# Foundation Mathematics Units 3 & 4

## Chapter 2 — Number and calculation



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## Theory

The same quantity can be written in several **forms** — as a whole number, a fraction, a decimal or a percentage. Being able to move between these forms, and knowing what kind of number you are dealing with, is the foundation of all the calculation you do in this course.

**Forms of a number.**

- An **integer** is a whole number, positive, negative or zero: $\ldots,-2,-1,0,1,2,\ldots$
- A **fraction** compares a part with a whole, written $\frac{a}{b}$ (with $b \neq 0$).
- A **decimal** uses place value after a point. It may **terminate** (stop), like $0.75$, or **recur** (repeat forever), like $0.444\ldots$
- A **percentage** is a number "out of $100$": $37\,\%$ means $\frac{37}{100}$.

A recurring decimal is written with a bar over the repeating digits:

$$0.\overline{4}=0.444\ldots,0.2\overline{7}=0.2777\ldots,0.\overline{18}=0.181818\ldots$$

**Rational numbers.**

A **rational number** is any number that can be written as a fraction $\frac{a}{b}$ where $a$ and $b$ are integers and $b \neq 0$. This is a large family: it includes every integer ($5=5/1$), every fraction, every **terminating** decimal ($0.9=9/10$) and every **recurring** decimal ($0.\overline{7}=7/9$).

**Irrational numbers.**

An **irrational number** is a number that **cannot** be written as a fraction of two integers. As a decimal it never terminates and never settles into a repeating pattern. The two kinds you meet most often are:

- **surds** — a square root (or cube root) that does not come out exactly, such as $\sqrt{2},\sqrt{3},\sqrt{11}$. (A root that *does* come out exactly, like $\sqrt{16}=4$, is rational.)
- $\pi$ — the ratio of a circle's circumference to its diameter, $\pi=3.14159\ldots$

Irrational numbers appear naturally in **measurement**: the circumference and area of a circle involve $\pi$, and the diagonal of a square or the longest side of a right-angled triangle often involves a surd (through Pythagoras' theorem). When an answer is irrational, you give either the **exact** value ($7\pi$, $2\sqrt{3}$) or a **rounded** decimal (21.99, 3.46) — never a fraction, because no fraction is exactly equal to it.

**Converting between forms.**

- **Fraction → decimal:** divide the top by the bottom. $\frac{3}{8}=3\div 8=0.375$.
- **Decimal → fraction:** read the place value, then simplify. $0.35=\frac{35}{100}=\frac{7}{20}$.
- **Fraction → percentage:** multiply by $100\,\%$. $\frac{3}{8}\times 100\,\%=37.5\,\%$.
- **Percentage → fraction:** write over $100$ and simplify. $44\,\%=\frac{44}{100}=\frac{11}{25}$.
- **Decimal ↔ percentage:** multiply or divide by $100$. $0.35=35\,\%$ and $44\,\%=0.44$.

**Ordering mixed numbers.**

To put a mixed list of fractions, decimals and percentages in order, convert them **all to decimals** first, then compare place by place.

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### Worked examples

#### *Example 1 — scaffolded*

A surveyor's field notes record six lengths, in metres. Classify each as rational or irrational:

$$\frac{3}{4}, \sqrt{9}, \sqrt{2}, \pi, \sqrt[3]{8}, \sqrt{5}.$$

| Working | Comment |
|---|---|
| $\frac{3}{4}$ is already a fraction of integers, so it is **rational**. | Any fraction $a/b$ is rational. |
| $\sqrt{9}=3=\frac{3}{1}$, so it is **rational**. | The root comes out exactly. |
| $\sqrt{2}=1.41421\ldots$ never terminates or repeats, so it is **irrational**. | A surd of a non-square is irrational. |
| $\pi=3.14159\ldots$ never terminates or repeats, so it is **irrational**. | $\pi$ is the classic irrational number. |
| $\sqrt[3]{8}=2$, so it is **rational**. | The cube root comes out exactly. |
| $\sqrt{5}=2.23606\ldots$ is **irrational**. | $5$ is not a perfect square. |

#### *Example 2 — scaffolded*

A hardware store's stocktake records three figures, each in a different form. (a) $\frac{2}{5}$ of the paint tins are white — write this as a decimal and a percentage. (b) $0.06$ of the timber lengths were rejected — write this as a fraction in simplest form and a percentage. (c) $24\,\%$ of the power tools sold came with a warranty — write this as a fraction in simplest form and a decimal.

| Working | Comment |
|---|---|
| (a) $\frac{2}{5}=2\div 5=0.4$; $0.4\times 100\,\%=40\,\%$. | Divide for the decimal, then $\times 100\,\%$. |
| (b) $0.06=\frac{6}{100}=\frac{3}{50}$; $0.06=6\,\%$. | Read place value, simplify; $\times 100$ for the percentage. |
| (c) $24\,\%=\frac{24}{100}=\frac{6}{25}$; $24\,\%=0.24$. | Write over $100$ and simplify; $\div 100$ for the decimal. |

#### *Example 3 — unscaffolded*

Four couriers report the proportion of parcels they delivered on time last month as

$$0.55, \frac{3}{5}, 58\,\%, \frac{5}{8}.$$

Write the four proportions in ascending order (smallest first).

Convert every value to a decimal: $0.55$ stays as $0.55$; $\frac{3}{5}=0.6$; $58\,\%=0.58$; $\frac{5}{8}=0.625$. Comparing the decimals,

$$0.55<0.58<0.6<0.625.$$

$\therefore$ in ascending order: $0.55, 58\%, \frac{3}{5}, \frac{5}{8}$ — so the courier reporting $\frac{5}{8}$ had the best on-time record.

### *Example 4 — unscaffolded*

A circular pond has diameter $4$ m and a square patio has side length $3$ m. (a) Find the exact circumference of the pond and state whether it is rational or irrational. (b) Find the exact length of the diagonal of the patio, and its value to two decimal places.

(a) The circumference is $C = \pi d = \pi \times 4 = 4\pi$ m. Because it is a multiple of $\pi$, it is **irrational**; as a decimal, $4\pi \approx 12.57$ m.

(b) By Pythagoras' theorem the diagonal is $d = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$ m. This is a surd, so it is **irrational**, and $3\sqrt{2} \approx 4.24$ m.

$\therefore$ both the circumference ($4\pi$ m) and the diagonal ($3\sqrt{2}$ m) are irrational; rounded, they are $12.57$ m and $4.24$ m.

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## Practice questions

### *Scaffolded*

**Q1.** A landscaping plan lists the six lengths below, in metres. Classify each as rational or irrational. (a) $\frac{5}{8}$ (b) $\sqrt{25}$ (c) $\sqrt{7}$ (d) $0.25$ (e) $\frac{\pi}{2}$ (f) $\sqrt[3]{27}$

**Q2.** A phone shows its battery charge as $\frac{7}{8}$ of full. (a) Write the charge as a decimal. (b) Write the charge as a percentage. (c) State whether $\frac{7}{8}$ is rational or irrational.

**Q3.** A netball team won $0.45$ of the matches it played this season. (a) Write $0.45$ as a fraction in simplest form. (b) Write $0.45$ as a percentage.

**Q4.** A shopping-centre survey found that $65\%$ of customers paid by card. (a) Write $65\%$ as a fraction in simplest form. (b) Write $65\%$ as a decimal.

