Foundation Mathematics Units 3 & 4

Chapter 1 — Working mathematically and mathematical investigations

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Chapter 1 Working mathematically and mathematical investigations — 1C Taking a problem-solving approach

Theory

Much of the mathematics in Foundation is about solving **real problems** — problems that do not arrive neatly labelled "use this formula". A reliable way to tackle any problem is to follow a **problem-solving cycle** and to have a toolkit of **strategies** to try.

The problem-solving cycle.

Good problem-solvers work through four stages, and loop back whenever they get stuck:

1. **Understand the problem.** What are you told? What are you asked to find? What are the units? Rephrase it in your own words, and note any conditions.
2. **Devise a plan.** Choose a strategy (see below). Decide what to calculate and in what order. Make a quick **estimate** of the answer so you have something to check against.
3. **Carry out the plan.** Do the working carefully, one step at a time, keeping track of units.
4. **Check and reflect.** Is the answer reasonable — the right size, the right units, does it fit the story? Could you solve it a shorter way? Did you answer the question asked?

Common strategies.

Most problems yield to one of a small set of strategies:

- **Draw a diagram** — a picture of the situation often shows the relationships.
- **Make a table** — organise the numbers, especially when something changes step by step.
- **Look for a pattern** — work out a few cases, spot the rule, then use it.
- **Guess, check and improve** — try a sensible value, see how far off it is, adjust.
- **Work backwards** — start from the end result and undo each step.
- **Break it into smaller parts** — solve pieces of a big problem, then combine.
- **Simplify the problem** — solve an easier version first, then scale up your method.

Estimate first, then check reasonableness.

An **estimate** (§2E) is your safety net: if a calculation gives an answer far from your estimate, you have probably made a mistake. Always finish by asking *"is this answer sensible in the real situation?"* — a single bus cannot carry 5400 students, and a part-time wage of $\$2000$ per hour is not believable.

Worked examples

Example 1 — scaffolded

Maya is saving for a bike that costs $\$540$. She saves $\$15$ in the first week and increases her saving by $\$5$ each week (so $\$15, \$20, \$25, \dots$). In which week does her **total** saving first reach $\$540$?

Working	Comment
Understand: weekly savings grow $\$15, \$20, \$25, \dots$; find the week when the running total first reaches $\$540$.	Identify what changes and what is asked.
Plan: make a table of the weekly saving and the running total; a total near $\$540$ is expected after about three months, so around week 12–13 (estimate).	"Make a table" suits a step-by-step change; estimate first.
Week: 1, 2, 3, … total: $15, 35, 60, \dots$	Carry out: add each week's saving to the running total.
By week 12 the total is $\$510$; by week 13 it is $\$585$.	Continue the table to where it passes $\$540$.

Working	Comment
Check: $\$510 < \$540 \leq \$585$, so week 13 is the first week the total reaches $\$540$. Reasonable — it matches the estimate.	Confirm the boundary and sanity-check against the estimate.

Example 2 — scaffolded

A pattern of triangles is built from matchsticks: 1 triangle uses 3 sticks, 2 triangles (in a row) use 5 sticks, 3 triangles use 7 sticks. (a) How many sticks are needed for 10 triangles? (b) How many triangles can be made from 41 sticks?

Working	Comment
Understand: sticks for $1, 2, 3$ triangles are $3, 5, 7$ — each extra triangle adds 2 sticks.	Look at the first few cases.
Look for a pattern: sticks $= 2 \times (\text{triangles}) + 1$.	The rule: start with 1, add 2 per triangle.
(a) 10 triangles: $2 \times 10 + 1 = 21$ sticks.	Apply the rule forwards.
(b) $2 \times (\text{triangles}) + 1 = 41$, so $\text{triangles} = \frac{41 - 1}{2} = 20$.	Work backwards from 41 sticks.
Check: 20 triangles $\rightarrow 2(20) + 1 = 41$ sticks. ✓	Verify the rule gives the stated total.

Example 3 — unscaffolded

At a market, Priya spent half of her money on plants, then $\$12$ on lunch, then half of what was left on a book, and went home with $\$9$. How much money did she start with?

The cleanest strategy here is to **work backwards**, undoing each step. She finished with $\$9$, which was half of what she had before the book, so before the book she had $\$9 \times 2 = \$18$. Before lunch she had $\$18 + \$12 = \$30$. That $\$30$ was half of her starting money, so she started with $\$30 \times 2 = \$60$.

Checking forwards: $\$60 \rightarrow$ half on plants leaves $\$30 \rightarrow$ minus $\$12$ leaves $\$18 \rightarrow$ half on a book leaves $\$9$. ✓

∴ Priya started with $\$60$.

Example 4 — unscaffolded

A rectangular chicken run is built against a straight wall, so only **three** sides need fencing. There is 24 m of fencing. What whole-number dimensions give the **largest** area?

Let the two equal sides (out from the wall) be x m each; the remaining side is $24 - 2x$ m, so the area is $A = x(24 - 2x)$. Use **guess, check and improve** over whole numbers:

x (m)	length $24 - 2x$ (m)	area (m²)
5	14	70
6	12	72
7	10	70

The area rises to $x = 6$ and then falls, so the maximum whole-number area is at $x = 6$.

∴ the run should be 6 m × 12 m, giving the largest area, 72 m².

Practice questions

Scaffolded

Q1. Liam does 20 push-ups on day 1 and adds 4 each day. (a) Make a table of the push-ups for days 1 to 5. (b) How many push-ups does he do on day 10? (c) On which day does he first do at least 60 push-ups?

Q2. A number is doubled, then 7 is added, giving 33. (a) Write down the steps in reverse (the "undo" steps). (b) Work backwards to find the original number.

Q3. A pattern of dots grows: figure 1 has 1 dot, figure 2 has 3, figure 3 has 6, figure 4 has 10. (a) How many dots are in figures 5 and 6? (b) How many dots are in figure 8? (c) Which figure has 55 dots?

Q4. Four friends each shake hands once with every other friend. (a) Draw a diagram with a dot for each friend and a line for each handshake. (b) How many handshakes take place?

Q5. Two whole numbers add to 15 and multiply to 54. (a) Guess a pair that adds to 15 and check the product. (b) Improve your guess until it works, and state the two numbers.

Q6. A concert hall has 38 rows of 42 seats. (a) Estimate the total number of seats by rounding each number to one significant figure. (b) Find the exact number of seats. (c) Find the percentage error of your estimate, correct to two decimal places.

Unscaffolded

Q7. Sam spent one-third of his money, then a further $\$20$, and had $\$60$ left. How much did he start with?

Q8. Café tables seat people around the edges: 1 table seats 4, two tables pushed end-to-end seat 6, three seat 8. (a) How many people can 10 tables seat? (b) How many tables are needed to seat 30 people?

Q9. A plumber charges an $\$80$ call-out fee plus $\$60$ per hour. For how many hours does a job cost $\$320$?

Q10. A rectangle has a perimeter of 34 cm and an area of 60 cm². Find its length and width.

Q11. A two-digit code is made from the digits $1, 2, 3, 4$ with no digit repeated. How many different codes are possible?

Q12. Ana is three times as old as Ben. In 4 years' time, the sum of their ages will be 32. How old are they now?

Q13. A square lawn has an area of 144 m². Find its side length and its perimeter.

Q14. A report claims that 12 buses, each carrying 45 students, moved 5400 students in total. Use estimation to decide whether this figure is reasonable, and give the correct total.

Q15. How many squares of **all sizes** can be found on a 3×3 grid of squares?

Q16. A recipe uses 750 g of flour to serve 6 people. How much flour is needed to serve 10 people?

Q17. A family buys 5 tickets for $\$84$. Adult tickets cost $\$18$ and child tickets cost $\$12$. How many adult and how many child tickets did they buy?

Q18. A phone plan costs $\$30$ per month and includes 100 calls; each call over 100 costs an extra $\$0.10$. (a) What is the monthly cost for 130 calls? (b) How many calls in a month would make the cost reach $\$40$?

Answers

Q1. (a) $20, 24, 28, 32, 36$. (b) 56. (c) day 11. **Q2.** (a) subtract 7, then halve. (b) 13. **Q3.** (a) 15 and 21. (b) 36. (c) figure 10. **Q4.** (b) 6 handshakes. **Q5.** 6 and 9. **Q6.** (a) ≈ 1600 (40×40). (b) 1596. (c) 0.25%. **Q7.** $\$120$. **Q8.** (a) 22 . (b) 14 tables. **Q9.** 4 hours. **Q10.** 12 cm by 5 cm. **Q11.** 12 codes. **Q12.** Ben is 6, Ana is 18. **Q13.** side 12 m, perimeter 48 m. **Q14.** Not reasonable: $12 \times 45 \approx 500$, and exactly 540 — the claim is about ten times too big. **Q15.** 14 squares. **Q16.** 1250 g. **Q17.** 4 adult and 1 child. **Q18.** (a) $\$33$. (b) 200 calls.

Fully-worked solutions

Q1. (a) Adding 4 each day: $20, 24, 28, 32, 36$. (b) Day n gives $20 + 4(n-1) = 16 + 4n$ push-ups, so day 10 gives $16 + 40 = 56$. (c) Solve $16 + 4n \geq 60$: $4n \geq 44$, so $n \geq 11$. On day 11 he does $16 + 44 = 60$, so day 11 is the first day he reaches 60.

Q2. Undo the operations in reverse order. The last thing done was "add 7", so first **subtract** 7: $33 - 7 = 26$. Before that the number was "doubled", so **halve**: $26 \div 2 = 13$. The original number was 13 (check: $13 \times 2 + 7 = 33$ ✓).

Q3. These are the triangular numbers, figure $n = \frac{n(n+1)}{2}$. (a) figure $5 = \frac{5 \times 6}{2} = 15$; figure $6 = \frac{6 \times 7}{2} = 21$. (b) figure $8 = \frac{8 \times 9}{2} = 36$. (c) Solve $\frac{n(n+1)}{2} = 55$, i.e. $n(n+1) = 110 = 10 \times 11$, so $n = 10$.

Q4. Draw 4 dots and join every pair: the handshakes are AB, AC, AD, BC, BD, CD. Each of the 4 people shakes 3 others' hands, giving $4 \times 3 = 12$, but each handshake is counted twice, so there are $\frac{12}{2} = 6$ handshakes.

Q5. Try pairs adding to 15: $7 + 8$ gives 56 (too big); $6 + 9$ gives 54. ✓ So the numbers are 6 and 9.

Q6. (a) Rounding to 1 significant figure: $38 \approx 40$ and $42 \approx 40$, so the estimate is $40 \times 40 = 1600$. (b) Exact: $38 \times 42 = 1596$. (c) Percentage error $= \frac{|1600 - 1596|}{1596} \times 100\% = \frac{4}{1596} \times 100\% = 0.25\%$ — a very good estimate.

Q7. Work backwards. After spending $\$20$ he had $\$60$, so before that he had $\$60 + \$20 = \$80$. That $\$80$ is the two-thirds left after spending one-third, so $\frac{2}{3}$ of his start is $\$80$, giving a start of $\$80 \div \frac{2}{3} = \$120$. (Check: $\$120 \rightarrow$ spend $\$40 \rightarrow \$80 \rightarrow$ spend $\$20 \rightarrow \$60$ ✓.)

Q8. Each extra table adds 2 seats to the 4 of the first table, so n tables seat $2n + 2$. (a) 10 tables seat $2 \times 10 + 2 = 22$. (b) Solve $2n + 2 = 30$: $2n = 28$, so $n = 14$ tables.

Q9. The cost is $80 + 60h$ for h hours. Solve $80 + 60h = 320$: $60h = 240$, so $h = 4$ hours.

Q10. The length and width add to half the perimeter, 17 cm, and multiply to 60 cm². Look for two numbers with sum 17 and product 60: 12 and 5 ($12 + 5 = 17$, $12 \times 5 = 60$). So the rectangle is 12 cm by 5 cm.

Q11. There are 4 choices for the first digit and, with no repeat, 3 for the second, giving $4 \times 3 = 12$ codes.

Q12. Let Ben be b; Ana is $3b$. In 4 years: $(b + 4) + (3b + 4) = 32$, so $4b + 8 = 32$, $4b = 24$, $b = 6$. Ben is 6 and Ana is $3 \times 6 = 18$.

Q13. The side is the square root of the area: $\sqrt{144} = 12$ m. The perimeter is $4 \times 12 = 48$ m.

Q14. Estimate $12 \times 45 \approx 10 \times 50 = 500$, and the exact value is $12 \times 45 = 540$. The claimed 5400 is about ten times too large, so it is **not** reasonable — the correct total is 540 students (someone likely added a zero).

Q15. Count by size: 1×1 squares — 9; 2×2 squares — 4; 3×3 squares — 1. Total $9 + 4 + 1 = 14$ squares.

Q16. Use the unitary method: $750 \div 6 = 125$ g per person, so 10 people need $125 \times 10 = 1250$ g.

Q17. Try combinations of 5 tickets. If there are 4 adults and 1 child: $4 \times \$18 + 1 \times \$12 = \$72 + \$12 = \$84$ ✓, and $4 + 1 = 5$ tickets ✓. So 4 adult and 1 child tickets.

Q18. (a) 130 calls is 30 over the included 100, costing an extra $30 \times \$0.10 = \$3$, so the monthly cost is $\$30 + \$3 = \$33$. (b) The extra charge must be $\$40 - \$30 = \$10$, which buys $\$10 \div \$0.10 = 100$ extra calls, so the total is $100 + 100 = 200$ calls.

Chapter 1 Working mathematically and mathematical investigations — 1D Finding a context for an investigation

Theory

A **mathematical investigation** starts with a real situation and a question worth answering. In Foundation Mathematics you carry out investigations for your assessment, so learning to **find and define a good investigation** is a skill in its own right. This section is about the very first step: turning something in the world around you into a clear, workable mathematical question.

Where investigations come from.

Mathematics is embedded in almost every everyday, work and community situation — often without any numbers in sight at first. Good starting points are places where something **varies**, where there is something to **count, measure or compare**, or where **money, data, rates, space or chance** are involved. Useful sources of a context include:

- your own life — travel, phone use, part-time work, sport, hobbies, spending;
- your work or a workplace you know — pricing, rosters, materials, waste;
- your community — a local club, the council, a school, the environment;
- the media — a news story, an advertisement, a published data set.

Recognising the mathematics in a situation.

