{10}$ is rational or irrational. 1 mark

Question 2 (5 marks)

a. A batch of concrete is made from cement, sand and gravel in the ratio $1:2:4$. Find the mass of each ingredient in a 350 kg batch. 2 marks

b. A recipe uses flour and sugar in the ratio $5:2$. If 750 g of flour is used, find the mass of sugar required. 1 mark

c. A map is drawn to a scale of $1:25\,000$. Show that 1 cm on the map represents 250 m in real life. 2 marks

Question 3 (5 marks)

The cost C (in dollars) of petrol varies directly with the number of litres L purchased. A motorist pays $\$54$ for 30 L.

a. Find the constant of proportionality (the cost per litre). 2 marks

b. Hence find the cost of 45 L of petrol. 1 mark

c. The time t to fill a pool varies inversely with the flow rate r. At a rate of 20 L/min it takes 90 minutes. Show that at a rate of 30 L/min it takes 60 minutes. 2 marks

Question 4 (5 marks)

a. Find 15% of $\$240$. 1 mark

b. A jacket costs $\$80$. In a sale it is reduced by 25%. Find the sale price. 2 marks

c. A café's daily takings rose from $\$1500$ to $\$1800$. Find the percentage increase. 2 marks

Question 5 (5 marks)

a. Solve the equation $4x+5=2x+17$. 2 marks

b. The simple interest formula is $I=\frac{Prt}{100}$. Make r the subject of the formula. 2 marks

c. Hence find r when $I=\$240$, $P=\$2000$ and $t=3$ years. 1 mark

Question 6 (5 marks)

A market stall has costs $C=150+3n$ dollars and revenue $R=8n$ dollars, where n is the number of items sold.

a. Show that the stall breaks even when $n=30$. 2 marks

b. Find the profit when 50 items are sold. 2 marks

c. State, with a reason, what happens financially if only 20 items are sold. 1 mark

End of Section B

Topic 1 Test B — Solutions

Question 1.

a. Move the decimal point 6 places: $3\,200\,000 = 3.2 \times 10^6$.

b. $\sqrt{144} + 2^3 = 12 + 8 = 20$.

c. The difference is $|128 - 119| = 9$ GB. As a percentage of the usable storage,

$$\frac{9}{119} \times 100\,\% = 7.56\ldots\% \approx 7.6\,\%.$$

d. $\sqrt{9} = 3$, which is rational. $\sqrt{10}$ is irrational (it is not a perfect square).

Question 2.

a. The ratio $1:2:4$ has $1+2+4=7$ parts, so one part $= 350 \div 7 = 50$ kg. The ingredients are cement $= 50$ kg, sand $= 2 \times 50 = 100$ kg and gravel $= 4 \times 50 = 200$ kg.

b. One part of flour $= 750 \div 5 = 150$ g, so sugar $= 2 \times 150 = 300$ g.

c. A scale of $1:25\,000$ means 1 cm on the map represents $25\,000$ cm in real life. Converting to metres,

$$25\,000\,\text{cm} = \frac{25\,000}{100}\,\text{m} = 250\,\text{m}.$$

So 1 cm represents 250 m, as required.

Question 3.

a. Direct variation gives $C = k\,L$. Using $C = 54$ when $L = 30$, $k = \frac{54}{30} = 1.8$, i.e. $\$1.80$ per litre.

b. $C = 1.8 \times 45 = \$81$.

c. Inverse variation gives $t = \frac{k}{r}$, so $k = t \times r = 90 \times 20 = 1800$. At $r = 30$,

$$t = \frac{1800}{30} = 60\,\text{minutes},$$

as required.

Question 4.

a. $15\,\%$ of $\$240 = 0.15 \times 240 = \$36$.

b. A $25\,\%$ reduction leaves $75\,\%$: sale price $= 0.75 \times 80 = \$60$.

c. The increase is $1800 - 1500 = \$300$. As a percentage of the original, $\frac{300}{1500} \times 100\,\% = 20\,\%$.

Question 5.

a. $4x + 5 = 2x + 17$. Subtract $2x$: $2x + 5 = 17$. Subtract 5: $2x = 12$. Divide by 2: $x = 6$.

b. $I = \frac{P\,r\,t}{100}$. Multiply both sides by 100: $100\,I = P\,r\,t$. Divide by $P\,t$:

$$r = \frac{100\,I}{P\,t}.$$

c. $r = \dfrac{100 \times 240}{2000 \times 3} = \dfrac{24\,000}{6000} = 4$, i.e. $4\,\%$ per year.

Question 6.

a. Break-even is where revenue equals cost: $8n = 150 + 3n$. Subtract $3n$: $5n = 150$. Divide by 5: $n = 30$, as required.

b. At $n = 50$: revenue $R = 8 \times 50 = \$400$ and cost $C = 150 + 3 \times 50 = \$300$, so profit $= 400 - 300 = \$100$.

c. At $n = 20$: revenue $= 8 \times 20 = \$160$ and cost $= 150 + 3 \times 20 = \$210$. Since cost exceeds revenue, the stall makes a **loss** of $\$50$ (it has not reached break-even).