**Q5.** A quality inspector records the fraction of items that passed inspection on three production lines. Write each fraction as a recurring decimal, using the bar notation. (a) $\frac{2}{3}$ (b) $\frac{5}{6}$ (c) $\frac{1}{9}$

**Q6.** Four classes recorded the proportion of their students who walk to school as

$$0.7, \frac{3}{4}, 68\%, \frac{4}{5}.$$

Write these proportions in ascending order.

### *Unscaffolded*

**Q7.** A builder's calculations produce the lengths $\sqrt{36}$, $\sqrt{10}$, 1.2, $\pi$, $\frac{8}{3}$ and $\sqrt[3]{64}$, all in metres. Classify each as rational or irrational.

**Q8.** At a secondary college, $\frac{7}{12}$ of the students travel to school by bus. Write $\frac{7}{12}$ as (a) a recurring decimal and (b) a percentage (to one decimal place).

**Q9.** In a batch of concrete, sand makes up $0.64$ of the total mass. Write $0.64$ as (a) a fraction in simplest form and (b) a percentage.

**Q10.** A café adds a $22.5\%$ surcharge to every bill on a public holiday. Write $22.5\%$ as (a) a fraction in simplest form and (b) a decimal.

**Q11.** Four apprentices have completed $\frac{5}{8}$, $0.6$, $62\%$ and $\frac{11}{20}$ of their required training hours. Write these in ascending order.

**Q12.** A circular garden bed has radius $5$ m. (a) Find its exact area (in terms of $\pi$). (b) State whether the area is rational or irrational. (c) Give the area correct to two decimal places.

**Q13.** Two structures are braced with a diagonal. (a) A loading ramp rises $5$ m over a horizontal run of $12$ m. Find the length of its sloping surface, and state whether it is rational or irrational. (b) A rectangular courtyard measures $2$ m by $3$ m. Find the exact length of the diagonal path across it, and state whether it is rational or irrational.

**Q14.** Classify each of $\sqrt{\frac{1}{4}}$, $\sqrt{0.16}$, $\sqrt{20}$ and $0.\overline{3}$ as rational or irrational, giving a reason for each.

**Q15.** A recipe uses $\frac{3}{4}$ cup of flour and $\frac{2}{3}$ cup of sugar. By converting each to a decimal, state which ingredient is used in the greater amount.

**Q16.** A market researcher recorded four survey results, each in a different form. Copy and complete the equivalence table so that every row shows all three forms.

| Fraction | Decimal | Percentage |
| --- | --- | --- |
| $\frac{1}{5}$ | | |
| | 0.85 | |
| | | $32\%$ |
| $\frac{1}{8}$ | | |

**Q17.** Five delivery drivers had on-time rates of

$$0.72, \frac{7}{10}, \frac{3}{4}, 71\%, 0.69.$$

Write the five values in ascending order.

**Q18.** In a basketball season, Mia made $\frac{5}{8}$ of her free throws and Ada made $62\%$ of hers. By converting to the same form, state who had the better free-throw record.

### Answers

**Q1.** (a) rational; (b) rational ($\sqrt{25}=5$); (c) irrational; (d) rational; (e) irrational; (f) rational ($\sqrt[3]{27}=3$). **Q2.** (a) $0.875$; (b) $87.5\%$; (c) rational. **Q3.** (a) $\frac{9}{20}$; (b) $45\%$. **Q4.** (a) $\frac{13}{20}$; (b) $0.65$. **Q5.** (a) $0.\bar{6}$; (b) $0.8\bar{3}$; (c) $0.\bar{1}$. **Q6.** $68\%, 0.7, \frac{3}{4}, \frac{4}{5}$. **Q7.** rational: $\sqrt{36}=6$, $1.2$, $\frac{8}{3}$, $\sqrt[3]{64}=4$; irrational: $\sqrt{10}$, $\pi$. **Q8.** (a) $0.58\bar{3}$; (b) $58.3\%$. **Q9.** (a) $\frac{16}{25}$; (b) $64\%$. **Q10.** (a) $\frac{9}{40}$; (b) $0.225$. **Q11.** $\frac{11}{20}, 0.6, 62\%, \frac{5}{8}$. **Q12.** (a) $25\pi$ m²; (b) irrational; (c) $78.54$ m². **Q13.** (a) 13 m, rational; (b) $\sqrt{13}$ m, irrational ($\approx 3.61$ m). **Q14.** $\sqrt{1/4}=1/2$ rational; $\sqrt{0.16}=0.4$ rational; $\sqrt{20}=2\sqrt{5}$ irrational; $0.\bar{3}=1/3$ rational. **Q15.** $\frac{3}{4}=0.75$ and $\frac{2}{3}\approx 0.67$, so more flour is used. **Q16.** $\frac{1}{5}=0.2=20\%$; $\frac{17}{20}=0.85=85\%$; $\frac{8}{25}=0.32=32\%$; $\frac{1}{8}=0.125=12.5\%$. **Q17.** $0.69, \frac{7}{10}, 71\%, 0.72, \frac{3}{4}$. **Q18.** $\frac{5}{8}=62.5\%$ versus $62\%$, so Mia had the better record.

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### Fully-worked solutions

**Q1.** (a) $\frac{5}{8}$ is a fraction of integers — **rational**. (b) $\sqrt{25}=5$ comes out exactly — **rational**. (c) $\sqrt{7}=2.6457\ldots$ does not terminate or repeat — **irrational**. (d) $0.25$ is a terminating decimal ($=1/4$) — **rational**. (e) $\frac{\pi}{2}$ is a multiple of $\pi$ — **irrational**. (f) $\sqrt[3]{27}=3$ — **rational**.

**Q2.** (a) $\frac{7}{8}=7\div 8=0.875$. (b) $0.875\times 100\%=87.5\%$. (c) It is a fraction of integers, so it is **rational**.

**Q3.** (a) $0.45=\frac{45}{100}=\frac{9}{20}$ (dividing top and bottom by 5). (b) $0.45=45\%$.

**Q4.** (a) $65\%=\frac{65}{100}=\frac{13}{20}$ (dividing by 5). (b) $65\%=0.65$.

**Q5.** (a) $\frac{2}{3}=2\div 3=0.666\ldots=0.\bar{6}$. (b) $\frac{5}{6}=5\div 6=0.8333\ldots=0.8\bar{3}$. (c) $\frac{1}{9}=1\div 9=0.111\ldots=0.\bar{1}$.

**Q6.** As decimals: $0.7$; $\frac{3}{4}=0.75$; $68\%=0.68$; $\frac{4}{5}=0.8$. Ordering, $0.68<0.7<0.75<0.8$, so the order is $68\%, 0.7, \frac{3}{4}, \frac{4}{5}$.

**Q7.** $\sqrt{36}=6$ (rational); $\sqrt{10}=3.162\ldots$ (irrational); $1.2=6/5$ (rational); $\pi$ (irrational); $\frac{8}{3}$ is a fraction of integers (rational); $\sqrt[3]{64}=4$ (rational).

**Q8.** (a) $\frac{7}{12}=7\div 12=0.58333\ldots=0.58\bar{3}$. (b) $0.58333\ldots\times 100\%=58.333\ldots\%=58.3\%$ (to one decimal place).