Look at a situation and ask: *what could I count, measure or compare here?* The answer usually points to an **area of study**:

- amounts of money, costs, income → **financial and consumer mathematics**;
- lengths, areas, volumes, plans, shapes → **space and measurement**;
- collected values, surveys, trends → **data analysis, probability and statistics**;
- rules, formulas, “how does X depend on Y” → **algebra, number and structure**.

Naming the mathematics early tells you what tools the investigation will need.

From a broad topic to an investigable question.

A **topic** such as “sport” or “the environment” is far too broad to investigate. You must **narrow** it, in stages, to a single specific question:

$$\text{broad topic} \rightarrow \text{a focus} \rightarrow \text{a specific question.}$$

For example: *the environment → household water use → “How much water could my household save by taking shorter showers, and what would that save in dollars per year?”*

What makes a question investigable.

A good investigation question is:

- **specific** — it asks one clear thing;
- **answerable with mathematics** — calculation, measurement or data will settle it;
- **manageable** — you can complete it in the time and with the resources you have;
- **data-accessible** — you can actually get the numbers you need;
- **worthwhile** — the answer is genuinely useful or interesting.

A question that fails any of these usually needs to be re-scoped before you start.

Scoping: deciding what is in and what is out.

Once you have a question, set its **scope** so it stays manageable:

- what you will **include** and **exclude** (e.g. one household, not the whole street);
- the **data** you need and **how** you will get it (measure, survey, or use published data);
- the **variables** — the quantities that change and how they might be related;
- the **mathematical tools** you expect to use (formulas, graphs, summary statistics, measurement).

Identifying variables.

A **variable** is a quantity that can take different values. Naming the key variables — and which ones you can control, measure or compare — shapes the whole investigation. In the shower example the variables are *shower length* (minutes), *water flow rate* (litres per minute), *water cost* (dollars per kilolitre) and *number of showers per week*.

A checklist for choosing an investigation.

Before committing, check your question is **specific, investigable with maths, manageable, data-accessible and worthwhile**, and that you can name the **variables** and the **mathematical tools** it will use. If you can, you have a context worth investigating.

Worked examples

Example 1 — scaffolded

A student is interested in the broad topic **"food waste at home"**. Turn this into a clear, investigable Foundation Mathematics question.

Working	Comment
Broad topic: *food waste at home*.	Far too broad to investigate directly.
Focus: the **cost** of food that gets thrown out each week.	Narrow to one measurable, money-related aspect.
Specific question: *"How much money does my household waste each week on food we throw out, and what would that add up to over a year?"*	One clear question, answerable with arithmetic.
Data needed: the value of food discarded each day for one or two weeks.	Data is accessible — you can record it yourself.
Variables: amount of food wasted ($ per day), number of days, weeks per year.	Names what will change and be calculated.
Maths tools: recording data, adding, averaging, scaling up to a year, percentages.	Confirms the areas of study (financial and consumer + data analysis).

Example 2 — scaffolded

Identify the mathematics embedded in running a **school canteen**, and list the variables and tools an investigation might use.

Working	Comment
Situation: a canteen buys stock and sells food and drinks.	Look for what can be counted, measured or compared.
Money appears: cost price, selling price, profit.	→ financial and consumer mathematics.
Data appears: which items sell best, sales per day.	→ data analysis, probability and statistics.
A rule appears: profit = income − expenditure.	→ algebra, number and structure.
Variables: items sold, price, cost, profit per item, day of week.	The quantities that vary.
Tools: tables, bar graphs, means, the profit formula, percentages.	The mathematics the investigation would use.

Example 3 — unscaffolded

A local **junior sports club** wants to know whether it should change its annual membership fee. Propose a clear mathematical investigation for this context.

The club's decision is really about money and members, so this is a financial-and-data investigation. A specific, investigable question is: *"How would the club's total membership income change if the annual fee rose from $120 to $140, assuming some members leave?"* The **data needed** are the current number of members and an estimate of how many would not renew at the higher fee (from a short survey). The **variables** are the fee (dollars), the number of members, and total income = fee × members. The **tools** are the income formula, a table comparing several fee levels, and a simple graph of income against fee. The scope is kept manageable by looking only at this one club and one season.

∴ a good investigation question is *"How would raising the annual fee from $120 to $140 change the club's total membership income?"*, using membership numbers and a short survey of members.

Example 4 — unscaffolded

A student proposes the investigation question *"Is public transport good?"*. Explain why this is not yet investigable, and rewrite it as a sound Foundation Mathematics question.

The question *"Is public transport good?"* is **not investigable** as written: "good" is not defined, no quantity is named, and no data would settle it — it is an opinion, not a mathematical question. To fix it, choose a specific, measurable aspect (for example cost or reliability) and make the question about numbers you can collect. A sound rewrite is: *"Over a month, is it cheaper for me to travel to work by train or by car, once fuel, parking and fares are included?"* This is specific, answerable by adding and comparing costs, manageable, and uses data you can gather.

∴ rewritten: *"Over a month, is it cheaper for me to commute by train or by car, including fuel, parking and fares?"* — a specific, data-based, comparable question.

Practice questions

Scaffolded

Q1. Consider the context **"the cost of running a car"**. (a) Identify two quantities you could measure or compare in this context. (b) Write one specific, investigable question about it. (c) State the data you would need to answer your question.

Q2. Start from the broad topic **"recycling in our suburb"**. (a) Narrow the topic to a single focus. (b) Write a specific investigable question about that focus. (c) Name the mathematical tools (from any area of study) your investigation would use.

Q3. A café owner notices some hours are much busier than others. (a) Name two variables in this situation. (b) Write an investigable question the owner could explore. (c) Explain how the owner could collect the data needed.

Q4. Three students suggest these investigation questions: (i) *"Is our school sporty?"* (ii) *"What is the most popular sport at our school, based on a survey of 60 students?"* (iii) *"Everything about sport."* (a) Which question is investigable? (b) For each of the other two, state one reason it is not yet investigable. (c) Suggest how question (i) could be rewritten.

Q5. Consider the context **"how students travel to school"**. (a) Identify the mathematics embedded in this context. (b) Explain what the *population* and a *sample* would be for an investigation of it. (c) Write one specific investigable question.

Q6. A student's question is *"How much would it cost to re-carpet the classrooms in our building?"*. (a) State one thing the investigation should **include** in its scope. (b) State one thing it could sensibly **exclude** to stay manageable. (c) Name a source of the data needed.

Unscaffolded

Q7. Turn the broad topic **“the weather”** into one clear, investigable Foundation Mathematics question, and name the data it would need.

Q8. Identify the mathematics embedded in **planning a road trip** between two country towns, listing at least three quantities that could be calculated.

Q9. Rewrite the vague question *“Is our town growing?”* as a specific, investigable mathematical question, and state what data would answer it.

Q10. For the context **“a market stall selling drinks on a hot day”**, propose an investigation: give the question, the variables, and the mathematical tools it would use.

Q11. Two questions are proposed: *“How do house prices across all of Australia compare over 20 years?”* and *“How have the median house prices in my suburb changed over the last 5 years?”*. State which is more **manageable** for a student investigation and give two reasons.

Q12. For an investigation of **“how much time students spend on their phones each day”**, identify the key variables and state how you would collect the data.

Q13. Propose a Foundation Mathematics investigation for the context **“energy use at home”**: give a specific question, the data needed, and the area(s) of study involved.

Q14. A student chooses the topic **“health”**. Narrow it to one specific investigable question and state what data you would collect and how.

Q15. For the context **“comparing two mobile phone plans”**, state a specific question, the variables involved, and the mathematical tools you would use.

Q16. A student asks *“Which is the fastest roller-coaster in the world?”* as their investigation. Explain why this is not a suitable **mathematical investigation** for them, and rewrite it into one that is.

Q17. Propose an investigation that combines **measurement and money** — for example, about tiling, painting or turfing an area. Give the question, the measurements needed, and the calculations involved.

Q18. A council publishes daily rainfall data for your town. Propose a **statistical** investigation question that uses this data, and state which summary statistics or graphs you would use.

Answers

(These are open-response tasks; each answer below is a model response — other well-scoped answers are acceptable if they meet the same requirements.)

Q1. (a) e.g. fuel cost per week; distance driven per week. (b) e.g. *"What does it cost me per week to run my car, including fuel, insurance and servicing?"* (c) weekly fuel spend, insurance and registration costs, servicing costs, distance travelled. **Q2.** (a) e.g. household recycling amounts. (b) e.g. *"What percentage of my household's weekly waste (by weight) is recyclable?"* (c) weighing/sorting data, percentages, a table or bar graph. **Q3.** (a) e.g. time of day; number of customers. (b) e.g. *"Which two hours of the day are busiest, based on customer counts over a week?"* (c) count customers each hour for a week and tabulate. **Q4.** (a) question (ii). (b) (i) "sporty" is undefined and not measurable; (iii) far too broad, no specific question or data. (c) e.g. *"What fraction of students play organised sport each week, based on a survey?"* **Q5.** (a) counting/categorising travel modes; percentages; comparison. (b) population = all students at the school; sample = the students actually surveyed. (c) e.g. *"What is the most common way students travel to school, based on a survey of 50 students?"* **Q6.** (a) e.g. the floor area of each classroom and the carpet price per m². (b) e.g. exclude corridors/offices; do one building only. (c) a flooring supplier's price per m², and measured or plan dimensions. **Q7.** e.g. *"How does the daily maximum temperature in my town vary over one month?"* — needs 30 days of maximum-temperature data. Any specific, data-based weather question is acceptable. **Q8.** distance (km), fuel used (L) and fuel cost ($), travel time at an average speed, number of stops/rest breaks; also possibly accommodation cost. **Q9.** e.g. *"By what percentage has my town's population changed over the last 10 years?"* — needs population figures for two or more years (e.g. from the ABS/Census). **Q10.** Question e.g. *"How does the number of drinks sold change with the day's maximum temperature?"*; variables: temperature, drinks sold, price, takings; tools: a table, scatter/line graph, and the takings = price × quantity relationship. **Q11.** The second (my suburb, 5 years) is more manageable: the data set is far smaller and easier to obtain, and the scope (one suburb, few years) fits a student investigation. **Q12.** variables: screen time per day (hours/minutes), day of week, possibly age/year level; collect by a survey or by students reading their phone's usage screen for a week. **Q13.** e.g. *"How much could my household cut its quarterly electricity bill by changing three habits?"*; data: past electricity bills and appliance usage; areas: financial and consumer + data analysis (and measurement of usage). **Q14.** e.g. *"How does the average daily number of steps compare between two year levels?"*; collect step counts from a sample of students over a week (fitness apps/step counters). **Q15.** Question e.g. *"Over 24 months, which of two phone plans is cheaper for my usage?"*; variables: monthly fee, included data, excess charges, months; tools: total-cost tables and a comparison graph. **Q16.** It relies on published records, not on data the student can collect or calculate — it is a look-up, not an investigation. Rewrite e.g. *"How does a roller-coaster's top speed relate to its maximum drop height, using data for 10 coasters?"* — now it compares two variables with data. **Q17.** e.g. *"What will it cost to turf my backyard?"*; measurements: the length and width (area) of the yard; calculations: area (m²), turf needed, and cost = area × price per m² (allowing for waste). **Q18.** e.g. *"How does monthly rainfall vary across the year in my town?"*; use the mean and range (or the five-number summary) per month, and a column graph or boxplots to compare months.

Fully-worked solutions

Q1. A car context is rich in money mathematics. (a) Two suitable quantities are the **weekly fuel cost** and the **weekly distance driven** (others: insurance, servicing). (b) A good specific question ties these together and is answerable by adding costs, e.g. *"What does it cost me per week to run my car, including fuel, insurance and servicing?"* (c) To answer it you would collect the weekly fuel spend, the annual insurance and registration converted to a weekly figure, and typical servicing costs — all quantities you can obtain. A strong answer names measurable, money-based quantities and a question that a calculation can settle.

Q2. (a) The broad topic narrows to a focus such as *how much household waste is recyclable*. (b) A specific question makes that measurable, e.g. *"What percentage of my household's weekly waste, by weight, is recyclable?"* (c) The tools are weighing and sorting the waste (data collection), then percentages and a table or bar graph to present the result. A good answer shows the topic → focus → question narrowing and names the maths.

Q3. (a) Two variables are the **time of day** and the **number of customers**. (b) An investigable question is *"Which two hours of the day are the busiest, based on customer counts over a week?"* (c) The owner collects the data by counting (or recording from the till) the number of customers in each hour, every day for a week, and tabulating it. The answer must name variables that vary and a countable way to gather data.

Q4. (a) Only **(ii)** is investigable — it is specific, has a defined measure (most popular sport) and states its data (a survey of 60 students). (b) Question (i) uses the undefined, unmeasurable word "sporty"; question (iii) is far too broad and states no question or data. (c) Rewriting (i) into a measurable form gives, e.g., *"What fraction of students play organised sport each week, based on a survey?"* This shows the test for investigability applied to real questions.

Q5. (a) The mathematics is **categorical data**: counting and comparing travel modes, then using fractions/percentages. (b) The **population** is every student at the school; a **sample** is the subset actually surveyed (used because surveying everyone is impractical). (c) A specific question is *"What is the most common way students travel to school, based on a survey of 50 students?"* The answer must connect the context to data handling and the population/sample idea.

Q6. The re-carpeting context is measurement plus money. (a) The scope should **include** the floor area of each classroom and the carpet's price per m². (b) It could **exclude** corridors, offices and stairs, or limit itself to one building, to stay manageable. (c) The data come from a flooring supplier's price list (price per m²) and from measuring the rooms or reading a scaled floor plan. A good answer sets a scope that is complete enough to answer the question yet small enough to finish.

Q7. "The weather" must be narrowed to one measurable variable over a set period. A sound question is *"How does the daily maximum temperature in my town vary over one month?"*, which needs the 30 daily maximum temperatures (recorded or from the Bureau of Meteorology). Any specific, data-based weather question — rainfall, temperature range, wind — that names its data is acceptable.

Q8. Planning a road trip embeds measurement, rates and money. At least three calculable quantities are the **distance** between the towns (km), the **fuel used** (litres = distance ÷ fuel economy) and its **cost** ($), and the **travel time** (distance ÷ average speed). Naming three such quantities, each computable, is the key.

Q9. "Is our town growing?" is vague and non-numerical. A specific version is *"By what percentage has my town's population changed over the last 10 years?"*, answered with population figures for the two years (from ABS/Census data) and a percentage-change calculation. The answer must convert a yes/no impression into a measured change.

