Foundation Mathematics Units 1 & 2

Chapter 11 Measurement: units, area and volume

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Theory

The **metric system** is built on powers of 10, which makes converting between units simply a matter of multiplying or dividing by 10, 100 or 1000. Every measurement is a **number and a unit** — the number only makes sense once you know the unit, so choosing and converting units correctly is a basic workplace and everyday skill.

The three common quantities.

Quantity	Units (small → large)	Key conversions
Length	mm, cm, m, km	$1\,\text{cm} = 10\,\text{mm}$, $1\,\text{m} = 100\,\text{cm} = 1000\,\text{mm}$, $1\,\text{km} = 1000\,\text{m}$
Mass	mg, g, kg, t	$1\,\text{g} = 1000\,\text{mg}$, $1\,\text{kg} = 1000\,\text{g}$, $1\,\text{t} = 1000\,\text{kg}$
Capacity	mL, L, kL	$1\,\text{L} = 1000\,\text{mL}$, $1\,\text{kL} = 1000\,\text{L}$

Metric prefixes.

The prefix tells you how the unit compares with the base unit (metre, gram, litre):

- **milli-** means one thousandth ($\div 1000$) — a millimetre is $1/1000$ of a metre;
- **centi-** means one hundredth ($\div 100$) — a centimetre is $1/100$ of a metre;
- **kilo-** means one thousand ($\times 1000$) — a kilometre is 1000 metres.

The golden rule.

When you convert a measurement, the *amount* does not change — only the unit and the number do.

- Changing to a **smaller** unit → the number gets **bigger** → **multiply**.
- Changing to a **larger** unit → the number gets **smaller** → **divide**.

For example, $2.4\,\text{m}$ is a length; in centimetres it is $2.4 \times 100 = 240\,\text{cm}$ (smaller unit, bigger number), and in kilometres it is $2.4 \div 1000 = 0.0024\,\text{km}$ (larger unit, smaller number).

Choosing sensible units.

Use units that keep the number convenient: a person's height in centimetres or metres (not millimetres), a car trip in kilometres, a dose of medicine in millilitres, a truckload in tonnes. A good check is whether the number "feels right" for the object.

Worked examples

Example 1 — scaffolded

Convert 3.5 km to metres, and 250 g to kilograms.

Working	Comment
3.5 km → m: metres are **smaller** than kilometres, so multiply by 1000.	Smaller unit ⇒ bigger number ⇒ multiply.
$3.5 \times 1000 = 3500\,\text{m}$.	Move the digits three places.
250 g → kg: kilograms are **larger** than grams, so divide by 1000.	Larger unit ⇒ smaller number ⇒ divide.
$250 \div 1000 = 0.25\,\text{kg}$.	250 g is a quarter of a kilogram.

Example 2 — scaffolded

A recipe needs 1.5 L of milk. How many millilitres is this? Also convert 4500 mL to litres.

Working	Comment
1.5 L → mL: millilitres are smaller, so multiply by 1000.	$1\,\text{L} = 1000\,\text{mL}$.
$1.5 \times 1000 = 1500\,\text{mL}$.	The recipe needs 1500 mL.
4500 mL → L: litres are larger, so divide by 1000.	Reverse of the first step.
$4500 \div 1000 = 4.5\,\text{L}$.	4500 mL is four and a half litres.

Example 3 — unscaffolded

A delivery van is loaded with 12 identical boxes, each of mass 8.5 kg. Find the total mass of the boxes in kilograms, and then in tonnes.

Total mass in kilograms is the number of boxes times the mass of each:

$$12 \times 8.5 = 102\,\text{kg}.$$

Tonnes are larger than kilograms, so divide by 1000:

$$102 \div 1000 = 0.102\,\text{t}.$$

∴ the boxes have a total mass of 102 kg, which is 0.102 t.

Example 4 — unscaffolded

A child is prescribed 5 mL of medicine, four times a day, for one week. Find the total amount of medicine given, in millilitres and in litres.

Doses per week: 4 times a day for 7 days is $4 \times 7 = 28$ doses. Total in millilitres: $5 \times 28 = 140\,\text{mL}$. Litres are larger than millilitres, so divide by 1000: $140 \div 1000 = 0.14\,\text{L}$.

∴ the child is given 140 mL of medicine, which is 0.14 L.

Practice questions

Scaffolded

Q1. Convert each length. (a) 4 m to centimetres. (b) 2500 m to kilometres. (c) 6.3 cm to millimetres.

Q2. Convert each mass. (a) 3 kg to grams. (b) 4500 mg to grams. (c) 2.5 t to kilograms.

Q3. Convert each capacity. (a) 2 L to millilitres. (b) 3500 mL to litres. (c) 5 kL to litres.

Q4. Convert each length, choosing multiply or divide carefully. (a) 0.75 km to metres. (b) 85 mm to centimetres. (c) 12 500 m to kilometres.

Q5. Convert each mass. (a) 1.25 kg to grams. (b) 750 g to kilograms. (c) 0.6 t to kilograms.

Q6. Convert each capacity. (a) 250 mL to litres. (b) 1.8 L to millilitres. (c) 4 kL to litres.

Unscaffolded

Q7. A ribbon 3.2 m long is cut into 8 equal pieces. Find the length of each piece in centimetres.

Q8. A bag holds 2 kg of flour. A recipe uses 250 g. How many full batches of the recipe can be made from one bag?

Q9. A rainwater tank holds 5 kL. It is emptied using a 20 L bucket. How many bucket-loads does it take to empty the full tank?

Q10. Along a 4.5 km walking track, distance markers are placed every 250 m. How many equal 250 m gaps are there along the track?