**Q9.** (a) $0.64=\frac{64}{100}=\frac{16}{25}$ (dividing top and bottom by 4). (b) $0.64\times 100\%=64\%$.

**Q10.** (a) $22.5\%=\frac{22.5}{100}=\frac{225}{1000}=\frac{9}{40}$. (b) $22.5\%=0.225$.

**Q11.** As decimals: $\frac{5}{8}=0.625$; $0.6$; $62\%=0.62$; $\frac{11}{20}=0.55$. Ordering, $0.55<0.6<0.62<0.625$, so the order is $\frac{11}{20}, 0.6, 62\%, \frac{5}{8}$.

**Q12.** (a) Area $=\pi r^2=\pi\times 5^2=25\pi$ m². (b) As a multiple of $\pi$ it is **irrational**. (c) $25\pi=78.5398\ldots\approx 78.54$ m².

**Q13.** (a) By Pythagoras' theorem the sloping surface is $\sqrt{5^2+12^2}=\sqrt{25+144}=\sqrt{169}=13$ m, which is **rational**. (b) The diagonal is $\sqrt{2^2+3^2}=\sqrt{4+9}=\sqrt{13}$ m. Since 13 is not a perfect square this is a surd, so it is **irrational** ($\approx 3.61$ m).

**Q14.** $\sqrt{\frac{1}{4}}=\frac{1}{2}$, a fraction — **rational**. $\sqrt{0.16}=0.4$, a terminating decimal — **rational**. $\sqrt{20}=\sqrt{4\times 5}=2\sqrt{5}$, a surd — **irrational**. $0.\overline{3}=\frac{1}{3}$, a recurring decimal that equals a fraction — **rational**.

**Q15.** $\frac{3}{4}=0.75$ and $\frac{2}{3}=0.666\ldots\approx 0.67$. Since $0.75>0.67$, more **flour** is used than sugar.

**Q16.** $\frac{1}{5}=1\div 5=0.2=20\%$; $0.85=\frac{85}{100}=\frac{17}{20}=85\%$; $32\%=\frac{32}{100}=\frac{8}{25}=0.32$; $\frac{1}{8}=1\div 8=0.125=12.5\%$.

**Q17.** As decimals: $0.72$; $\frac{7}{10}=0.7$; $\frac{3}{4}=0.75$; $71\%=0.71$; $0.69$. Ordering, $0.69<0.7<0.71<0.72<0.75$, so the order is $0.69, \frac{7}{10}, 71\%, 0.72, \frac{3}{4}$.

**Q18.** Converting Mia's record to a percentage: $\frac{5}{8}=0.625=62.5\%$. Ada made $62\%$. Since $62.5\%>62\%$, **Mia** had the better free-throw record.

# Chapter 2 Number and calculation — 2D Order of operations, powers and roots

## Theory

A calculation that mixes additions, multiplications, brackets and powers has only **one** correct answer, because mathematics follows an agreed **order of operations**. Getting the order wrong is one of the most common causes of a wrong answer — and of a wrong figure on an invoice or a quote.

**The order of operations.**

Work through a calculation in this order:

1. **Brackets** — do anything inside brackets first.
2. **Indices** — powers and roots (e.g. $3^2$, $\sqrt{16}$).
3. **Division and multiplication** — working from left to right.
4. **Addition and subtraction** — working from left to right.

A common memory aid is **BIDMAS** (Brackets, Indices, Division/Multiplication, Addition/Subtraction). The key traps are that division and multiplication rank **equally** (so you go left to right), and likewise for addition and subtraction. For example,

$$20-6\div 2+1=20-3+1=18,$$

where the division is done before the subtraction and addition.

**Powers (indices).**

A **power** is a short way to write repeated multiplication. In $5^3$, the **base** is $5$ and the **index** (or exponent) is $3$, and it means $5\times 5\times 5=125$. The special names are **squared** for an index of 2 ($7^2=49$) and **cubed** for an index of 3 ($2^3=8$). On a scientific calculator, use the $x^2$ key for squares and the $x^{\square}$ (or $\wedge$) key for any other power.

**Roots.**

A **root** undoes a power. The **square root** $\sqrt{a}$ is the positive number that, when squared, gives $a$: since $11^2=121$, $\sqrt{121}=11$. The **cube root** $\sqrt[3]{a}$ undoes a cube: since $6^3=216$, $\sqrt[3]{216}=6$. Use the square-root and cube-root keys on your calculator, and remember that a root counts as an **index**, so it is evaluated at the same stage as powers.

**Where powers and roots come from.**

Squares, cubes and their roots appear throughout measurement:

- the **area** of a square of side $s$ is $s^2$, so the side of a square of known area is $s=\sqrt{\text{area}}$;
- the **volume** of a cube of edge $s$ is $s^3$, so the edge of a cube of known volume is $s=\sqrt[3]{\text{volume}}$;
- **Pythagoras' theorem** gives the longest side of a right-angled triangle as $c=\sqrt{a^2+b^2}$.

**Using a scientific calculator.**

Enter a calculation the way it is written and let the calculator apply the order of operations, but use **brackets** generously to make your meaning clear — especially around a numerator, a denominator, or the contents of a root. For instance $\sqrt{25+144}$ must be entered as $\sqrt{(25+144)}$, giving $\sqrt{169}=13$; without the brackets the calculator would read $\sqrt{25}+144$.

## Worked examples

### *Example 1 — scaffolded*

Evaluate $3+4\times(7-2)^2\div 5$.

| Working | Comment |
|---|---|
| $(7-2)=5$. | Brackets first. |
| $5^2=25$. | Indices next. |
| $4\times25=100$, then $100\div5=20$. | Multiplication and division, left to right. |
| $3+20=23$. | Addition last. |

### *Example 2 — scaffolded*

Evaluate $2^3+\sqrt{144}-\sqrt[3]{27}$.

| Working | Comment |
|---|---|
| $2^3=8$. | A power. |
| $\sqrt{144}=12$. | A square root (since $12^2=144$). |
| $\sqrt[3]{27}=3$. | A cube root (since $3^3=27$). |
| $8+12-3=17$. | Add and subtract, left to right. |

### *Example 3 — unscaffolded*

A square courtyard has an area of $45$ m². Find the length of one side, correct to one decimal place.

The area of a square of side $s$ is $s^2$, so $s^2=45$ and therefore

$$s=\sqrt{45}=6.708\ldots\,\mathrm{m}.$$

Rounding to one decimal place gives $6.7$ m.

∴ each side of the courtyard is about $6.7$ m long.

### *Example 4 — unscaffolded*

A right-angled garden bed has its two shorter sides measuring $1.2$ m and $1.6$ m. Find the length of the longest side (the hypotenuse).

By Pythagoras' theorem the hypotenuse is $c=\sqrt{a^2+b^2}$ with $a=1.2$ and $b=1.6$:

$$c=\sqrt{1.2^2+1.6^2}=\sqrt{1.44+2.56}=\sqrt{4}=2\,\mathrm{m}.$$

∴ the longest side of the garden bed is $2$ m.