Q10. A market-stall context suggests a relationship investigation. A good question is *"How does the number of cold drinks sold change with the day's maximum temperature?"* The **variables** are temperature, drinks sold, price and takings; the **tools** are a table of temperature vs sales, a scatter or line graph, and the relationship takings = price × quantity. The response should give a question, its variables and its tools.

Q11. The second question — median house prices in one suburb over 5 years — is far more **manageable**. Two reasons: the data set is small and readily available (a handful of yearly medians for one suburb), whereas comparing *all* of Australia over 20 years needs a huge, hard-to-obtain data set; and the narrow scope can be completed in the time available. Judging manageability by data size and scope is the point.

Q12. The key **variables** are the daily screen time (in hours or minutes) and, usefully, the day of week and the student's year level. Data can be collected by a survey in which students report their phone's own "screen time" figure for each day of a week, or by having them record it. A good answer names the quantity being measured and a realistic collection method.

Q13. Energy use at home is a financial-and-data context. A specific question is *"How much could my household cut its quarterly electricity bill by changing three habits?"* The **data** are past electricity bills and estimates of appliance usage; the **areas of study** are financial and consumer mathematics (costs) and data analysis, probability and statistics (comparing before/after), with some measurement of usage. The response must give a specific question, its data and its maths.

Q14. "Health" is far too broad. Narrowed, a good question is *"How does the average daily number of steps compare between Year 11 and Year 12 students?"* You would collect a week of step-count data from a sample of students in each year (from fitness apps or step counters) and compare the averages. The answer must show narrowing and name accessible data.

Q15. Comparing phone plans is a money investigation. A specific question is *"Over 24 months, which of two plans is cheaper for my level of use?"* The **variables** are each plan's monthly fee, included data, any excess charges, and the number of months; the **tools** are total-cost tables for each plan and a graph comparing the two. A complete answer states the question, variables and tools.

Q16. A "fastest roller-coaster" question only requires looking up a record, so the student collects no data and does no mathematics — it is not an investigation. Made investigable, it compares two variables with data, e.g. *"How does a roller-coaster's top speed relate to its maximum drop height, using data for 10 coasters?"* — which needs a data table and a scatter graph. The key idea is turning a look-up into a data comparison.

Q17. A measurement-and-money investigation could be *"What will it cost to lay turf on my backyard?"* The **measurements** needed are the length and width of the yard (to get its area in m^2, allowing for its shape); the **calculations** are the area, the amount of turf required (with an allowance for waste), and the cost = area × price per m^2. A good answer links a measured area to a money calculation.

Q18. With published daily rainfall data, a sound **statistical** question is *"How does monthly rainfall vary across the year in my town?"* You would summarise each month's rainfall using the **mean** and **range** (or a full five-number summary) and compare the months with a **column graph** or **parallel boxplots**. The answer must pose a data-driven question and name suitable summary statistics and displays.

Chapter 1 Working mathematically and mathematical investigations — 1E Key issues in mathematical problem-solving

Theory

When you use mathematics to solve a real problem — rather than a tidy textbook exercise — you meet issues that a clean exercise never raises: the information is incomplete, some quantities have to be guessed, and there is rarely a single "exact" answer. Handling these issues well is what separates a useful piece of mathematics from a meaningless one. This section covers the key issues to manage in **every** investigation.

Making and stating assumptions.

A real situation is usually too complicated to model exactly, so we **simplify** it by making **assumptions** — reasonable statements we take to be true so the maths becomes workable. For example, to estimate the cost of carpeting a room we might assume the room is a perfect rectangle and ignore the doorway. Good practice is to **state each assumption clearly**, because the answer is only as trustworthy as the assumptions behind it. Different reasonable assumptions give different (but defensible) answers.

Identifying variables and constraints.

A **variable** is a quantity that can change (the number of guests, the price per litre, the speed). A **constraint** is a limit or condition the solution must respect (a budget of $500, a room only 3 m wide, a maximum of 40 hours). Before calculating, list what can vary and what limits apply — this tells you what to work out and what to check your answer against.

Estimation and reasonableness.

Always form a rough **estimate** before (and after) a detailed calculation. An estimate is a quick, sensible value found by rounding and simple arithmetic; it is your best guard against a gross error. After calculating, ask *is this answer reasonable?* — check the **order of magnitude** (tens, hundreds, thousands?), the **units**, and whether it makes **physical sense**. An answer that a car uses $5 of petrol to drive from Melbourne to Sydney fails every one of these checks.

Working with messy, real data.

Real data is incomplete, rounded, or collected imperfectly. Round your **final** answer to a sensible accuracy and do not claim more precision than the data supports: reporting a paddock's area as $4\,381.762$ m² from tape measurements read to the nearest metre is false precision. Carry full precision through the working, then round once at the end.

"Good enough" — choosing an appropriate accuracy.

The accuracy a problem needs depends on its **purpose**. A tradesperson quoting a job needs a quick estimate to the nearest $100; the final invoice must be exact to the cent. Deciding "how accurate is accurate enough?" is itself part of solving the problem.

Recognising limitations.

Every model and method has **limitations** — assumptions that may not hold, data that may be biased, a trend that may not continue. A good solution ends by stating what could make the answer wrong and how confident we should be. Acknowledging limitations is a strength, not a weakness.

Worked examples

Example 1 — scaffolded

Estimate how many 20-litre buckets of water are needed to fill a backyard swimming pool that is about 8 m long and 4 m wide, with an average depth of 1.5 m.

Working	Comment
Assume the pool is a rectangular box of uniform average depth 1.5 m.	State the simplifying assumption; a real pool has sloping sides, so this is an estimate.
Volume $= 8 \times 4 \times 1.5 = 48$ m³.	The variables are length, width and average depth.
48 m³ $= 48 \times 1000 = 48\,000$ L.	Convert to litres (1 m³ $= 1000$ L).
Buckets $= 48\,000 \div 20 = 2400$.	Divide the volume by the bucket capacity.
Reasonableness: a domestic pool holding tens of thousands of litres is sensible.	Check the order of magnitude against everyday knowledge.

Example 2 — scaffolded

A community group is planning a sausage sizzle for about 150 people. Sausages come in packs of 12 at $6.50 a pack. Estimate how many packs to buy and the cost.

Working	Comment
Assume each person eats about 2 sausages.	A stated assumption; the organiser could revise it.
Sausages needed $\approx 150 \times 2 = 300$.	Variable: servings per person.
Packs $= 300 \div 12 = 25$ exactly, so buy 25 packs.	Here it divides evenly; otherwise round **up** so you don't run short.
Cost $= 25 \times \$6.50 = \$162.50$.	The budget is the constraint to check this against.
Reasonableness: about $1 per person for sausages is believable.	A quick per-person check confirms the total.

Example 3 — unscaffolded

A student claims that a car uses only 5 litres of petrol to drive the 900 km from Melbourne to Sydney. Explain why this is unreasonable, and give a better estimate.

A typical car uses roughly 8 litres of petrol per 100 km. Over 900 km that is

$$\frac{900}{100} \times 8 = 72\,\text{litres},$$

so a realistic figure is about 72 L — more than **fourteen times** the claim. The claim of 5 L would mean the car travels 180 km per litre, which is far beyond any real car, so it fails the reasonableness check.

∴ the claim is not reasonable; a sensible estimate is about 70–75 litres (roughly $135 of petrol at $1.90 per litre).

Example 4 — unscaffolded

A phone's battery drops from $100\,\%$ to $76\,\%$ after 2 hours of video. A student models the battery as falling by a constant $12\,\%$ per hour and concludes it will reach $0\,\%$ after about $8\,1/3$ hours. State two assumptions in this model and one limitation.

The model **assumes** (1) the battery drains at a **constant** rate, and (2) the phone keeps doing the **same** activity (video) the whole time. A key **limitation** is that real batteries do not drain linearly — the rate changes with the task, screen brightness, temperature and battery age — so the true time to $0\,\%$ will differ from the model's prediction.

∴ the linear model gives a rough guide only; its constant-rate assumption is its main weakness.

Practice questions

Scaffolded

Q1. You want to estimate how many jelly beans fill a 1-litre jar. (a) State one assumption you would make. (b) Identify the main variable your estimate depends on. (c) If you assume each jelly bean, together with the gaps around it, takes up about 2 cm^3, estimate the number in the 1000 cm^3 jar.

Q2. A family is planning a 600 km road trip. (a) List two variables that affect how long the drive takes. (b) State one constraint they might have. (c) Assuming an average speed of 80 km/h, estimate the driving time.

Q3. A bricklayer needs to estimate the bricks for a wall 12 m long and 2.5 m high. (a) State the assumption behind using "bricks per square metre". (b) Find the wall's area. (c) Assuming about 50 bricks per square metre, estimate the number of bricks.

Q4. At a checkout, a shopper's receipt for a loaf of bread, 2 litres of milk and a dozen eggs totals \$142. (a) Estimate a reasonable total for these three items. (b) State whether \$142 is reasonable. (c) Suggest what error might explain the receipt.

Q5. A tradesperson is asked for the cost of tiling a bathroom. (a) Explain why a quick estimate to the nearest \$100 is appropriate at the quoting stage. (b) Explain why the final invoice needs to be exact to the cent.

Q6. A council estimates a festival crowd by assuming 2 people stand on each square metre of a 1500 m^2 area. (a) Estimate the crowd size. (b) State one assumption in this method. (c) State one limitation of the estimate.

Unscaffolded

Q7. A room's four walls have a total area of about 72 m^2. One tin of paint covers 12 m^2. Estimate how many tins are needed, and state any assumption you make.

Q8. A household of 4 people uses about 150 litres of water per person per day. Estimate the household's water use for a full year, and comment on how accurate the estimate is.

Q9. A rectangular kitchen floor is 6 m by 4 m and is to be covered with square tiles of area 0.25 m^2 each. Estimate the number of tiles needed, and state one assumption.

Q10. Explain, using order of magnitude and units, why an answer of "the school hall holds 2 000 000 people" must be wrong, and suggest a realistic figure for a hall 30 m by 20 m.

Q11. A student estimates that a marathon runner (42 km) finishes in 3 minutes. Identify which reasonableness check this fails, and give a realistic finishing time.

Q12. To estimate the number of breaths a person takes in a day, state the assumptions you would make and outline (without a final exact figure required) how you would calculate it.

Q13. A driver estimates the petrol cost of a 500 km trip, assuming the car uses 7 litres per 100 km and petrol costs \$1.85 per litre. Estimate the cost, and state one thing that could make the real cost different.

Q14. A weather report predicts "about 30 mm of rain tomorrow". Explain why this is given as a rounded estimate rather than an exact figure such as 29.6183 mm.

Q15. A caterer prepares finger food for 200 guests, assuming 3 servings each, with servings supplied on trays of 20. Estimate the number of trays needed, and state how the answer changes if guests eat more than assumed.

Q16. A shop's sign says "everything reduced by up to 70 %". A customer expects every item to be 70 % off. Explain why this expectation is unreasonable, referring to the wording.

Q17. A recipe needs about 2.5 kg of apples, and there are roughly 8 apples in a kilogram. Estimate the number of apples to buy, and state whether you would round up or down and why.

Q18. A small business models its monthly profit as growing by a constant \$500 each month. State two assumptions in this model and one reason the real profit might not follow it.

Answers

Q1. (a) e.g. the beans are all a similar size / they settle the same way right through the jar / the jar is filled to the brim (accept any sensible assumption). (b) the size (volume) of one jelly bean. (c) about 500 jelly beans. **Q2.** (a) any two of: average speed, traffic, number/length of stops, road conditions. (b) e.g. a budget for fuel, a time they must arrive, needing rest breaks. (c) about $7\,1/2$ hours of driving. **Q3.** (a) that the wall is covered evenly with no large gaps/openings. (b) 30 m². (c) about 1500 bricks. **Q4.** (a) roughly \$13 (about \$4 bread + \$3 milk + \$6 eggs — accept \$8–\$15). (b) not reasonable — \$142 is about ten times too large. (c) a decimal-point or scanning error (e.g. \$1.42 read as \$142, or a wrong quantity). **Q5.** (a) the customer only needs a ballpark figure to decide, and exact measurements aren't known yet. (b) the invoice is the actual amount charged, so it must match the real cost exactly. **Q6.** (a) about 3000 people. (b) e.g. that people are spread evenly at 2 per m² over the whole area. (c) e.g. crowds are denser in some places and sparser in others, so the true number differs. **Q7.** about 6 tins (assuming even coverage and no second coat). **Q8.** about 219 000 litres per year; only a rough estimate — real use varies by season and habits, so it is accurate to maybe the nearest ten thousand litres. **Q9.** 96 tiles (assuming no tiles are wasted by cutting). **Q10.** 2 000 000 is the wrong order of magnitude (millions, not the hundreds or low thousands a hall holds) and ignores the space each person needs; a 600 m² hall holds at most about 1200 people at 2 per m². **Q11.** it fails the "physical sense" check — no one runs 42 km in 3 minutes; a realistic time is about 3 to 5 hours. **Q12.** assume about 15 breaths per minute, all day; then breaths $\approx 15 \times 60 \times 24$ (about 21 600) — a stated method with assumptions is what is required. **Q13.** about \$64.75; the real cost could differ with driving conditions, the true fuel-use rate, or a change in the petrol price. **Q14.** rainfall cannot be predicted to four decimal places; the data and the forecast are uncertain, so a rounded estimate honestly reflects that uncertainty. **Q15.** 30 trays; if guests eat more than 3 servings each, more trays are needed, so round up / prepare a few spare. **Q16.** "up to 70 %" means 70 % is the **maximum** discount, not the discount on every item; most items are reduced by less than 70 %, and only some reach the full 70 %. **Q17.** about 20 apples; round **up** (buy 20–21) so there is enough for the recipe. **Q18.** assumes profit rises by the **same** amount every month and that conditions (sales, costs) stay steady; real profit can be affected by seasons, competition or rising costs, so it may not grow by a constant \$500.

Fully-worked solutions

Q1. (a) A reasonable assumption is that the jelly beans are all roughly the same size and settle the same way throughout the jar, so one packing rate applies everywhere. (b) The estimate depends mainly on the volume of a single jelly bean (and on how tightly the beans pack). (c) Allowing about 2 cm³ per bean once the gaps around it are counted, a 1000 cm³ jar holds

$$\frac{1000}{2} = 500 \text{ jelly beans}$$

(an estimate, not an exact count).

Q2. (a) Two variables: the average speed and the time spent stopped (also traffic, road type). (b) A constraint could be a fuel budget, an arrival deadline, or needing a rest break every 2 hours. (c) At 80 km/h, driving time $= \frac{600}{80} = 7.5$ hours (excluding stops).

Q3. (a) Using "bricks per square metre" assumes the wall is covered evenly with no large openings (like a window). (b) Area $= 12 \times 2.5 = 30$ m². (c) Bricks $\approx 30 \times 50 = 1500$.

Q4. (a) Bread about \$4, milk about \$3, eggs about \$6, so a sensible total is about \$13. (b) \$142 is roughly ten times too large, so it is **not** reasonable. (c) A likely cause is a decimal-point error (a \$1.42 item scanned as \$142) or a wrong quantity entered.

Q5. (a) At the quoting stage the exact measurements and material prices are not yet fixed, and the customer only needs a ballpark figure to decide whether to proceed, so an estimate to the nearest \$100 is appropriate. (b) The invoice is the amount actually charged and paid, so it must equal the true cost exactly, to the cent.

Q6. (a) Crowd $\approx 1500 \times 2 = 3000$ people. (b) It assumes people are spread evenly at 2 per square metre across the whole area. (c) In reality density varies — crowds bunch near stages and thin out elsewhere — so the estimate could be well out.

Q7. Assuming the paint covers evenly and only one coat is needed, tins $= \dfrac{72}{12} = 6$. (If a second coat is needed, double it.)

Q8. Per day the household uses $4 \times 150 = 600$ litres. Over a year, $600 \times 365 = 219\,000$ litres. This is only an estimate: real use rises in summer and with gardening, so it is trustworthy to about the nearest ten thousand litres, not to the litre.

Q9. Floor area $= 6 \times 4 = 24$ m². With each tile covering 0.25 m², tiles $= \dfrac{24}{0.25} = 96$, assuming no tiles are lost to cutting at the edges (in practice buy a few extra).

Q10. "Two million" is the wrong order of magnitude — hall capacities are in the hundreds or low thousands — and the claim ignores that each person needs space. A $30 \times 20 = 600$ m² hall at about 2 people per m² holds at most about 1200 people.

Q11. This fails the **physical-sense** check: a 42 km marathon cannot be run in 3 minutes (that is 840 km/h). A realistic finishing time is about 3 to 5 hours.

Q12. Assume a person breathes about 15 times per minute and that this rate is roughly constant through the day. Then breaths per day $\approx 15 \times 60 \times 24 = 21\,600$. The method — state the per-minute rate, then scale up by minutes per day — is the point; the figure is only an estimate.

Q13. Fuel used $= \dfrac{500}{100} \times 7 = 35$ litres, costing $35 \times \$1.85 = \$64.75$. The real cost could differ if the car uses more/less fuel in the actual conditions, or if the petrol price changes.

Q14. Tomorrow's rainfall is genuinely uncertain and cannot be known to four decimal places, so quoting "29.6183 mm" would be false precision. "About 30 mm" honestly reflects the uncertainty in both the forecast and the measurement.

Q15. Servings needed $= 200 \times 3 = 600$; trays $= \dfrac{600}{20} = 30$. If guests eat more than 3 servings on average, 600 servings is too few, so the caterer should round up and prepare a few spare trays.

Q16. "Reduced by **up to** $70\,\%$" states $70\,\%$ as the largest discount in the sale, not the discount on every item. The wording promises that everything carries *some* reduction and that no reduction exceeds $70\,\%$; most items will be discounted by less than that, with only some at the full $70\,\%$. So expecting every item to be $70\,\%$ off misreads "up to".

Q17. With about 8 apples per kilogram, 2.5 kg gives $2.5 \times 8 = 20$ apples. Since running short would spoil the recipe, round **up** and buy 20–21 apples.

Q18. The model assumes profit grows by the **same** \$500 every month and that trading conditions stay steady. In reality, seasonal demand, new competitors, or rising costs can change the monthly increase, so the real profit may rise faster, slower, or even fall — the constant-\$500 assumption is the model's main weakness.

Theory

A mathematical investigation has three components — **formulation**, **exploration** and **communication**. This section is about the first of them. **Formulation** means setting out the context of a problem and then turning it into mathematics: taking a situation described in words and writing it in a **mathematical form** you can work with. You are *not* doing the calculations yet — that belongs to *exploration* — you are setting the problem up.

The four steps of formulating.

To formulate a model, work through these steps:

1. **Identify the variables** — the quantities that can change — and give each a letter and a **unit**. For example "let C be the total cost in dollars and t the time in minutes".
2. **State your assumptions** — the simplifying decisions you are making so the maths stays manageable (e.g. "assume the price stays constant", "assume the tank drains at a steady rate"). Good assumptions are reasonable and clearly stated.
3. **Find the relationship** between the variables — how the quantities depend on each other, often as "output = fixed part + rate × input".
4. **Choose a representation and state the goal** — write the relationship as a **formula, equation, table or graph**, and say clearly what you want to find.

Fixed part + variable part.

Many everyday situations have a **fixed** amount plus an amount that **varies** with some quantity. This gives a linear model

$$\text{output} = (\text{fixed amount}) + (\text{rate}) \times (\text{input}).$$

For instance, a tradesperson who charges a \$90 call-out fee plus \$70 per hour has a total cost $C = 90 + 70h$, where h is the number of hours. The \$90 is the fixed part; the \$70 per hour is the rate.

Choosing good variables and assumptions.

- Use a letter that reminds you of the quantity (C for cost, t for time, n for a number of items) and always attach a unit.
- List only the assumptions that matter. Stating "assume no discount is applied" makes your model honest about what it does and does not include.
- Check that your relationship makes sense: put in an easy value (like the input $= 0$) and see whether the output is sensible.

Formulating well makes the rest of an investigation straightforward: once the model is written down, *exploration* is carrying out the calculations, and *communication* is explaining what the results mean.

Worked examples

Example 1 — scaffolded

A plumber charges a fixed call-out fee of \$90 plus \$70 for each hour of work. Formulate a model for the total cost of a job.

Working	Comment
Let C be the total cost in dollars and h the number of hours worked.	Identify the variables and give each a unit.

Working	Comment
Assume: one call-out fee per job; time is charged at \$70 per hour for whole and part hours; the cost of any parts is not included.	State the assumptions clearly.
Total cost = call-out fee + hourly rate × hours.	Describe the relationship in words first.
$C = 90 + 70h$.	Write the model as a formula (fixed part \$90, rate \$70/h).
Goal: find C for a given number of hours, or find h that fits a given budget.	State what is to be found.

Example 2 — scaffolded

A 1000 L water tank is drained to clean it. Water leaves at a steady 25 L per minute. Formulate a model for the volume of water left in the tank.

Working	Comment
Let V be the volume of water in litres and t the time in minutes since draining began.	Variables with units.
Assume: the tank starts full at 1000 L; the flow rate stays constant at 25 L/min; no water is added.	Assumptions that keep the model linear.
Volume left = starting volume − rate × time.	The relationship.
$V = 1000 - 25t$.	The model. Check: at $t = 0$, $V = 1000$ L — correct.
Goal: find V at a given time, or the time when the tank is empty ($V = 0$).	State the goal.

Example 3 — unscaffolded

A stall sells homemade jam. There are fixed costs of \$120 for the stall and equipment. Each jar sells for \$8 and costs \$3 in ingredients to make. Formulate a model for the profit.

Let P be the profit in dollars and n the number of jars made and sold. Assume every jar made is sold, the selling price and ingredient cost per jar stay constant, and the only fixed cost is the \$120. The revenue is $8n$ and the total cost is $120 + 3n$, so

$$P = \text{revenue} - \text{cost} = 8n - (120 + 3n) = 5n - 120.$$

∴ the model is $P = 5n - 120$, and the goal is to find the profit for a given number of jars, or the number of jars needed to break even ($P = 0$).

Example 4 — unscaffolded

A car uses fuel at a rate of 8 litres per 100 km. Petrol costs \$1.90 per litre. Formulate a model for the fuel cost of a trip.

Let F be the fuel cost in dollars and d the distance in kilometres. Assume the fuel-use rate and the petrol price stay constant for the whole trip. The fuel used is $\frac{8}{100}d = 0.08d$ litres, so the cost is

$$F = 1.90 \times 0.08d = 0.152d.$$

∴ the model is $F = 0.152d$, and the goal is to find the fuel cost for a given trip distance.

Practice questions

Scaffolded

Q1. A taxi charges a flagfall of $4.20 plus $2.10 per kilometre. (a) Define suitable variables, with units, for the total fare and the distance travelled. (b) State one assumption you are making. (c) Write a formula for the fare and state what the goal of the investigation might be.

Q2. A casual worker is paid $25 per hour and also receives a fixed $40 travel allowance each shift. (a) Define variables for the total pay and the hours worked. (b) State an assumption about the pay. (c) Write a model for the total pay for a shift.

Q3. A rectangular paddock is to be fenced. Its length is twice its width. (a) Let W be the width in metres; write the length in terms of W. (b) State an assumption (e.g. about gates). (c) Write a model for the perimeter P in terms of W.

Q4. A hall is hired for a party for $150, and catering costs $12 per guest. (a) Define variables for the total cost and the number of guests. (b) State one assumption. (c) Write a formula for the total cost.

Q5. A student wants to save $800 for a laptop by putting away $50 each week. (a) Define variables for the amount saved and the number of weeks. (b) State an assumption. (c) Write a model for the amount saved, and state the goal.

Q6. A car travels a long trip at an average speed of 90 km/h. (a) Define variables for the travel time and the distance. (b) State an assumption about the speed. (c) Write a model for the travel time T in terms of the distance d.

Unscaffolded

For each of the following, **formulate a model**: define the variables (with units), state a reasonable assumption, write the relationship as a formula, and state what the goal would be.

Q7. A gym offers two options: pay $15 for each casual visit, or pay a $65 monthly membership for unlimited visits. Formulate a model for the cost of each option.

Q8. A part-time employee is paid $25 for every hour worked, with no other payments. Formulate a model for the employee's pay.

Q9. A swimming pool already holds 50 L of water. A hose adds water at a steady 30 L per minute. Formulate a model for the volume of water in the pool.

Q10. A school sausage sizzle has fixed costs of $180 (for the equipment and site). Each sausage sold makes a profit of $3.50. Formulate a model for the total profit.

Q11. A rainwater tank collects about 0.8 L of water for each square metre of roof area during a certain storm. Formulate a model for the volume of water collected in terms of the roof area.

Q12. A phone plan costs $30 per month plus $0.15 for each minute of calls. Formulate a model for the monthly cost.

Q13. A ladder of length 5 m leans against a wall, with its foot a distance d metres from the wall. Formulate a model (using Pythagoras' theorem) for the height h the ladder reaches up the wall.

Q14. One litre of paint covers about 12 square metres of wall. Formulate a model for the number of litres of paint needed to cover a wall of area A square metres.

Q15. A traveller converts Australian dollars to US dollars at an exchange rate of 0.66 US dollars per Australian dollar. Formulate a model for the number of US dollars received.

Q16. A household's rooftop solar panels save about $0.28 for every kilowatt-hour (kWh) of electricity they generate. Formulate a model for the yearly saving in terms of the energy generated.

Q17. A market stall can hire a site for $60 for the day, or pay $4 per hour instead. Formulate a model for the cost of each option, and write the condition (equation) for the two options to cost the same.

Q18. A cake recipe for 8 people uses 320 g of flour. Assuming the amount of flour is proportional to the number of people, formulate a model for the flour needed for p people.

Answers

Q1. (a) C = fare ($), k = distance (km). (b) e.g. the rate is the same for the whole trip / no booking fee. (c) $C=4.20+2.10k$; goal: find the fare for a distance, or the distance affordable for a set fare. **Q2.** (a) W = pay ($), h = hours. (b) e.g. one allowance per shift; whole and part hours paid at $25/h. (c) $W=25h+40$. **Q3.** (a) length $=2W$. (b) e.g. ignore gaps for gates. (c) $P=2(W+2W)=6W$. **Q4.** (a) C = cost ($), g = guests. (b) e.g. every guest is catered for at $12. (c) $C=150+12g$. **Q5.** (a) A = amount saved ($), w = weeks. (b) e.g. $50 saved every week without fail. (c) $A=50w$; goal: the number of weeks until $A=800$. **Q6.** (a) T = time (h), d = distance (km). (b) e.g. the average speed stays 90 km/h. (c) $T=\frac{d}{90}$. **Q7.** $C_{\text{casual}}=15n$ (where n = visits) and $C_{\text{member}}=65$; goal: which is cheaper for a given number of visits. **Q8.** $P=25h$, where P = pay ($) and h = hours. **Q9.** $V=50+30t$, where V = volume (L) and t = time (min). **Q10.** $P=3.5n-180$, where P = profit ($) and n = sausages sold. **Q11.** $V=0.8A$, where V = volume (L) and A = roof area (m²). **Q12.** $C=30+0.15m$, where C = cost ($) and m = minutes of calls. **Q13.** $h=\sqrt{5^2-d^2}=\sqrt{25-d^2}$, where h = height (m) and d = distance of the foot from the wall (m). **Q14.** $L=\frac{A}{12}$, where L = litres of paint and A = area (m²). **Q15.** $U=0.66a$, where U = US dollars and a = Australian dollars. **Q16.** $S=0.28e$, where S = saving ($) and e = energy generated (kWh). **Q17.** $C_{\text{flat}}=60$ and $C_{\text{hourly}}=4t$ (where t = hours); same cost when $60=4t$. **Q18.** $f=\frac{320}{8}p=40p$, where f = flour (g) and p = number of people.

Fully-worked solutions

Q1. (a) Let C be the fare in dollars and k the distance in kilometres. (b) Assume the per-kilometre rate is constant for the whole trip and there is no separate booking fee. (c) The fare is a fixed flagfall plus a rate per kilometre: $C=4.20+2.10k$. A goal could be to find the fare for a 12 km trip, or how far you can travel for $30.

Q2. (a) Let W be the total pay in dollars and h the hours worked. (b) Assume one $40 allowance per shift and that all hours are paid at $25/h. (c) Pay = hourly pay + allowance $=25h+40$.

Q3. (a) With width W metres, the length is $2W$ metres. (b) Assume the whole boundary is fenced with no gaps left for gates. (c) The perimeter is $P=2(\text{length}+\text{width})=2(2W+W)=6W$ metres.