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## Practice questions

### *Scaffolded*

**Q1.** Evaluate, using the order of operations: (a) $8+3\times4$; (b) $(8+3)\times4$; (c) $30-12\div4+5$.

**Q2.** Evaluate: (a) $5^2$; (b) $2^4$; (c) $10^3$.

**Q3.** Evaluate: (a) $\sqrt{81}$; (b) $\sqrt{0.25}$; (c) $\sqrt[3]{64}$.

**Q4.** Evaluate, using the order of operations: (a) $3^2+4^2$; (b) $\sqrt{36+64}$; (c) $50-2\times3^3$.

**Q5.** Use a calculator to substitute into each formula (round to one decimal place where needed): (a) the area of a circle $A=\pi r^2$ when $r=6$ cm; (b) the volume of a cube $V=s^3$ when $s=4$ cm; (c) the hypotenuse $c=\sqrt{a^2+b^2}$ when $a=7$ and $b=24$.

**Q6.** Find the missing length: (a) a square has area 36 m² — find its side length; (b) a cube has volume 125 cm³ — find its edge length; (c) a square has area 30 m² — find its side length, correct to one decimal place.

### *Unscaffolded*

**Q7.** Evaluate $12+6\times5-8$.

**Q8.** Evaluate $(15-7)^2\div4$.

**Q9.** Evaluate $4\times3^2-10$.

**Q10.** Evaluate $\sqrt{100-36}$.

**Q11.** Evaluate $2^3+3^3$.

**Q12.** Evaluate $\sqrt[3]{1000}+\sqrt{49}$.

**Q13.** Evaluate $100\div(2+3)^2$.

**Q14.** Evaluate $-5+2\times(-3)^2$.

**Q15.** Evaluate $6.5+1.5\times4^2$.

**Q16.** A right-angled paddock has its two shorter sides measuring 80 m and 150 m. Using $c=\sqrt{a^2+b^2}$, find the length of the longest side.

**Q17.** A square lawn has an area of 52 m². Find the length of one side, correct to one decimal place.

**Q18.** A cube-shaped gift box has a volume of 343 cm³. Find the length of one edge.

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### Answers

**Q1.** (a) 20; (b) 44; (c) 32. **Q2.** (a) 25; (b) 16; (c) 1000. **Q3.** (a) 9; (b) 0.5; (c) 4. **Q4.** (a) 25; (b) 10; (c) $-4$. **Q5.** (a) 113.1 cm²; (b) 64 cm³; (c) 25. **Q6.** (a) 6 m; (b) 5 cm; (c) 5.5 m. **Q7.** 34. **Q8.** 16. **Q9.** 26. **Q10.** 8. **Q11.** 35. **Q12.** 17. **Q13.** 4. **Q14.** 13. **Q15.** 30.5. **Q16.** 170 m. **Q17.** 7.2 m. **Q18.** 7 cm.

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### Fully-worked solutions

**Q1.** (a) Multiplication before addition: $8+3\times4=8+12=20$. (b) Brackets first: $(8+3)\times4=11\times4=44$. (c) Division first, then left to right: $30-12\div4+5=30-3+5=32$.

**Q2.** (a) $5^2=5\times5=25$. (b) $2^4=2\times2\times2\times2=16$. (c) $10^3=10\times10\times10=1000$.

**Q3.** (a) $\sqrt{81}=9$ (since $9^2=81$). (b) $\sqrt{0.25}=0.5$ (since $0.5^2=0.25$). (c) $\sqrt[3]{64}=4$ (since $4^3=64$).

**Q4.** (a) $3^2+4^2=9+16=25$. (b) Work inside the root first: $\sqrt{36+64}=\sqrt{100}=10$. (c) Indices, then multiplication, then subtraction: $50-2\times3^3=50-2\times27=50-54=-4$.

**Q5.** (a) $A=\pi\times6^2=\pi\times36=113.097\ldots\approx113.1$ cm². (b) $V=4^3=64$ cm³. (c) $c=\sqrt{7^2+24^2}=\sqrt{49+576}=\sqrt{625}=25$ .

**Q6.** (a) side = $\sqrt{36}=6$ m. (b) edge = $\sqrt[3]{125}=5$ cm. (c) side = $\sqrt{30}=5.477\ldots\approx5.5$ m.

**Q7.** Multiplication before addition and subtraction: $12+6\times5-8=12+30-8=34$.

**Q8.** Brackets, then the index, then the division: $(15-7)^2\div4=8^2\div4=64\div4=16$.

**Q9.** Index first, then multiply, then subtract: $4\times3^2-10=4\times9-10=36-10=26$.

**Q10.** Work inside the root first: $\sqrt{100-36}=\sqrt{64}=8$.

**Q11.** $2^3+3^3=8+27=35$.

**Q12.** $\sqrt[3]{1000}+\sqrt{49}=10+7=17$.

**Q13.** Brackets, then the index, then the division: $100\div(2+3)^2=100\div5^2=100\div25=4$.

**Q14.** The index applies to $-3$: $(-3)^2=9$, so $-5+2\times9=-5+18=13$.

**Q15.** Index first, then multiply, then add: $6.5+1.5\times4^2=6.5+1.5\times16=6.5+24=30.5$.

**Q16.** $c=\sqrt{80^2+150^2}=\sqrt{6400+22\,500}=\sqrt{28\,900}=170$ m.

**Q17.** The side of a square is the square root of its area: $s=\sqrt{52}=7.211\ldots\approx7.2$ m.

**Q18.** The edge of a cube is the cube root of its volume: $s=\sqrt[3]{343}=7$ cm (since $7^3=343$).

## Theory

In everyday life an **exact** answer is often unnecessary, or even impossible — you rarely need to know that a shopping trolley holds $147.836 of groceries, only that it is "about $150". An **estimate** is a quick, sensible value found without a full calculation, and an **approximation** is a value rounded to a convenient level of accuracy. Both are essential for **checking** that a calculator answer is reasonable: a mistyped key can turn $18 into $180, and only an estimate will catch it.

**Rounding to a number of decimal places.**

To round to a given number of decimal places, look at the **next** digit after the place you are keeping. If it is $5$ or more, round the last kept digit **up**; if it is $4$ or less, leave it unchanged. For example $3.7482$ rounded to $2$ decimal places is $3.75$ (the next digit is $8$), and to $1$ decimal place is $3.7$ (the next digit is $4$).

**Rounding to significant figures.**

The **significant figures** of a number are its meaningful digits, counted from the first non-zero digit. In $48\,273$ the first significant figure is $4$; in $0.00612$ it is $6$ (leading zeros are **not** significant). To round to $n$ significant figures, keep the first $n$ significant digits and use the next digit to decide whether to round up, then replace any remaining whole-number digits with zeros to hold place value. So $48\,273$ to $2$ significant figures is $48\,000$, and $0.00612$ to $1$ significant figure is $0.006$.

**Estimating by rounding.**

A fast way to estimate a calculation is to round every number to **one significant figure** and then compute. For example

$$78 \times 43 \approx 80 \times 40 = 3200,$$

which is close to the exact value $3354$. This is accurate enough to check a calculator answer and to spot a gross error.

**Leading-digit approximation.**

The **leading digit** of a number is its first non-zero digit. A **leading-digit approximation** rounds the number to the place value of that digit: find the leading digit, apply the usual rounding rule at its place value, then replace the remaining whole-number digits with zeros. So $387$ becomes $400$ (leading digit $3$, in the hundreds), $28.084$ becomes $30$ (leading digit $2$, in the tens) and $0.0000456$ becomes $0.00005$ (leading digit $4$, in the hundred-thousandths). This is the same operation as rounding to one significant figure, and it is the standard way to describe a very large or very small number quickly. Applied term by term, $387 + 612 + 245 \approx 400 + 600 + 200 = 1200$, against an exact sum of $1244$. Note that each number is rounded to the **nearest** multiple of its leading place value, so a leading-digit estimate may finish either above or below the exact value — do **not** simply drop the digits after the first.