Q4. (a) Let C be the total cost in dollars and g the number of guests. (b) Assume every guest is catered for at $12 and the hire fee is a single $150. (c) $C=150+12g$.

Q5. (a) Let A be the amount saved in dollars and w the number of weeks. (b) Assume exactly $50 is saved each week. (c) $A=50w$; the goal is the number of weeks w for which $A=800$.

Q6. (a) Let T be the travel time in hours and d the distance in kilometres. (b) Assume a constant average speed of 90 km/h (ignoring stops). (c) Since time = distance ÷ speed, $T=\frac{d}{90}$.

Q7. Let n be the number of visits in a month and C the cost in dollars. Assume the prices are fixed and the membership gives unlimited visits. Casual: $C_{\text{casual}}=15n$. Membership: $C_{\text{member}}=65$ (a fixed cost, independent of n). The goal is to decide which is cheaper for a given number of visits.

Q8. Let P be the pay in dollars and h the hours worked. Assume a constant $25/h and no other payments. Then $P=25h$.

Q9. Let V be the volume in litres and t the time in minutes. Assume the pool starts with 50 L and the hose runs steadily at 30 L/min. Then $V=50+30t$. Check: at $t=0$, $V=50$ L — correct.

Q10. Let P be the profit in dollars and n the number of sausages sold. Assume each sausage gives a constant $3.50 profit and the fixed cost is $180. Then $P=3.5n-180$. The goal might be the break-even number ($P=0$).

Q11. Let V be the volume collected in litres and A the roof area in square metres. Assume a steady collection of 0.8 L per square metre for this storm. Then $V=0.8\,A$.

Q12. Let C be the monthly cost in dollars and m the total minutes of calls. Assume the \$30 base and \$0.15 per minute are fixed. Then $C=30+0.15\,m$.

Q13. Let h be the height reached in metres and d the distance of the ladder's foot from the wall in metres. The wall is vertical, so the ladder, wall and ground form a right-angled triangle with the 5 m ladder as the hypotenuse. By Pythagoras' theorem $h^2+d^2=5^2$, so $h=\sqrt{25-d^2}$.

Q14. Let L be the number of litres and A the wall area in square metres. Assume a constant coverage of 12 m² per litre. Then $L=\frac{A}{12}$.

Q15. Let U be the number of US dollars and a the number of Australian dollars. Assume the rate stays at 0.66 and there are no conversion fees. Then $U=0.66\,a$.

Q16. Let S be the yearly saving in dollars and e the energy generated in kilowatt-hours. Assume a constant saving of \$0.28 per kWh. Then $S=0.28\,e$.

Q17. Let t be the number of hours at the market and C the cost in dollars. The flat option is a fixed $C_{\text{flat}}=60$; the hourly option is $C_{\text{hourly}}=4\,t$. The two options cost the same when $60=4\,t$ (which would be explored to find $t=15$ hours).

Q18. Let f be the flour needed in grams and p the number of people. Assuming the flour is proportional to the number of people, the rate is $\frac{320}{8}=40$ g per person, so $f=40\,p$. Check: at $p=8$, $f=320$ g — correct.

Chapter 1 Working mathematically and mathematical investigations — 1G Stage 2: Explore

Theory

A mathematical investigation has three stages: **Formulate** (set up the mathematics — Section 1F), **Explore** (do the mathematics — this section), and **Communicate** (report the results — Section 1H). Once you have *formulated* a model — chosen your variables, stated your assumptions and written a rule, equation or table — the **Explore** stage is where you actually **work it out**.

What "exploring" a model involves.

Exploring means carrying out the mathematics on the model you have set up. The main things you do are:

- **Calculate** — substitute values into your rule to get results.
- **Build a table of values** — a systematic table shows how one quantity changes as another does, and makes patterns visible.
- **Draw or read a graph** — a graph of the model shows the overall behaviour at a glance (where it is increasing, where it crosses zero, where two models meet).
- **Solve** the equation when you need a particular result (for example, "when does the balance reach $\$2000$?").
- **Use technology** — a scientific calculator or spreadsheet does the repeated arithmetic quickly and reliably, which lets you try many values.

Test cases and try different values.

A good explorer does not stop at one calculation. **Test particular cases** — try a small value, a large value, and a value in between — to see how the model behaves and to build confidence that it is working. Trying several values is often how you find a break-even point, a maximum, or the moment a target is reached.

Check for errors and reasonableness.

Every result should be **checked**. Ask: is the arithmetic right (does substituting the answer back in work)? and is the result **reasonable** in the real context (a price of $\$3$, not $\$3000$; a time of 20 minutes, not -5 minutes)? An unreasonable result is a signal to look for an error — or to **refine the model**.

Refine the model when the results do not fit.

Exploration often reveals that a first model is too simple or gives impossible answers. When that happens you **go back and refine** it — restrict the values it applies to, add a condition, or change an assumption — then explore again. Refining is a normal, expected part of investigating, not a failure.

Computational thinking and algorithms.

Working through a model step by step is an example of **computational thinking** — a way of organising a problem so that you, a calculator or a computer can solve it reliably. It has four parts:

- **Decomposition** — break a large problem into smaller, manageable parts.
- **Pattern recognition** — look for repetition, or a rule connecting the parts.
- **Abstraction** — focus only on the information that matters and ignore the rest.
- **Algorithm** — a clear, step-by-step procedure that solves the problem.

An **algorithm** is a precise list of steps. Written in **pseudocode** — using ← for "becomes" (assignment) — it can be **traced** by hand (work through the steps, keeping track of each value) or run on a calculator or spreadsheet. For example, to find the total cost of n items at $\$4$ each plus a $\$5$ delivery fee:

```
INPUT n
total ← 4 × n
```

```
total ← total + 5
OUTPUT total
```

Tracing this for $n=3$: total ← $4\times3=12$, then total ← $12+5=17$, so the algorithm outputs $\$17$.

Worked examples

Example 1 — scaffolded

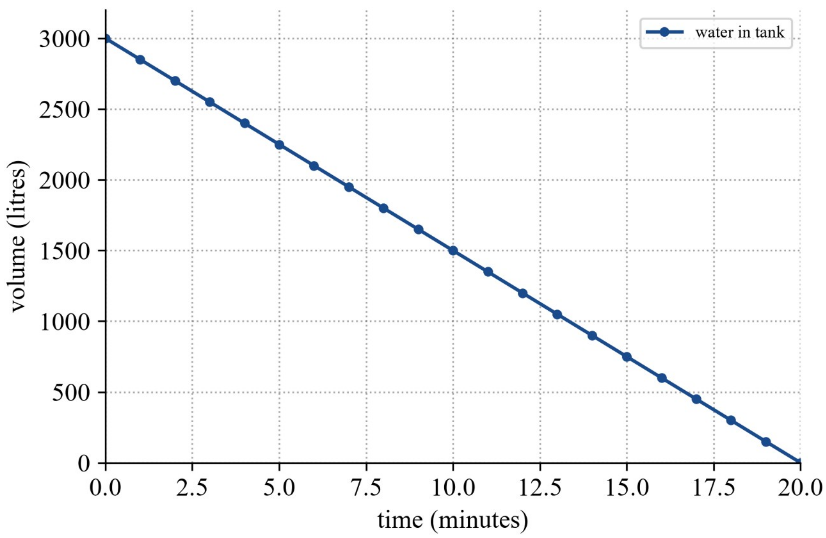
A food-truck owner has formulated a model for the weekly profit, $P=5n-200$ dollars, where n is the number of meals sold in a week. Explore the model: build a table of values, find the break-even point, and state the profit when 100 meals are sold.

Working	Comment
Substitute $n=0,20,40,60,80,100$ into $P=5n-200$.	Choose a spread of values to see the behaviour.
n: 0, 20, 40, 60, 80, 100 → P: $-200,-100,0,100,200,300$.	A table of values makes the pattern clear: profit rises by $\$100$ for every extra 20 meals.
Break-even is where $P=0$: $5n-200=0$, so $n=40$.	Break-even is the value that makes profit zero — read it from the table or solve the equation.
At $n=100$: $P=5(100)-200=\$300$.	Substitute the required value.
A profit of $\$300$ for 100 meals is reasonable for a food truck.	Always check the result makes sense in context.

Example 2 — scaffolded

A water tank drains steadily. Its volume is modelled by $V=3000-150t$ litres, where t is the time in minutes. Explore the model to find when the tank is empty, and draw its graph.

Working	Comment
Table for $t=0,5,10,15,20$: $V=3000,2250,1500,750,0$.	Systematic values reveal a steady fall of 750 L every 5 minutes.
The tank is empty when $V=0$: $3000-150t=0$, so $t=20$ minutes.	Solve for the time that makes the volume zero.
The graph is a straight line from $(0,3000)$ down to $(20,0)$.	A graph shows the whole story at a glance.
Check: at $t=10$, $V=3000-1500=1500$ L — half empty at half the time, as expected.	Test a middle case to confirm the model behaves sensibly.



Example 3 — unscaffolded

A student saving for a laptop formulates the model $A=500+45w$ dollars after w weeks, starting from $\$500$ and adding $\$45$ each week, with a goal of $\$2000$. Explore the model to find how many whole weeks are needed to reach the goal.

Solving $500+45w=2000$ gives $45w=1500$, so $w=\dfrac{1500}{45}=33.3\ldots$ weeks. Because saving happens in whole weeks, 33 weeks would give only $500+45\times33=\$1985$, which is short of the goal, so the value must be rounded **up**. Checking $w=34$: $A=500+45\times34=\$2030\geq\$2000$.

$\therefore$ the student needs 34 weeks to reach the $\$2000$ goal.

Example 4 — unscaffolded

Exploring the draining-tank model $V=3000-150t$ from Example 2, a student calculates the volume at $t=25$ minutes and gets $V=3000-150\times25=-750$ litres. Explain why this is unreasonable, and refine the model.

A tank cannot hold a **negative** volume of water, so -750 litres is impossible — the result does not fit the real situation. Exploring further shows the tank reaches $V=0$ at $t=20$ minutes (Example 2) and simply stays empty after that; the rule $V=3000-150t$ only describes the real tank while there is still water in it.

$\therefore$ the model must be **refined** to say it is valid only for $0\leq t\leq20$ minutes (with $V=0$ for $t>20$); outside that interval the straight-line rule no longer applies.

Practice questions

Scaffolded

Q1. A plumber's charge is modelled by $C=60+25h$ dollars for a job lasting h hours. (a) Build a table of values for $h=0,1,2,3,4$. (b) Find the charge for a 3-hour job. (c) Solve to find how many hours a job lasts if the charge is $\$210$.

Q2. A car travels at a steady speed, modelled by $D=80t$ kilometres in t hours. (a) Build a table of values for $t=0,1,2,3,4,5$. (b) Find the distance after 3.5 hours. (c) Solve to find the time taken to travel 600 km.

Q3. A savings plan is modelled by $A=200+30w$ dollars after w weeks. (a) Build a table of values for $w=0,5,10,15,20$. (b) Find the amount saved after 12 weeks. (c) Find how many whole weeks are needed to reach $\$1000$.

Q4. A phone plan costs $C=20+0.5g$ dollars, where g is the number of gigabytes of extra data used. (a) Build a table of values for $g=0,5,10,15,20$. (b) Find the cost when 8 GB of extra data is used. (c) Solve to find the extra data used if the bill is $\$35$.

Q5. A market stall's profit is modelled by $P=12x-500$ dollars for x items sold. (a) Find the profit when $x=30$. (b) The owner says "the stall always makes a loss". Explore the model further to show this is wrong. (c) Find the smallest whole number of items that gives a profit.

Q6. An investment is modelled by $A=1000\times1.1^t$ dollars after t years. (a) Build a table of values for $t=0,1,2,3$ (to the nearest dollar). (b) Find the value after 3 years. (c) By trying values, find the first whole year in which the investment exceeds $\$1500$.

Unscaffolded

Q7. A taxi fare is modelled by $C=45+1.2k$ dollars for a trip of k kilometres. Find the fare for a 15 km trip, and the distance of a trip costing $\$99$.

Q8. A tank drains according to $V=5000-200t$ litres after t minutes. Build a table of values for $t=0,5,10,15,20,25$ and find when the tank is empty.

Q9. A savings model is $A = 300 + 50w$ dollars after w weeks. Find how many whole weeks are needed to reach $\$1500$.

Q10. A business profit is modelled by $P = 6n - 240$ dollars for n sales. Find the break-even number of sales and the profit when $n = 100$.

Q11. A recipe uses 2.5 kg of flour to serve 40 people. A caterer models the flour needed as proportional to the number of people. Explore the model to find the flour needed to serve 100 people.

Q12. A candle's length while burning is modelled by $L = 50 - 8n$ centimetres after n hours. Calculate L at $n = 7$, explain why the result is unreasonable, and refine the model by stating the values of n for which it is valid.

Q13. Two messaging plans are modelled by Plan A: $\$30$ (flat) and Plan B: $C = 15 + 0.1t$ dollars for t texts. Explore to find the number of texts at which the two plans cost the same, and state which plan is cheaper for fewer texts.

Q14. An investment is modelled by $A = 2000 \times 1.05^t$ dollars after t years. Find the value after 10 years (to the nearest cent), and the first whole year in which it exceeds $\$3000$.

Q15. The rule $F = 1.8C + 32$ converts a temperature of C degrees Celsius to F degrees Fahrenheit. Build a table of values for $C = 0, 10, 20, 30, 40$.

Q16. A rectangular pen uses 40 m of fencing around its four sides, so its area is modelled by $A = x(20 - x)$ square metres, where x is the width. Build a table of values for $x = 2, 5, 8, 10, 12, 15$ and state the width that gives the largest area.

Q17. A school models the number of buses needed for 118 students as $118 \div 45$, where each bus seats 45. Calculate the value, then refine the model to give a sensible number of buses.

Q18. A stall's profit is modelled by $P = 8n - 320$ dollars for n items sold. Find the break-even number of items and the profit when $n = 60$, and state whether making 60 items is worthwhile.

Q19. A committee asks, "How much profit will our sausage sizzle make?" Use **decomposition** to list three smaller questions you would need to answer first in order to work out the profit.

Q20. A tiler lays a single border of tiles around square garden beds. A 1×1 bed needs 8 border tiles, a 2×2 bed needs 12, and a 3×3 bed needs 16. (a) Describe the **pattern** and write a rule for the number of border tiles around an $n \times n$ bed. (b) Use the rule to find the number of border tiles for a 10×10 bed.