**Interval estimates.**

Sometimes the most honest estimate is a **range** rather than a single value. If each of $4$ items costs between $1.95 and $2.15, the total lies between $4 \times 1.95 = \$7.80$ and $4 \times 2.15 = \$8.60$. Writing "between $7.80 and $8.60" is an **interval estimate**.

**Floor and ceiling values.**

In many real problems the answer must be a **whole** number, and the context decides which way to round:

- A **floor value** rounds **down** to the nearest whole number — the largest whole number that is not more than the value. Use it for "how many whole ones fit / can I afford": with $50 and $12 tickets you can buy $50 \div 12 = 4.16\ldots$, floored to $4$ tickets.

- A **ceiling value** rounds **up** to the nearest whole number — the smallest whole number that is not less than the value. Use it for "how many are needed to cover everything": seating $75$ guests at tables of $6$ needs $75 \div 6 = 12.5$ tables, which must be rounded up to $13$ tables (twelve tables would leave three guests standing).

**Percentage error.**

When a measured or estimated value differs from the true (actual) value, the **percentage error** measures how far off it is, as a percentage of the actual value:

$$\text{percentage error} = \frac{|\text{measured} - \text{actual}|}{\text{actual}} \times 100\,\%.$$

The vertical bars mean "take the size of the difference" (always positive), so percentage error is never negative.

**Checking reasonableness.**

After any calculation, ask: *is this answer sensible?* Check the **order of magnitude** (is it in the right ballpark — tens, hundreds, thousands?), the **units** (metres or kilometres?), and whether it makes **physical sense** (a single apple cannot cost \$162). A quick estimate is the best reasonableness check of all.

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## Worked examples

### *Example 1 — scaffolded*

A length is measured as $12.7649$ m. Round this length to (a) 2 decimal places, (b) $1$ decimal place and (c) $3$ significant figures.

| Working | Comment |
|---|---|
| (a) The 3rd decimal digit is $4$, so leave the 2nd unchanged: $12.76$ m. | Round to 2 d.p.; $4 \le 4$, round down. |
| (b) The 2nd decimal digit is $6$, so round the 1st up: $12.8$ m. | Round to 1 d.p.; $6 \ge 5$, round up. |
| (c) The first three significant figures are $1, 2, 7$; the next digit is $6$, so round up: $12.8$ m. | The $7$ rounds up to $8$ because the next digit is $6$. |

### *Example 2 — scaffolded*

A rectangular room measures $6.2$ m by $3.9$ m. Estimate its floor area by rounding each measurement to one significant figure, then find the exact area and the percentage error of the estimate (to $2$ decimal places).

| Working | Comment |
|---|---|
| Round each measurement to 1 s.f.: $6.2 \approx 6$ and $3.9 \approx 4$. | Rounding for a quick estimate. |
| Estimated area $= 6 \times 4 = 24$ m². | Multiply the rounded values. |
| Exact area $= 6.2 \times 3.9 = 24.18$ m². | Full calculation for comparison. |
| Percentage error $= \frac{\lvert 24 - 24.18 \rvert}{24.18} \times 100\,\% = \frac{0.18}{24.18} \times 100\,\% \approx 0.74\,\%$. | Difference over actual, as a percentage. |

### *Example 3 — unscaffolded*

A school is organising an excursion for $143$ students. Each bus seats $45$ students. Tickets to the venue cost \$38 each and the excursion budget for tickets is \$500. (a) How many buses are needed? (b) How many tickets can be bought within the budget?

(a) The number of buses is $143 \div 45 = 3.17\ldots$. Three buses would seat only $3 \times 45 = 135$ students, leaving 8 behind, so the value must be rounded **up** (a ceiling value) to $4$ buses.

(b) The number of tickets is $500 \div 38 = 13.15\ldots$. Only whole tickets can be bought, and $14$ tickets would cost $14 \times 38 = \$532 > \$500$, so the value must be rounded **down** (a floor value) to $13$ tickets.

$\therefore$ the school needs $4$ buses and can buy $13$ tickets within the budget.

### *Example 4 — unscaffolded*

A student measures the length of a bench as $1.18$ m. The actual length is $1.25$ m. Find the percentage error in the student's measurement, correct to one decimal place.

The size of the difference is $|1.18 - 1.25| = 0.07$ m. As a percentage of the actual length,

$$\text{percentage error} = \frac{0.07}{1.25} \times 100\,\% = 5.6\,\%.$$

$\therefore$ the measurement has a percentage error of $5.6\,\%$.

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## Practice questions

### *Scaffolded*

**Q1.** Round the number $63.4827$ to (a) 2 decimal places; (b) 1 decimal place; (c) the nearest whole number.

**Q2.** Round $36\,749$ to (a) 1 significant figure; (b) 2 significant figures; (c) 3 significant figures.

**Q3.** (a) Use a leading-digit approximation for each number to estimate $472 + 318 + 265$, and compare with the exact sum. (b) Estimate $64 \times 73$ by rounding each number to one significant figure, and compare with the exact product.

**Q4.** Decide whether each situation needs a floor value or a ceiling value, then answer it. (a) $178$ people are to travel in minibuses that each seat $45$. How many minibuses are needed? (b) A caterer has \$250 and each fruit platter costs \$32. How many platters can be bought? (c) A $2.3$ m timber board is cut into $0.4$ m pieces. How many whole pieces are obtained?

**Q5.** A market stall estimates its takings for a day as \$2600 by rounding. The exact takings were \$2565. (a) Find the size of the error. (b) Find the percentage error, correct to two decimal places. (c) State whether the estimate was an over-estimate or an under-estimate.

**Q6.** The crowd at a community festival was estimated at $4800$ people. The actual attendance, from ticket sales, was $5127$. (a) Find the percentage error of the estimate, correct to two decimal places. (b) The organisers expect each person to spend between \$15 and \$20. Give an interval estimate for the total spending, based on the actual attendance.

### *Unscaffolded*

**Q7.** Round $8.06519$ to (a) 2 decimal places and (b) 3 decimal places.

**Q8.** Round $0.0473921$ to (a) 2 significant figures and (b) 3 significant figures, and round $725\,480$ to (c) 3 significant figures.

**Q9.** Estimate $613 \times 28$ by rounding each number to one significant figure. Find the exact product and the percentage error of your estimate, correct to two decimal places.

**Q10.** A shopper buys items costing \$4.85, \$11.20, \$6.75 and \$22.40. Use a leading-digit approximation for each amount to estimate the total, then find the exact total and state whether the estimate is too high or too low.

**Q11.** A farm collects $500$ eggs. The eggs are packed into cartons that hold $24$ eggs each. How many cartons can be completely filled?

**Q12.** A warehouse must move $1375$ kg of sand using containers that each hold $200$ kg. How many containers are needed?

**Q13.** A digital scale reads a parcel's mass as 2.4 kg. The true mass is 2.5 kg. Find the percentage error of the scale's reading.

**Q14.** Estimate $39 \times 51$ by rounding each number to one significant figure, and find the percentage error of the estimate, correct to two decimal places. Comment on how good the estimate is.

**Q15.** A tiler needs 7 boxes of tiles, each costing between $28.50 and $31.00. Give an interval estimate for the total cost of the tiles.

**Q16.** A receipt claims that 18 apples were bought at $0.90 each for a total of $162. Use estimation to explain why this total cannot be correct, and find the correct total.

**Q17.** A wall measures 5.8 m long and 2.7 m high. One tin of paint covers 8 $m^2$. (a) Estimate the wall's area by rounding each measurement to one significant figure. (b) Find the exact area, and hence the number of tins that must be bought.

**Q18.** A garden path is 47.3 m long and is to be edged with timber lengths that are each 2 m long. (a) How many whole 2 m lengths fit along the path end to end? (b) How many 2 m lengths must be bought to edge the whole path?

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## Answers

**Q1.** (a) $63.48$; (b) $63.5$; (c) $63$. **Q2.** (a) $40\,000$; (b) $37\,000$; (c) $36\,700$. **Q3.** (a) $\approx 1100$ (exact $1055$); (b) $\approx 4200$ (exact $4672$). **Q4.** (a) ceiling, $4$ minibuses; (b) floor, $7$ platters; (c) floor, $5$ pieces. **Q5.** (a) \$35; (b) $1.36\,\%$; (c) over-estimate. **Q6.** (a) $6.38\,\%$; (b) between $\$76\,905$ and $\$102\,540$. **Q7.** (a) $8.07$; (b) $8.065$. **Q8.** (a) $0.047$; (b) $0.0474$; (c) $725\,000$. **Q9.** estimate $18\,000$, exact $17\,164$, percentage error $4.87\,\%$. **Q10.** estimate \$42 (leading digits: $5+10+7+20$), exact \$45.20; the estimate is too low. **Q11.** $20$ cartons. **Q12.** $7$ containers. **Q13.** $4\,\%$. **Q14.** estimate $2000$, exact $1989$, percentage error $0.55\,\%$; a very good estimate. **Q15.** between \$199.50 and \$217. **Q16.** $18\times\$0.90\approx 20\times\$1=\$20$, so \$162 is far too large; the correct total is \$16.20. **Q17.** (a) $\approx 18$ m² ($6\times 3$); (b) exact $15.66$ m², so 2 tins. **Q18.** (a) $23$ lengths; (b) $24$ lengths.

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## Fully-worked solutions

**Q1.** (a) The 3rd decimal digit of $63.4827$ is $2$, so the 2nd stays: $63.48$. (b) The 2nd decimal digit is $8$, so round the 1st up: $63.5$. (c) The first decimal digit is $4$, so round down: $63$.

**Q2.** The first significant figure of $36\,749$ is $3$. (a) 1 s.f.: the next digit is $6$, so $3$ rounds to $4$: $40\,000$. (b) 2 s.f.: keep $3,6$; the next digit is $7$, so $6$ rounds to $7$: $37\,000$. (c) 3 s.f.: keep $3,6,7$; the next digit is $4$, so no change: $36\,700$.

**Q3.** (a) Each number has its leading digit in the hundreds, so round each to the nearest hundred: $472\approx 500$, $318\approx 300$ and $265\approx 300$, giving $500+300+300=1100$. The exact sum is $472+318+265=1055$, so this estimate is a little **high** — two of the three numbers rounded up. (b) Rounding each factor to $1$ s.f.: $64\approx 60$ and $73\approx 70$, so $60\times 70=4200$. The exact product is $64\times 73=4672$, so $4200$ is a reasonable check, a little low.

**Q4.** (a) $178\div 45=3.95\ldots$; four minibuses seat $180$ and three seat only $135$, so round **up** — a ceiling value gives $4$ minibuses. (b) $250\div 32=7.8\ldots$; eight platters cost $8\times 32=\$256>\$250$, so round **down** — a floor value gives $7$ platters. (c) $2.3\div 0.4=5.75$; only whole pieces count, so round **down** to $5$ pieces ($0.3$ m of board is left over).

**Q5.** (a) The size of the error is $|2600-2565|=\$35$. (b) Percentage error $=\dfrac{35}{2565}\times 100\,\%=1.36\,\%$ (to 2 d.p.). (c) Since $2600>2565$, the estimate is an **over-estimate**.

**Q6.** (a) The difference is $|4800-5127|=327$, so percentage error $=\dfrac{327}{5127}\times 100\,\%=6.38\,\%$ (to 2 d.p.). (b) With $5127$ people spending between \$15 and \$20, the total lies between $5127\times 15=\$76\,905$ and $5127\times 20=\$102\,540$.

**Q7.** (a) The 3rd decimal digit of $8.06519$ is $5$, so round the 2nd up: $8.07$. (b) The 4th decimal digit is $1$, so the 3rd stays: $8.065$.

**Q8.** (a) The first significant figure of $0.0473921$ is $4$. To 2 s.f. keep $4,7$; the next digit is $3$, so no change: $0.047$. (b) To 3 s.f. keep $4,7,3$; the next digit is $9$, so $3$ rounds to $4$: $0.0474$. (c) $725\,480$ to 3 s.f.: keep $7,2,5$; the next digit is $4$, so no change, and the remaining digits become zeros: $725\,000$.

**Q9.** Rounding to 1 s.f.: $613\approx 600$ and $28\approx 30$, so the estimate is $600\times 30=18\,000$. The exact product is $613\times 28=17\,164$, so percentage error $=\dfrac{|18\,000-17\,164|}{17\,164}\times 100\,\%=\dfrac{836}{17\,164}\times 100\,\%=4.87\,\%$ (to 2 d.p.).

**Q10.** Round each amount at its leading digit: $\$4.85\approx\$5$ and $\$6.75\approx\$7$ (leading digits in the units, so round to the nearest dollar), while $\$11.20\approx\$10$ and $\$22.40\approx\$20$ (leading digits in the tens, so round to the nearest ten dollars). The estimate is $\$5+\$10+\$7+\$20=\$42$. The exact total is $4.85+11.20+6.75+22.40=\$45.20$, so the estimate is **too low** — here the two larger amounts both rounded down by more than the smaller two rounded up.

**Q11.** $500\div 24=20.83\ldots$. Only completely filled cartons count, so take the floor value: 20 cartons (with $500-20\times 24=20$ eggs left over).

**Q12.** $1375\div 200=6.875$. Six containers hold only 1200 kg, so a seventh is needed for the remaining sand — take the ceiling value: 7 containers.

**Q13.** The difference is $|2.4-2.5|=0.1$ kg, so percentage error $=\frac{0.1}{2.5}\times 100\,\% = 4\,\%$.

**Q14.** Rounding to 1 s.f.: $39\approx 40$ and $51\approx 50$, so the estimate is $40\times 50 = 2000$. The exact product is $39\times 51 = 1989$, so percentage error $=\frac{|2000-1989|}{1989}\times 100\,\% = \frac{11}{1989}\times 100\,\% = 0.55\,\%$ (to 2 d.p.). The estimate is very good — the percentage error is well under $1\,\%$.

**Q15.** The lowest possible total is $7\times \$28.50 = \$199.50$ and the highest is $7\times \$31.00 = \$217$, so the total cost lies between \$199.50 and \$217.

**Q16.** Estimating, $18\times \$0.90 \approx 20\times \$1 = \$20$, so a total of \$162 is about eight times too large and cannot be correct. The exact total is $18\times 0.90 = \$16.20$ (the claimed figure has the decimal point in the wrong place).