Q21. The following **algorithm** works out a taxi fare, where k is the distance in kilometres:

```
INPUT k
fare ← 4
fare ← fare + 2 × k
OUTPUT fare
```

(a) **Trace** the algorithm for $k = 6$ to find the fare. (b) Write the algorithm as a formula for the fare C in terms of k.

Answers

Q1. (a) $60, 85, 110, 135, 160$. (b) $\$135$. (c) 6 hours. **Q2.** (a) $0, 80, 160, 240, 320, 400$. (b) 280 km. (c) 7.5 hours. **Q3.** (a) $200, 350, 500, 650, 800$. (b) $\$560$. (c) 27 weeks. **Q4.** (a) $20, 22.50, 25, 27.50, 30$. (b) $\$24$. (c) 30 GB. **Q5.** (a) $-\$140$ (a loss). (b) e.g. at $x=50$, $P=\$100$, a profit, so the stall does not *always* lose. (c) 42 items. **Q6.** (a) $1000, 1100, 1210, 1331$. (b) $\$1331$. (c) year 5 ($\$1610.51$). **Q7.** $\$63$; 45 km. **Q8.** $5000, 4000, 3000, 2000, 1000, 0$; empty at $t=25$ minutes. **Q9.** 24 weeks. **Q10.** break-even 40 sales; $\$360$. **Q11.** 6.25 kg. **Q12.** $L=-6$ cm, impossible; valid for $0 \le n \le 6$ (whole hours), i.e. the candle burns out during the 7th hour. **Q13.** equal at 150 texts; Plan B is cheaper for fewer than 150 texts. **Q14.** $\$3257.79$; year 9 ($\$3102.66$). **Q15.** $32, 50, 68, 86, 104$. **Q16.** $36, 75, 96, 100, 96, 75$; largest area at width $x=10$ m (100 m²). **Q17.** $118 \div 45 = 2.62\ldots$, rounded up to 3 buses. **Q18.** break-even 40 items; $\$160$; yes — $60 > 40$, so it makes a profit. **Q19.** e.g. (1) How many sausages will we sell? (2) What are the total costs (sausages, bread, onions, gas)? (3) What price do we charge each? Then profit = income − costs. **Q20.** (a) tiles $=4n+4$. (b) 44 tiles. **Q21.** (a) $\$16$. (b) $C=4+2k$.

Fully-worked solutions

Q1. (a) Substituting $h=0, 1, 2, 3, 4$ into $C=60+25h$ gives $60, 85, 110, 135, 160$. (b) $C=60+25\times 3=\$135$. (c) $60+25h=210 \Rightarrow 25h=150 \Rightarrow h=6$ hours.

Q2. (a) $D=80t$ gives $0, 80, 160, 240, 320, 400$. (b) $D=80\times 3.5=280$ km. (c) $80t=600 \Rightarrow t=7.5$ hours.

Q3. (a) $A=200+30w$ gives $200, 350, 500, 650, 800$. (b) $A=200+30\times 12=\$560$. (c) $200+30w=1000 \Rightarrow 30w=800 \Rightarrow w=26.6\ldots$; 26 weeks gives only $\$980$, so round up to 27 weeks.

Q4. (a) $C=20+0.5g$ gives $20, 22.50, 25, 27.50, 30$. (b) $C=20+0.5\times 8=\$24$. (c) $20+0.5g=35 \Rightarrow 0.5g=15 \Rightarrow g=30$ GB.

Q5. (a) $P=12\times 30-500=-\$140$, a loss. (b) Trying a larger value, at $x=50$: $P=12\times 50-500=\$100$, a profit — so the claim that the stall *always* loses money is wrong; it depends on how many items are sold. (c) $12x-500>0 \Rightarrow x>41.6\ldots$, so the smallest whole number is 42 items.

Q6. (a) $A=1000\times 1.1^t$ gives $1000, 1100, 1210, 1331$. (b) $A=1000\times 1.1^3=\$1331$. (c) Trying values: $t=4$ gives $\$1464.10$ (still under $\$1500$), $t=5$ gives $\$1610.51$, so year 5 is the first year it exceeds $\$1500$.

Q7. $C=45+1.2\times 15=\$63$. For $C=99$: $45+1.2k=99 \Rightarrow 1.2k=54 \Rightarrow k=45$ km.

Q8. $V=5000-200t$ gives $5000, 4000, 3000, 2000, 1000, 0$. The tank is empty when $V=0$: $5000-200t=0 \Rightarrow t=25$ minutes.

Q9. $300+50w=1500 \Rightarrow 50w=1200 \Rightarrow w=24$ weeks (exactly).

Q10. Break-even: $6n-240=0 \Rightarrow n=40$ sales. At $n=100$: $P=6\times 100-240=\$360$.

Q11. The flour per person is $\frac{2.5}{40}=0.0625$ kg. For 100 people: $0.0625\times 100=6.25$ kg (equivalently $\frac{2.5}{40}\times 100$).

Q12. $L=50-8\times 7=-6$ cm. A candle cannot have a negative length, so the result is unreasonable. The candle reaches $L=0$ when $50-8n=0$, i.e. $n=6.25$ hours, so the model is valid only for $0 \le n \le 6.25$; for whole hours it applies up to $n=6$, and the candle burns out during the 7th hour.

Q13. Setting the costs equal: $15+0.1t=30 \Rightarrow 0.1t=15 \Rightarrow t=150$ texts. For fewer than 150 texts $15+0.1t<30$, so Plan B is cheaper; beyond 150 texts Plan A (flat $\$30$) is cheaper.

Q14. $A=2000\times 1.05^{10}=\$3257.79$ (to the nearest cent). Trying values: $t=8$ gives $\$2954.91$ (under $\$3000$), $t=9$ gives $\$3102.66$, so year 9 is the first year it exceeds $\$3000$.

Q15. $F=1.8C+32$ gives, for $C=0, 10, 20, 30, 40$: $F=32, 50, 68, 86, 104$.

Q16. $A = x(20 - x)$ gives, for $x = 2, 5, 8, 10, 12, 15$: $36, 75, 96, 100, 96, 75$. The largest area in the table is 100 m² at $x = 10$ m; the values rise to $x = 10$ and then fall, so the width giving the largest area is 10 m.

Q17. $118 \div 45 = 2.62\ldots$ buses. Since part of a bus is not possible and 2 buses seat only 90 students, the model must be refined by rounding **up**: 3 buses are needed.

Q18. Break-even: $8n - 320 = 0 \Rightarrow n = 40$ items. At $n = 60$: $P = 8 \times 60 - 320 = \$160$. Because 60 is more than the break-even of 40, making 60 items returns a profit, so it is worthwhile.

Q19. Decomposition breaks the profit question into smaller parts that can each be answered. Three suitable sub-questions are: (1) *How many sausages will we sell?* (the quantity); (2) *What does it cost us to run the stall?* (buying sausages, bread, onions and gas — the total expenditure); and (3) *What price will we charge per sausage?* (which, with the quantity, gives the income). The profit is then income − expenditure.

Q20. (a) The number of border tiles increases by 4 each time the bed gets one unit wider ($8, 12, 16, \ldots$), so the rule is tiles $= 4n + 4$. (Checking: $n = 1$ gives $4(1) + 4 = 8$; $n = 2$ gives 12; $n = 3$ gives 16.) (b) For a 10×10 bed, tiles $= 4 \times 10 + 4 = 44$.

Q21. (a) Tracing the steps: fare $\leftarrow 4$, then fare $\leftarrow 4 + 2 \times 6 = 16$, so the algorithm outputs $\$16$. (b) The two assignment steps combine to give $C = 4 + 2k$.

Chapter 1 Working mathematically and mathematical investigations — 1H Stage 3: Communicate

Theory

An investigation is only finished when someone else can understand what you found and why it matters. The third stage of the modelling and problem-solving process is to **communicate**: to present your results clearly, interpret them in the real context, and draw a conclusion that answers the original question. This section is about turning correct mathematics into a clear, honest and useful message.

Answer the question that was asked.

The most important rule of communication is that your conclusion must **answer the original question**, in words, in the context of the problem — not just leave a number sitting on the page. If the question was "Which phone plan is cheaper over a year?", the answer is not "960"; it is "**Plan A is cheaper, by $180 (about 16%) over the year**". Always finish with a sentence a non-mathematician could act on.

Choose the right representation.

Match the way you present results to the message you want to send:

- a **table** is best for exact values and for letting the reader look up a figure;
- a **line graph** is best for showing a trend or change over time;
- a **bar or column chart** compares categories;
- a **pie chart** shows parts of a whole (proportions);
- a **diagram, map or plan** communicates spatial information.

A good representation makes the key point obvious at a glance. If a reader has to work hard to find the message, choose a different representation.

Interpret results in context.

Numbers must be translated back into the real situation, **with units**. "The mean is 35.7" means little; "the average commute took about 36 **minutes**" is a statement a reader understands. Interpretation also means commenting on what the result *means* for the people or decision involved — for example, "so the new route would save most staff about ten minutes each morning".

State assumptions and limitations.

Honest communication reports the **assumptions** you made and the **limitations** of your work: the size and fairness of any sample, values you estimated or rounded, and factors you left out. A conclusion such as "based on a survey of 30 students, most preferred …" tells the reader how much to trust it. Never present an estimate as though it were exact.

Know your audience.

Write for the person who will read it. A council report, a business owner and a classmate need different amounts of technical detail, but all need a clear conclusion, correct units, and no unexplained jargon. Round to a sensible accuracy for the context — cents for money, whole minutes for travel time.

Structure of a short investigation report.

A clear report usually has: the **question**; a brief **method** (what data, how collected); the **results** (a well-chosen table or graph); the **interpretation and conclusion** (answering the question, with units); and the **assumptions and limitations**. Keeping this structure makes your reasoning — formulation, solution, interpretation — easy to follow.

Worked examples

Example 1 — scaffolded

An investigation compared two phone plans over a year: Plan A costs $960 for the year and Plan B costs $1140. Write a clear conclusion that communicates the result to a customer.

Working	Comment
Identify the message: Plan A costs less than Plan B.	Decide the single main point before writing.
Difference $= 1140 - 960 = \$180$.	Quantify the difference so the conclusion is specific.
As a percentage of Plan B: $\frac{180}{1140} \times 100\% \approx 16\%$.	A percentage helps a reader judge how big the saving is.
Conclusion: "Over a year, **Plan A is cheaper than Plan B by $180, about 16%**, so Plan A is the better value for a customer with this usage."	Answers the question, in context, with units, and notes it depends on the usage assumed.

Example 2 — scaffolded

A study recorded a city's average monthly rainfall for each of the twelve months. The investigator wants to show how rainfall **changes through the year**. Recommend the best representation and justify the choice.

Working	Comment
The data is a value (rainfall) measured over time (months).	Identify the data type and what the reader should see.
The message is a **trend** across the year, not a comparison of categories or a share of a whole.	The message decides the representation.
A **line graph** (rainfall on the vertical axis, months across the bottom) shows the rise and fall clearly.	Line graphs are made for change over time.
A pie chart would be wrong (rainfall months are not parts of one whole) and a plain table hides the shape of the trend.	Justify by ruling out poorer choices.

Example 3 — unscaffolded

A canteen surveyed 400 students and found that 260 would buy a new vegetarian meal. Write a one-sentence conclusion that interprets this result in context, and add a sentence stating one limitation.

The proportion is $\frac{260}{400} \times 100\% = 65\%$. In context: "**About 65% of the 400 students surveyed said they would buy the vegetarian meal, so there is likely enough demand to add it to the menu.**" A limitation: the result depends on the survey being a fair sample of the whole school — if only students near the canteen were asked, or if far fewer actually buy it than said they would, the true demand could be lower.

∴ the result supports adding the meal, but the conclusion should be treated as an estimate based on a single survey.

Example 4 — unscaffolded

A student reports the result of a traffic study as: "$28, 31, 33, 35, 38, 41, 44$." Explain why this is poor communication and rewrite it as a clear conclusion.

A bare list of numbers is not a conclusion: it does not say what the numbers are, give units, summarise them, or answer any question. A reader cannot tell what was found. The data are seven daily commute times in minutes; their mean is $\frac{28+31+33+35+38+41+44}{7} = 35.7$ minutes. A clear version is: "**Over the week, the commute took on average about 36 minutes, ranging from 28 to 44 minutes**, so a traveller should allow around 45 minutes to be safe."

∴ good communication summarises the data, adds units and context, and states a usable conclusion.

Practice questions

Scaffolded

Q1. An investigation found that a household could save \$312 a year by switching energy plans. (a) Why is "the answer is 312" not an adequate conclusion? (b) Rewrite it as a clear conclusion, in context and with units. (c) State one assumption the saving depends on.

Q2. A study recorded these daily commute times (minutes): $28, 31, 33, 35, 38, 41, 44$. (a) Find the mean commute time, correct to one decimal place. (b) Write a one-sentence conclusion interpreting the mean in context (with units). (c) State one limitation of concluding from just one week of data.

Q3. For each message, name the best representation (table, line graph, bar chart or pie chart) and give a reason: (a) how a town's population changed each year from 2015 to 2024; (b) the share of a household budget spent on rent, food, transport and other; (c) a comparison of average rainfall in four different cities.

Q4. A report concludes: "Most people preferred Option B." (a) Explain why this conclusion is too vague. (b) The survey found 180 of 300 people preferred Option B — rewrite the conclusion with a supporting figure. (c) What percentage preferred Option B?

Q5. A council report will be read by residents who are not mathematicians. (a) Give two things the report should include so residents can trust and use the conclusion. (b) A draft says "the mean is $\mu = 47.3$". Rewrite this sentence for the audience, assuming it refers to average weekly waste in kilograms.

Q6. An investigation into a school fundraiser reports income \$4200 and costs \$3650. (a) Find the profit. (b) Write a conclusion that answers "Was the fundraiser worthwhile financially?" in context. (c) State one non-financial factor the conclusion should acknowledge as a limitation.

Unscaffolded

Q7. A council found the recycling rate rose from 62% to 74% after a new program. Write a clear conclusion that communicates the change, quoting both the percentage-point rise and the result in context.

Q8. A survey of 50 shoppers is used to make a claim about all 8000 customers of a store. Write a sentence stating the main limitation that any conclusion must acknowledge.

Q9. You must present the monthly sales of one product across a year so a manager can see the **trend** at a glance. State which representation you would use and why, and name one representation you would avoid.