**Q17.** (a) Rounding to 1 s.f.: $5.8\approx 6$ and $2.7\approx 3$, so the estimated area is $6\times 3 = 18$ m². (b) The exact area is $5.8\times 2.7 = 15.66$ m². The number of tins is $15.66\div 8 = 1.95\ldots$; since part of a second tin is still needed, round **up** to 2 tins.

**Q18.** (a) $47.3\div 2 = 23.65$. Twenty-three whole 2 m lengths reach 46 m, so 23 lengths fit end to end (a floor value). (b) The last 1.3 m still needs edging, so a 24th length must be bought — the ceiling value is 24 lengths.

### Theory

Some quantities are far too large or too small to write comfortably in ordinary form. The distance to the Sun is about $150\,000\,000\,000$ m, and a human hair is about $0.000\,07$ m thick. Numbers like these are clumsy to write, easy to miscount and hard to compare. **Scientific notation** (also called **standard form**) gives a compact, reliable way to write and compare them.

**Powers of 10.**

Scientific notation is built on powers of $10$. A **positive** power tells you how many times to multiply by $10$; a **negative** power tells you how many times to divide.

| Power | $10^3$ | $10^2$ | $10^1$ | $10^0$ | $10^{-1}$ | $10^{-2}$ | $10^{-3}$ |
|---|---|---|---|---|---|---|---|
| Value | 1000 | 100 | 10 | 1 | 0.1 | 0.01 | 0.001 |

**Scientific notation.**

A number is in **scientific notation** when it is written as

$$a \times 10^n,$$

where $a$ is a number with $1 \le a < 10$ (one non-zero digit before the decimal point) and $n$ is an integer. For example, $150\,000\,000\,000 = 1.5 \times 10^{11}$ and $0.000\,07 = 7 \times 10^{-5}$.

**Converting an ordinary number to scientific notation.**

Place the decimal point just after the first non-zero digit to get $a$, then count how many places the point has moved to find $n$:

- for a **large** number ($\ge 10$) the point moves **left**, so $n$ is **positive**: $86\,000\,000 = 8.6 \times 10^7$ (the point moved $7$ places left);
- for a **small** number ($< 1$) the point moves **right**, so $n$ is **negative**: $0.000\,58 = 5.8 \times 10^{-4}$ (the point moved $4$ places right).

**Converting scientific notation back to an ordinary number.**

Move the decimal point $n$ places — right if $n$ is positive, left if $n$ is negative — and fill with zeros: $4.7 \times 10^6 = 4\,700\,000$ and $6.4 \times 10^{-5} = 0.000\,064$.

**SI prefixes.**

Metric (SI) prefixes are shorthand for powers of $10$ and are used constantly for data, distances and masses.

| Prefix | tera (T) | giga (G) | mega (M) | kilo (k) | milli (m) | micro (μ) | nano (n) |
|---|---|---|---|---|---|---|---|
| Power | $10^{12}$ | $10^9$ | $10^6$ | $10^3$ | $10^{-3}$ | $10^{-6}$ | $10^{-9}$ |

So $7$ megabytes $= 7 \times 10^6$ bytes, and $40$ milligrams $= 40 \times 10^{-3} = 0.04$ grams.

**Using a scientific calculator.**

Enter scientific notation with the $\times 10^x$ key (labelled **EE** or **EXP** on some calculators) — **not** by typing "$\times 10\ \wedge$". To enter $3 \times 10^8$, press `3`, then the $\times 10^x$ key, then `8`. A calculator with a natural display shows a result such as $1.5 \times 10^{11}$ exactly as it is written here; a calculator with a line display shortens it to `1.5E11`, where the number after the `E` is the power of $10$.

**Calculating in scientific notation.**

To multiply, multiply the number parts and **add** the powers; to divide, divide the number parts and **subtract** the powers:

$$(3\times10^4)\times(2\times10^6)=6\times10^{10},(8\times10^9)\div(4\times10^3)=2\times10^6.$$

Your calculator does this automatically once the numbers are entered correctly.

**Comparing magnitudes.**

To compare two numbers in scientific notation, compare the **powers of 10** first — the larger power means the larger number. Only if the powers are equal do you compare the number parts $a$. So $2.9\times10^{-6}>7.5\times10^{-7}$, because $-6>-7$.

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## Worked examples

### *Example 1 — scaffolded*

Write (a) $45\,000\,000$ and (b) $0.000\,32$ in scientific notation.

| Working | Comment |
|---|---|
| (a) First non-zero digit gives $a=4.5$. | Put the decimal point after the first digit. |
| The point moved 7 places **left**, so $n=7$: $4.5\times10^7$. | Large number ⇒ positive power. |
| (b) First non-zero digit gives $a=3.2$. | The first non-zero digit is the 3. |
| The point moved 4 places **right**, so $n=-4$: $3.2\times10^{-4}$. | Small number ⇒ negative power. |

### *Example 2 — scaffolded*

Write (a) $2.5\times10^9$ as an ordinary number, and (b) express it using a suitable SI prefix for bytes.

| Working | Comment |
|---|---|
| (a) Move the point 9 places right: $2\,500\,000\,000$. | Positive power ⇒ move right. |
| (b) $10^9$ is the prefix **giga**, so $2.5\times10^9$ bytes $=2.5$ GB. | Match the power of 10 to its prefix. |

### *Example 3 — unscaffolded*

Light travels at about $3\times10^8$ metres per second. How far does light travel in 90 seconds? Give your answer in scientific notation.

Distance = speed × time, so

$$\text{distance}=(3\times10^8)\times90=3\times10^8\times9\times10^1=27\times10^9=2.7\times10^{10}\,\text{m}.$$

(The result $27\times10^9$ is rewritten as $2.7\times10^{10}$ so that $1\le a<10$.)

∴ light travels $2.7\times10^{10}$ m in 90 seconds.

### *Example 4 — unscaffolded*

A bacterium is about $2\times10^{-6}$ m long and a virus is about $5\times10^{-8}$ m long. How many times longer than the virus is the bacterium?

Divide the two lengths:

$$\frac{2\times10^{-6}}{5\times10^{-8}}=\frac{2}{5}\times10^{-6-(-8)}=0.4\times10^2=40.$$

∴ the bacterium is 40 times longer than the virus.

## Practice questions

### *Scaffolded*

**Q1.** Write each number in scientific notation. (a) $6\,200\,000$ (b) $91\,000$ (c) $0.000\,45$

**Q2.** Write each number in ordinary form. (a) $3.4 \times 10^{5}$ (b) $7 \times 10^{8}$ (c) $2.9 \times 10^{-3}$

**Q3.** Use the SI prefixes to convert each quantity. (a) $5$ GB into bytes. (b) $250$ mg into grams. (c) $8$ km into metres.

**Q4.** Evaluate, giving each answer in scientific notation. (a) $\left(2 \times 10^{5}\right) \times \left(4 \times 10^{3}\right)$ (b) $\left(9 \times 10^{10}\right) \div \left(3 \times 10^{4}\right)$

**Q5.** Consider the numbers $3.2 \times 10^{7}$ and $9.5 \times 10^{6}$. (a) State which number is larger. (b) How many times larger is it, correct to one decimal place?

**Q6.** A hard drive holds $1.8 \times 10^{12}$ bytes. One video file is $4.5 \times 10^{6}$ bytes. (a) How many such files fit on the drive? (b) Write your answer in scientific notation.

### *Unscaffolded*

**Q7.** Write $384\,000$ and $0.000\,007$ in scientific notation.

**Q8.** Write $1.2 \times 10^{10}$ and $6.5 \times 10^{-4}$ in ordinary form.

**Q9.** The population of Australia is about $2.8 \times 10^{7}$ people. Write this in ordinary form.

**Q10.** A red blood cell is about $8 \times 10^{-6}$ m across. Write this length in ordinary form, and then in micrometres (μm).

**Q11.** Evaluate $\left(5 \times 10^{7}\right) \times \left(3 \times 10^{-2}\right)$, giving your answer in scientific notation.

**Q12.** Evaluate $\left(6 \times 10^{-3}\right) \div \left(2 \times 10^{-6}\right)$, giving your answer in scientific notation.

**Q13.** A $2$ TB hard drive holds $2 \times 10^{12}$ bytes. Each photo is $8 \times 10^{6}$ bytes. How many photos fit on the drive? Give your answer in scientific notation.

**Q14.** Which is larger, $4.1 \times 10^{-5}$ or $3.8 \times 10^{-4}$? Explain your reasoning.

**Q15.** The Sun is about $1.5 \times 10^{11}$ m from Earth. Light travels at $3 \times 10^{8}$ m/s. How many seconds does light take to reach Earth from the Sun?

**Q16.** A grain of sand has a mass of about $2 \times 10^{-7}$ kg. About how many grains of sand are there in $1$ kg? Give your answer in scientific notation.

**Q17.** A memory card is advertised as 32 GB. Write its capacity in bytes, in scientific notation.

**Q18.** The mass of Earth is about $6 \times 10^{24}$ kg and the mass of the Moon is about $7.3 \times 10^{22}$ kg. About how many times heavier than the Moon is the Earth, correct to the nearest whole number?

### Answers

**Q1.** (a) $6.2 \times 10^6$; (b) $9.1 \times 10^4$; (c) $4.5 \times 10^{-4}$. **Q2.** (a) $340\,000$; (b) $700\,000\,000$; (c) $0.0029$. **Q3.** (a) $5 \times 10^9$ bytes; (b) $0.25$ g; (c) $8000$ m. **Q4.** (a) $8 \times 10^8$; (b) $3 \times 10^6$. **Q5.** (a) $3.2 \times 10^7$; (b) about $3.4$ times. **Q6.** (a) $400\,000$ files; (b) $4 \times 10^5$. **Q7.** $3.84 \times 10^5$ and $7 \times 10^{-6}$. **Q8.** $12\,000\,000\,000$ and $0.000\,65$. **Q9.** $28\,000\,000$. **Q10.** $0.000\,008$ m $= 8$ µm. **Q11.** $1.5 \times 10^6$. **Q12.** $3 \times 10^3$. **Q13.** $2.5 \times 10^5$ photos. **Q14.** $3.8 \times 10^{-4}$ is larger (its power $-4$ is greater than $-5$ ). **Q15.** 500 s. **Q16.** $5 \times 10^6$ grains. **Q17.** $3.2 \times 10^{10}$ bytes. **Q18.** about 82 times.

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### Fully-worked solutions

**Q1.** (a) $a = 6.2$; the point moves 6 places left, so $6\,200\,000 = 6.2 \times 10^6$. (b) $91\,000 = 9.1 \times 10^4$ (point moves 4 places left). (c) $0.000\,45 = 4.5 \times 10^{-4}$ (point moves 4 places right).

**Q2.** (a) Move the point 5 places right: $3.4 \times 10^5 = 340\,000$. (b) $7 \times 10^8 = 700\,000\,000$. (c) Move the point 3 places left: $2.9 \times 10^{-3} = 0.0029$.

**Q3.** (a) Giga $= 10^9$, so 5 GB $= 5 \times 10^9$ bytes. (b) Milli $= 10^{-3}$, so 250 mg $= 250 \times 10^{-3}$ g $= 0.25$ g. (c) Kilo $= 10^3$, so 8 km $= 8 \times 1000 = 8000$ m.

**Q4.** (a) Multiply the number parts and add the powers: $(2 \times 4) \times 10^{5+3} = 8 \times 10^8$. (b) Divide the number parts and subtract the powers: $(9 \div 3) \times 10^{10-4} = 3 \times 10^6$.

**Q5.** (a) Both have the same number part range, but $10^7 > 10^6$, so $3.2 \times 10^7$ is larger. (b) $\frac{3.2 \times 10^7}{9.5 \times 10^6} = \frac{3.2}{9.5} \times 10^1 = 0.3368 \times 10 = 3.4$ (to 1 d.p.), so it is about 3.4 times larger.

**Q6.** (a) Number of files $= \frac{1.8 \times 10^{12}}{4.5 \times 10^6} = \frac{1.8}{4.5} \times 10^{12-6} = 0.4 \times 10^6 = 400\,000$. (b) $400\,000 = 4 \times 10^5$ files.

**Q7.** $384\,000 = 3.84 \times 10^5$ (point moves 5 places left); $0.000\,007 = 7 \times 10^{-6}$ (point moves 6 places right).

**Q8.** $1.2 \times 10^{10} = 12\,000\,000\,000$ (point moves 10 places right); $6.5 \times 10^{-4} = 0.000\,65$ (point moves 4 places left).

**Q9.** $2.8 \times 10^7$: move the point 7 places right to get $28\,000\,000$ people.

**Q10.** $8 \times 10^{-6}$ m $= 0.000\,008$ m. Since micro $= 10^{-6}$, $8 \times 10^{-6}$ m $= 8$ µm.

**Q11.** $(5 \times 3) \times 10^{7+(-2)} = 15 \times 10^5 = 1.5 \times 10^6$ (rewriting $15 \times 10^5$ so that $1 \le a < 10$).

**Q12.** $(6 \div 2) \times 10^{-3-(-6)} = 3 \times 10^3$.

**Q13.** Number of photos $= \frac{2 \times 10^{12}}{8 \times 10^6} = \frac{2}{8} \times 10^6 = 0.25 \times 10^6 = 2.5 \times 10^5$ photos.

**Q14.** Compare the powers of 10: $-4$ is greater than $-5$, so $3.8 \times 10^{-4}$ is the larger number (it equals $0.000\,38$, while $4.1 \times 10^{-5} = 0.000\,041$).

**Q15.** Time $= \frac{\text{distance}}{\text{speed}} = \frac{1.5 \times 10^{11}}{3 \times 10^8} = \frac{1.5}{3} \times 10^{11-8} = 0.5 \times 10^3 = 500$ s.

**Q16.** Number of grains $= \frac{1}{2 \times 10^{-7}} = \frac{1}{2} \times 10^7 = 0.5 \times 10^7 = 5 \times 10^6$ grains.

**Q17.** Giga $= 10^9$, so 32 GB $= 32 \times 10^9 = 3.2 \times 10^{10}$ bytes (rewriting $32 \times 10^9$ so that $1 \le a < 10$).

**Q18.** $\dfrac{6 \times 10^{24}}{7.3 \times 10^{22}} = \dfrac{6}{7.3} \times 10^{24-22} = 0.822 \times 10^{2} = 82.2$, so the Earth is about 82 times heavier than the Moon.