Q10. A water-saving trial reduced average daily household use from 185 L to 148 L. Write a conclusion that reports the reduction as both an amount and a percentage, in context.

Q11. A student's report ends with a graph but no words. Explain what is missing and write a suitable concluding sentence for an investigation that compared two brands of battery and found Brand X lasted longer on average.

Q12. Rewrite this conclusion so it is honest about uncertainty: "Everyone will save \$500 a year by installing solar panels."

Q13. An investigation used the assumption that "petrol stays at \$2.00 per litre for the whole year". Explain why stating this assumption is important when communicating the result, and what should be added if petrol prices are volatile.

Q14. A market-stall investigation reports income \$4200 and costs \$3650 for the day. Write a two-sentence conclusion: one stating the financial result (with the profit), and one interpreting whether the stall was worthwhile and under what assumption.

Q15. Choose the better of these two conclusions for a report on average class test scores, and explain why: (i) "$\bar{x} = 63.2$"; (ii) "The class averaged about 63 out of 100, a solid result, though two very low scores pulled the average down."

Q16. A report presents twelve months of temperature data as a long paragraph of numbers. Suggest a better way to communicate it and explain what the reader will be able to see that they could not before.

Q17. An investigation concludes that a new bus route “saves time”. Rewrite this as a precise, quantified conclusion, inventing a reasonable saving, and state the context and one limitation.

Q18. Explain, in two or three sentences, why an investigation report should always include its assumptions and limitations, using an example of a conclusion that would mislead a reader if these were left out.

Answers

Q1. (a) It is a bare number with no units or context and does not answer the question. (b) e.g. "Switching plans would save the household about **\$312 a year** on electricity." (c) The saving assumes the household's usage stays roughly the same. **Q2.** (a) 35.7 minutes. (b) e.g. "The commute averaged about 36 **minutes** over the week." (c) One week may not be typical (weather, roadworks, school holidays could change it). **Q3.** (a) line graph (shows change over time); (b) pie chart (parts of a whole); (c) bar chart (compares categories). **Q4.** (a) "Most" gives no figure and no margin. (b) e.g. "180 of the 300 people surveyed ($60\,\%$) preferred Option B." (c) $60\,\%$. **Q5.** (a) e.g. a clear plain-language conclusion, correct units, and the assumptions/limitations. (b) e.g. "On average each household put out about 47 **kg** of waste per week." **Q6.** (a) profit $= 4200 - 3650 = \$550$. (b) e.g. "The fundraiser made a profit of **\$550**, so it was worthwhile financially." (c) e.g. volunteer time/effort, or whether it can be repeated. **Q7.** e.g. "The recycling rate rose by 12 **percentage points, from** $62\,\%$ **to** $74\,\%$, so about three-quarters of waste is now recycled — a clear improvement after the program." **Q8.** e.g. The sample of 50 is very small relative to 8000 customers and may not represent them, so any conclusion is only an estimate. **Q9.** A **line graph** (sales over the twelve months) shows the trend; avoid a pie chart (months are not parts of one whole). **Q10.** Reduction $= 185 - 148 = 37$ L, a $20\,\%$ decrease: e.g. "Average daily use fell by 37 **L (about** $20\,\%$**)**, from 185 L to 148 L per household." **Q11.** A conclusion in words is missing: e.g. "On average Brand X lasted longer than Brand Y, so Brand X is the better choice for this use." **Q12.** e.g. "Households **could save up to about \$500 a year**, depending on their electricity use and the amount of sunlight — some will save less." **Q13.** Stating it lets the reader know the result only holds while petrol is near \$2.00; if prices are volatile, give a range or note the result will change if prices rise/fall. **Q14.** e.g. "The stall made a profit of **\$550** on the day (income \$4200 − costs \$3650). This makes it worthwhile, assuming a typical day's sales and no unpaid costs such as the seller's own time." **Q15.** (ii) is better — it uses words, units and context, and honestly notes the effect of the low scores; (i) is a bare symbol and number. **Q16.** Use a **line graph** (or a neat table); the reader can then see the yearly pattern — the hottest and coldest months and the overall shape — which a paragraph of numbers hides. **Q17.** e.g. "The new route saved on average about 8 **minutes** per trip for commuters between the suburb and the city, though the saving may be smaller in peak-hour traffic." **Q18.** Model answer in solutions.

Fully-worked solutions

Q1. (a) A conclusion must answer the question in context; "312" alone has no units and does not say what it refers to or what to do. (b) "Switching to the cheaper plan would save the household about **\$312 a year** on its electricity bill." (c) The figure assumes the household's electricity usage stays about the same as in the period studied.

Q2. (a) $\bar{x} = \dfrac{28+31+33+35+38+41+44}{7} = \dfrac{250}{7} \approx 35.7$ minutes. (b) "The commute took on average about 36 **minutes** over the week." (c) A single week may not be representative — holidays, weather or roadworks could make a typical week different.

Q3. (a) a **line graph**, because population is a value changing over time and a line shows the trend; (b) a **pie chart**, because the budget categories are parts of one whole (the total budget); (c) a **bar chart**, because four cities are separate categories being compared.

Q4. (a) "Most" gives no number, so the reader cannot judge how strong the preference is or how many were asked. (b) "180 of the 300 people surveyed ($60\,\%$) preferred Option B." (c) $\dfrac{180}{300} \times 100\,\% = 60\,\%$.

Q5. (a) A clear plain-language conclusion with correct units, and a statement of the assumptions/limitations so residents know how much to trust it (accept any two of: conclusion, units, method, assumptions, limitations). (b) "On average, each household put out about 47 **kg** of waste per week."

Q6. (a) profit $= 4200 - 3650 = \$550$. (b) "The fundraiser made a profit of **\$550**, so financially it was worthwhile." (c) It should note a non-financial factor such as the volunteer time required, or whether the result can be repeated.

Q7. The rise is $74 - 62 = 12$ percentage points. "The recycling rate rose by 12 **percentage points, from** $62\,\%$ **to** $74\,\%$, after the new program — about three-quarters of waste is now recycled, a clear improvement." (Stating "percentage points" avoids confusing it with a $19\,\%$ relative increase.)

Q8. "Because only 50 of the 8000 customers were surveyed — under $1\,\%$ — the sample may not represent all customers, so the conclusion is an estimate that could be wrong if the sample was not chosen fairly."

Q9. A **line graph** with the months across the bottom and sales up the side: it shows the trend through the year at a glance. Avoid a **pie chart**, because the twelve months are not parts of a single whole and a pie would hide the trend.

Q10. Reduction $= 185 - 148 = 37$ L; as a percentage $\frac{37}{185} \times 100\,\% = 20\,\%$. "The trial cut average daily household water use by 37 **litres (a** $20\,\%$ **reduction)**, from 185 L to 148 L."

Q11. The report has no written conclusion — a graph alone does not state the finding. A suitable sentence: "On average, Brand X batteries lasted longer than Brand Y, so Brand X is the better choice for this device."

Q12. The word "everyone" and the exact "$500" overstate certainty. Honest version: "Households **could save up to about $500 a year**, though the actual saving depends on their electricity use and how much sun their roof gets, so some will save less."

Q13. The result was calculated assuming petrol stays at $2.00/L, so it is only valid while prices are near that. Stating the assumption warns the reader; if prices are volatile, the report should give a range of results for different prices, or note the conclusion will change if the price rises or falls.

Q14. "The stall made a profit of **$550** on the day (income $4200 minus costs $3650). This makes the stall worthwhile, assuming the day's takings are typical and no costs — such as the seller's own time — have been left out."

Q15. Conclusion (ii) is better. It reports the average with units and context ("about 63 out of 100"), judges it ("a solid result"), and is honest about a limitation (two very low scores lowered the mean). Conclusion (i) is only a symbol and a number and communicates nothing to a general reader.

Q16. Present the data as a **line graph** (months across the bottom, temperature up the side) or a tidy table. The reader can then see the yearly pattern — which months are hottest and coldest and the overall shape of the change — which is impossible to see in a long list of numbers.

Q17. "Saves time" is vague. A precise version: "The new bus route saved on average about 8 **minutes** per trip for commuters travelling from the suburb to the city centre. This saving may be smaller during peak hour when traffic is heavier." (Any reasonable quantified saving with context and a limitation is acceptable.)

Q18. A report should state its assumptions and limitations because a conclusion can be correct arithmetically yet misleading without them. For example, "solar panels save $500 a year" would mislead a reader whose house is shaded or who uses little electricity; adding "assuming average sunlight and usage" lets the reader judge whether the conclusion applies to them.

Chapter 1 Working mathematically and mathematical investigations — 1I A worked sample investigation

Theory

This section brings together the whole investigation cycle — **Formulate**, **Explore** and **Communicate** (sections 1F–1H) — by working through one complete investigation from start to finish. Read it as a model for the structure and reasoning your own investigations should show.

The problem.

> A household is thinking about installing a rainwater tank to water the garden. Roughly **how much water can the roof collect in a year**, is it enough for the garden, and **what size tank** should they choose?

Stage 1 — Formulate

We turn the everyday question into a mathematical one by choosing the quantities, stating assumptions, and finding a relationship.

Variables (with units).

- A = the roof catchment area, in square metres (m^2)
- R = the average yearly rainfall, in millimetres (mm)
- e = the collection efficiency (a decimal between 0 and 1; some water is lost to splashing, evaporation and the first-flush diverter)
- V = the volume of water collected in a year, in litres (L)

Key fact. 1 mm of rain falling on 1 m^2 of roof gives 1 L of water. So the yearly collected volume is

$$V = A \times R \times e.$$

Assumptions. The roof area is $A = 120$ m^2; the local average rainfall is $R = 650$ mm per year; the efficiency is $e = 0.85$; the garden uses about 5000 L per month all year. We assume an average year and steady garden use.

What we want to find. (i) the yearly collected volume, (ii) whether it meets the yearly garden use, and (iii) a sensible tank size.

Stage 2 — Explore

Now we do the mathematics.

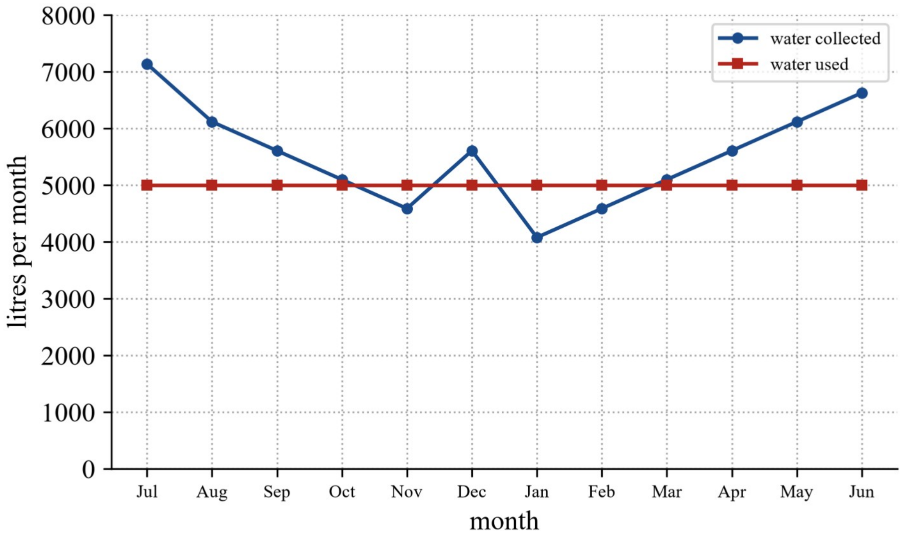
Yearly total.

$$V = A \times R \times e = 120 \times 650 \times 0.85 = 66\,300\,\mathrm{L}.$$

The garden uses $5000 \times 12 = 60\,000$ L per year, so **on average the roof collects more than the garden needs**, with a surplus of $66\,300 - 60\,000 = 6300$ L.

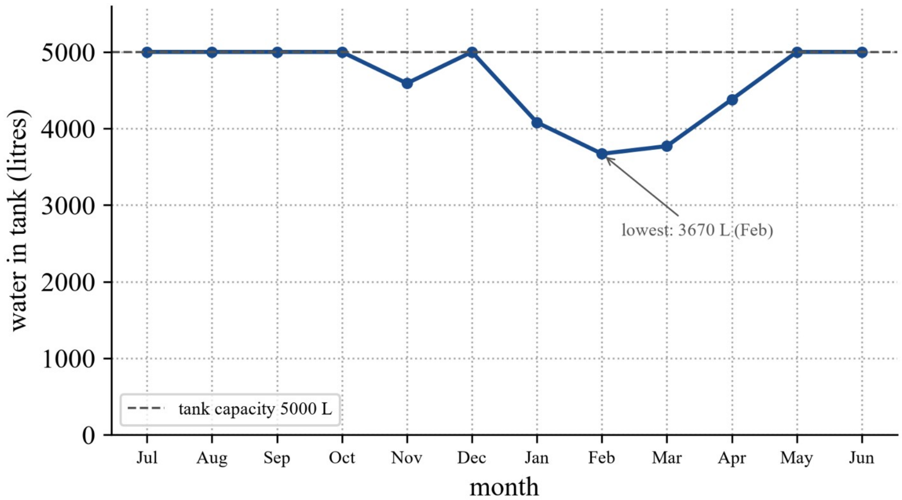
But rainfall is not the same every month. Using the monthly rainfall for the area, the water collected each month is $(\text{monthly mm}) \times A \times e = (\text{monthly mm}) \times 102$ L. The table shows the result against the steady 5000 L monthly use.

Month	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun
Rain (mm)	70	60	55	50	45	55	40	45	50	55	60	65
Collected (L)	7140	6120	5610	5100	4590	5610	4080	4590	5100	5610	6120	6630
Used (L)	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000



The wet winter months collect **more** than is used; the driest months (November, January and February) collect **less**. A tank is needed to store the winter surplus and carry it into summer.

Testing a tank size. Start a 5000 L tank full in July and, each month, add what is collected and subtract what is used (the tank cannot go above 5000 L or below 0). The level each month is:



The tank stays full through winter and is drawn down over summer to a **lowest level of** 3670 **L in February**, then refills. It **never runs dry**, so a 5000 L tank is enough to bridge the dry months with a comfortable buffer.

Stage 3 — Communicate

We interpret the results back in the real situation and state a clear conclusion.

> **Conclusion.** The 120 m² roof collects about 66 300 **L per year**, more than the garden's 60 000 L. Because rainfall is seasonal, a tank is needed to store winter water for summer. A 5000 **L tank** never falls below 3670 L in an average year, so it meets the garden's needs with a safety margin. A larger tank (say 10 000 L) would add security in a dry year but costs more and takes more space.

Assumptions and limitations. The model uses *average* rainfall — a drought year would collect less and could empty a small tank. It assumes steady garden use, a fixed efficiency, and ignores overflow lost when the tank is already full in winter. A next step would be to test the model against a dry year's rainfall.

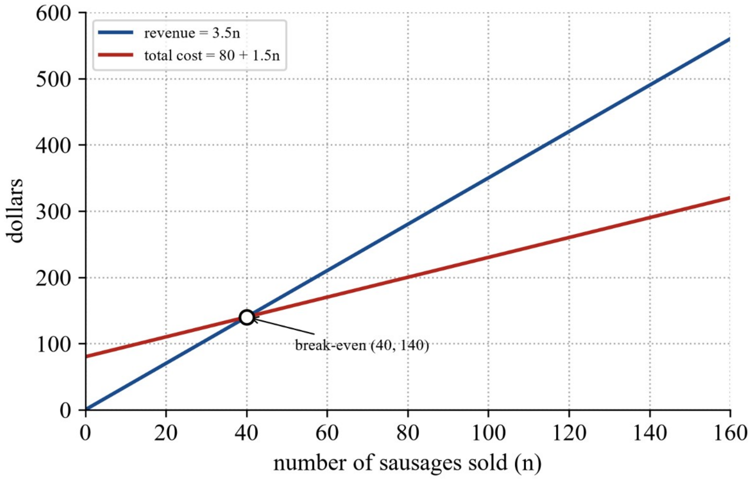
The cycle in one line. *Formulate* the model $V = A\,R\,e$ and its assumptions → *Explore* with calculations, a table and graphs → *Communicate* a conclusion, in context, with its limitations.

Worked examples

Example 1 — scaffolded

A school committee plans a **sausage sizzle** to raise money. A gas bottle costs \$30 and the stall site costs \$50. Each sausage in bread costs \$1.50 to make and sells for \$3.50. Investigate whether the stall can raise \$500.

Working	Comment
Formulate. Let n = number of sausages sold. Fixed cost $=30+50=\$80$; each sale adds \$1.50 cost and \$3.50 revenue.	Identify the variable and the cost/revenue model.
Revenue $R=3.5n$; total cost $C=80+1.5n$; profit $P=R-C=2n-80$.	Write the relationships (Formulate).
Explore. Break-even ($P=0$): $2n-80=0$, so $n=40$ sausages.	Solve for where revenue meets cost.
Profit if 150 are sold: $P=2(150)-80=\$220$.	Test the expected sales.
To raise \$500: $2n-80=500$, so $n=290$ sausages.	Solve for the target.
Communicate. The stall breaks even at 40 sausages and makes \$220 if 150 sell, but must sell 290 to raise \$500 — likely too many for one event.	Interpret in context and conclude.



Example 2 — scaffolded

A family plans a 720 **km road trip**. Their car uses 7.5 L of petrol per 100 km and petrol costs \$1.85 per litre. Investigate the fuel cost and the driving time at an average 100 km/h.

Working	Comment
Formulate. Fuel used = distance × rate per 100 km; cost = fuel × price; time = distance ÷ speed.	Set up the relationships.
Explore. Fuel $=\dfrac{720}{100}\times 7.5=54$ L.	Carry out the calculation.
Cost $=54\times 1.85=\$99.90$.	Fuel litres times price per litre.
Time $=\dfrac{720}{100}=7.2$ h $=7$ h 12 min.	Distance over average speed.
Communicate. The trip needs about 54 L of petrol costing **\$99.90**, and about 7 **h** 12 **min** of driving (plus rest stops).	State the results in context.

Example 3 — unscaffolded

A student uses about 12 GB of mobile data a month. **Plan A** is $45/month with unlimited data. **Plan B** is $30/month and includes 5 GB, with each extra 5 GB block costing $10. Investigate which plan is cheaper over a year.

Formulating the cost of each plan: Plan A costs $45 \times 12 = \$540$ per year regardless of use. For Plan B at 12 GB, the student needs the base 5 GB plus **two** extra 5 GB blocks (reaching 15 GB), so a month costs $30 + 2 \times 10 = \$50$, giving $50 \times 12 = \$600$ per year. Comparing, $\$540 < \$600$.

∴ Plan A is cheaper by $60 over the year for this student. (If their use dropped below 5 GB, Plan B at $30/month would win — the answer depends on the assumed usage.)

Example 4 — unscaffolded

A room is 5 m long, 4 m wide and 2.4 m high. Its doors and windows cover about 4 m². Investigate how much paint is needed to give the walls two coats if 1 L covers 12 m².

The four walls have area $2(5+4) \times 2.4 = 43.2$ m². Subtracting the openings gives $43.2 - 4 = 39.2$ m² of wall. Two coats cover $39.2 \times 2 = 78.4$ m². The paint needed is $\frac{78.4}{12} = 6.53$ L, so the painter must buy 7 **L** (paint is sold in whole litres, and part of a seventh litre is still needed).

∴ about 6.53 L of paint is required, so buy 7 L (or two 4 L tins to allow for waste).

Practice questions

Scaffolded

Q1. A rainwater tank investigation uses a roof of area $A = 150$ m², rainfall $R = 700$ mm and efficiency $e = 0.80$. (a) State what each variable means and its units. (b) Using $V = A\,R\,e$, find the water collected in a year. (c) The garden uses 50 000 L a year. State, with a reason, whether the roof collects enough.

Q2. A market stall selling candles has fixed costs of $120 and each candle costs $1.50 to make and sells for $4.00. (a) Write expressions for the revenue R, total cost C and profit P for n candles. (b) Find the break-even number of candles. (c) Find the profit if 100 candles are sold.

Q3. A driver plans a 540 km trip. The car uses 8 L per 100 km and petrol is $1.90 per litre. (a) Formulate: write how to find the fuel used and its cost. (b) Find the fuel used. (c) Find the fuel cost.

Q4. A concert needs $900 to cover the hall and equipment. Tickets sell for $25 and each attendee costs $7 in catering. (a) Write the profit P for n tickets sold. (b) Find the break-even number of tickets. (c) State one assumption you have made.

Q5. A garden bed of area 30 m² needs 10 L of water per square metre each week over a 12-week summer. (a) Formulate an expression for the total water needed. (b) Find the total water needed for the summer. (c) State one limitation of your model.

Q6. A student investigates whether to buy lunch ($9 each school day) or bring it from home (about $4 each day), over a 40-week year of 5 school days each. (a) Write expressions for the yearly cost of each option. (b) Find the yearly cost of each. (c) State the yearly saving of the cheaper option.

Unscaffolded

Q7. A roof of area 200 m² is in an area with 700 mm of rain a year and efficiency 0.80. Find the water it can collect in a year, and state whether it meets a garden use of 100 000 L.

Q8. A cake stall has fixed costs of $120; each cake costs $1.50 to make and sells for $4.00. Find the break-even number of cakes.

Q9. A 540 km trip uses petrol at 8 L per 100 km, costing $1.90 per litre. Find the total fuel cost.

Q10. A community event must raise its \$900 hire cost. Tickets are \$25 and each guest costs \$7. Find the number of tickets needed to break even.

Q11. A phone user needs 8 GB a month. Plan A is \$40/month unlimited; Plan B is \$25/month including 5 GB with extra 5 GB blocks at \$12 each. Investigate which is cheaper per year, and state the assumption your answer depends on.

Q12. A garden of area 30 m² is watered at 10 L per square metre each week for 12 weeks. Find the total water used over the summer.

Q13. A painter must give two coats to 96 m² of wall, with paint covering 12 m² per litre. Find how many whole litres must be bought.

Q14. A food van sells burgers for \$8 each; ingredients cost \$3 per burger and the day's fixed costs (gas, site, wages) are \$250. Find the break-even number of burgers and the profit if 120 are sold.

Q15. A house has a 150 m² roof in an area with 600 mm annual rainfall and efficiency 0.90. The garden uses $40\,000$ L a year. Find the yearly surplus or shortfall of collected water, and comment on whether a tank is worthwhile.

Q16. Two removalists quote for a job: Company A charges \$90 call-out plus \$70 per hour; Company B charges \$150 plus \$55 per hour. Investigate for what length of job each is cheaper.

Q17. A student investigates whether a \$60 annual pool membership (then \$2 per visit) is cheaper than paying \$8 per visit with no membership. Find the number of visits at which the two cost the same, and state which is cheaper for someone who visits 40 times a year.

Q18. A café owner plans to sell a new \$5 smoothie. Ingredients cost \$2 each and the new blender and setup cost \$450. Investigate how many smoothies must be sold to cover the setup, and how many to make a \$300 profit.

Answers

Q1. (a) A roof area (m²), R yearly rainfall (mm), e efficiency (decimal); (b) 84 000 L; (c) yes — $84\,000 > 50\,000$ L. **Q2.** (a) $R=4n$, $C=120+1.5n$, $P=2.5n-120$; (b) 48 candles; (c) \$130. **Q3.** (a) fuel $=540/100\times 8$, cost = fuel $\times 1.90$; (b) 43.2 L; (c) \$82.08. **Q4.** (a) $P=18n-900$; (b) 50 tickets; (c) e.g. every attendee is catered for / all tickets sold in advance. **Q5.** (a) $30\times 10\times 12$; (b) 3600 L; (c) e.g. assumes the same use each week / ignores rain. **Q6.** (a) buy $=9\times 5\times 40$, home $=4\times 5\times 40$; (b) \$1800 vs \$800; (c) \$1000 saved. **Q7.** 112 000 L; yes, it exceeds 100 000 L. **Q8.** 48 cakes. **Q9.** \$82.08. **Q10.** 50 tickets. **Q11.** Plan A \$480/yr; Plan B needs one extra block (8 GB) $=\$37$/month $=\$444$/yr, so **Plan B** is cheaper (by \$36), assuming steady 8 GB use. **Q12.** 3600 L. **Q13.** 16 L. **Q14.** break-even 50 burgers; profit at 120 is \$350. **Q15.** collected 81 000 L, use 40 000 L → surplus 41 000 L; a tank is well worthwhile. **Q16.** equal at 4 hours; A cheaper under 4 h, B cheaper over 4 h. **Q17.** equal at 10 visits; membership is cheaper for 40 visits (\$140 vs \$320). **Q18.** 150 smoothies to cover setup; 250 to make \$300 profit.

Fully-worked solutions

Q1. (a) A = roof area (m²), R = yearly rainfall (mm), e = efficiency (a decimal). (b) $V=A\,R\,e=150\times 700\times 0.80=84\,000$ L. (c) Since $84\,000>50\,000$, the roof collects more than the garden needs, so on average there is enough.

Q2. (a) $R=4n$; $C=120+1.5n$; $P=R-C=4n-(120+1.5n)=2.5n-120$. (b) Break-even: $2.5n-120=0$, so $n=\frac{120}{2.5}=48$ candles. (c) $P=2.5(100)-120=250-120=\$130$.

Q3. (a) Fuel used $=\frac{540}{100}\times 8$ litres; cost = fuel $\times\ \$1.90$. (b) Fuel $=5.4\times 8=43.2$ L. (c) Cost $=43.2\times 1.90=\$82.08$.

Q4. (a) Revenue $25n$, cost $900+7n$, so $P=25n-(900+7n)=18n-900$. (b) $18n-900=0\Rightarrow n=\frac{900}{18}=50$ tickets. (c) An assumption: every ticket is sold and every guest is catered for.

Q5. (a) Total water $=30\times 10\times 12$ litres (area × rate × weeks). (b) $=3600$ L. (c) It assumes the garden is watered the same amount every week and ignores any rain that falls.

Q6. (a) Buying $=9\times 5\times 40=\$1800$; home $=4\times 5\times 40=\$800$ (cost per day × 5 days × 40 weeks). (b) \$1800 versus \$800. (c) Bringing lunch saves $1800-800=\$1000$ a year.

Q7. $V=200\times 700\times 0.80=112\,000$ L. Since $112\,000>100\,000$, the roof meets the garden's yearly use.

Q8. Profit $P=(4-1.5)n-120=2.5n-120$. Break-even: $n=\frac{120}{2.5}=48$ cakes.

Q9. Fuel $=\frac{540}{100}\times 8=43.2$ L; cost $=43.2\times 1.90=\$82.08$.

Q10. $P=(25-7)n-900=18n-900$; break-even $n=\frac{900}{18}=50$ tickets.

Q11. Plan A: $40\times 12=\$480$ a year. Plan B at 8 GB needs the 5 GB base plus one extra 5 GB block, so $25+12=\$37$ a month $=37\times 12=\$444$ a year. Since $\$444<\$480$, **Plan B** is cheaper by \$36 — provided the user stays near 8 GB each month (heavier use adding another block would change the result).

Q12. Water $=30\times 10\times 12=3600$ L.

Q13. Two coats: $96\times 2=192$ m². Litres $=\frac{192}{12}=16$ L exactly, so buy 16 L.

Q14. Profit $P=(8-3)n-250=5n-250$. Break-even: $n=\frac{250}{5}=50$ burgers. At 120: $P=5(120)-250=600-250=\$350$.

Q15. Collected $=150\times600\times0.90=81\,000$ L. Surplus $=81\,000-40\,000=41\,000$ L. The roof collects about twice the garden's use, so a tank to store the surplus is well worthwhile (limited by the tank size chosen).

Q16. Company A costs $90+70h$; Company B costs $150+55h$ for h hours. Equal when $90+70h=150+55h$, i.e. $15h=60$, so $h=4$ hours. For jobs under 4 hours A is cheaper; over 4 hours B is cheaper (its higher call-out is offset by the lower hourly rate).

Q17. Membership costs $60+2v$ for v visits; pay-per-visit costs $8v$. Equal when $60+2v=8v$, i.e. $6v=60$, so $v=10$ visits. For 40 visits: membership $=60+2(40)=\$140$ versus $8(40)=\$320$, so the **membership** is much cheaper.

Q18. Each smoothie makes $5-2=\$3$ toward costs. To cover the \$450 setup: $n=\frac{450}{3}=150$ smoothies. For a \$300 profit on top: $n=\frac{450+300}{3}=\frac{750}{3}=250$ smoothies.