Q11. A bottle contains 60 tablets, each with 500 mg of an active ingredient. Find the total mass of active ingredient in the bottle, in grams.

Q12. A jug holds 2.5 L of juice. It is poured into glasses that each hold 200 mL. How many glasses can be filled completely?

Q13. A truck is carrying a load of 3.4 t made up of identical pallets, each of mass 85 kg. How many pallets are on the truck?

Q14. Three offcuts of timber measure 0.8 m, 85 cm and 790 mm. By converting them all to millimetres, decide which offcut is the longest.

Q15. A nurse gives a patient a 2.5 mL dose four times a day for 5 days. Find the total amount of medicine given, in millilitres and in litres.

Q16. A straight fence runs along one side of a paddock that is 0.45 km long. A post is placed every 3 m, including one at each end. How many posts are needed?

Q17. A sauce recipe for 4 people makes 600 mL. A caterer scales the same recipe up for 10 people. How much sauce will this make, in litres?

Q18. A parcel has a mass of 1.85 kg. Postage is charged per 500 g (any part of a 500 g block is charged as a whole block). Find how many 500 g blocks the parcel is charged for, and the total postage if each block costs $\$3.20$.

Answers

Q1. (a) 400 cm. (b) 2.5 km. (c) 63 mm. **Q2.** (a) 3000 g. (b) 4.5 g. (c) 2500 kg. **Q3.** (a) 2000 mL. (b) 3.5 L. (c) 5000 L. **Q4.** (a) 750 m. (b) 8.5 cm. (c) 12.5 km. **Q5.** (a) 1250 g. (b) 0.75 kg. (c) 600 kg. **Q6.** (a) 0.25 L. (b) 1800 mL. (c) 4000 L. **Q7.** 40 cm. **Q8.** 8 batches. **Q9.** 250 bucket-loads. **Q10.** 18 gaps. **Q11.** 30 g. **Q12.** 12 glasses. **Q13.** 40 pallets. **Q14.** 85 cm ($=850$ mm) is the longest. **Q15.** 50 mL $=0.05$ L. **Q16.** 151 posts. **Q17.** 1.5 L. **Q18.** 4 blocks; $\$12.80$.

Fully-worked solutions

Q1. (a) cm are smaller than m: $4\times100=400$ cm. (b) km are larger than m: $2500\div1000=2.5$ km. (c) mm are smaller than cm: $6.3\times10=63$ mm.

Q2. (a) $3\times1000=3000$ g. (b) g are larger than mg: $4500\div1000=4.5$ g. (c) $2.5\times1000=2500$ kg.

Q3. (a) $2\times1000=2000$ mL. (b) $3500\div1000=3.5$ L. (c) $5\times1000=5000$ L.

Q4. (a) $0.75\times1000=750$ m. (b) $85\div10=8.5$ cm. (c) $12\,500\div1000=12.5$ km.

Q5. (a) $1.25\times1000=1250$ g. (b) $750\div1000=0.75$ kg. (c) $0.6\times1000=600$ kg.

Q6. (a) $250\div1000=0.25$ L. (b) $1.8\times1000=1800$ mL. (c) $4\times1000=4000$ L.

Q7. First convert to centimetres: $3.2\,\text{m}=3.2\times100=320$ cm. Then divide into 8 equal pieces: $320\div8=40$ cm each.

Q8. Convert the bag to grams: $2\,\text{kg}=2\times1000=2000$ g. Number of 250 g batches: $2000\div250=8$ batches.

Q9. Convert the tank to litres: $5\,\text{kL}=5\times1000=5000$ L. Number of 20 L buckets: $5000\div20=250$ bucket-loads.

Q10. Convert the track to metres: $4.5\,\text{km}=4.5\times1000=4500$ m. Number of 250 m gaps: $4500\div250=18$ gaps.

Q11. Total mass in milligrams: $60\times500=30\,000$ mg. Convert to grams (larger unit): $30\,000\div1000=30$ g.

Q12. Convert the jug to millilitres: $2.5\,\text{L}=2500$ mL. Dividing, $2500\div200=12.5$, so 12 glasses can be filled completely (the last 0.5 of a glass is not full).

Q13. Convert the load to kilograms: $3.4\,\text{t}=3.4\times1000=3400$ kg. Number of 85 kg pallets: $3400\div85=40$ pallets.

Q14. In millimetres: $0.8\,\text{m}=0.8\times1000=800$ mm; $85\,\text{cm}=85\times10=850$ mm; the third is 790 mm. Since $850>800>790$, the 85 cm offcut is the longest.

Q15. Doses: 4 a day for 5 days is $4\times5=20$ doses. Total: $2.5\times20=50$ mL. In litres: $50\div1000=0.05$ L.

Q16. Convert the fence length to metres: $0.45\,\text{km}=0.45\times1000=450$ m. Posts every 3 m give $450\div3=150$ equal gaps. A straight fence has one more post than gaps (a post at each end), so $150+1=151$ posts.

Q17. Amount per person: $600\div4=150$ mL. For 10 people: $150\times10=1500$ mL. In litres: $1500\div1000=1.5$ L.

Q18. Convert the parcel to grams: $1.85\,\text{kg}=1850$ g. Dividing by 500 gives $1850\div500=3.7$, and because any part of a block is charged as a whole block, this rounds up to 4 blocks. Total postage: $4\times\$3.20=\$12.80$.

Theory

Good measurement is a skill: you choose the right **instrument** and **unit**, read the scale carefully, and record the result with sensible **accuracy**. This section is about doing that reliably.

Choosing the instrument and the unit.

Match the tool and the unit to the job:

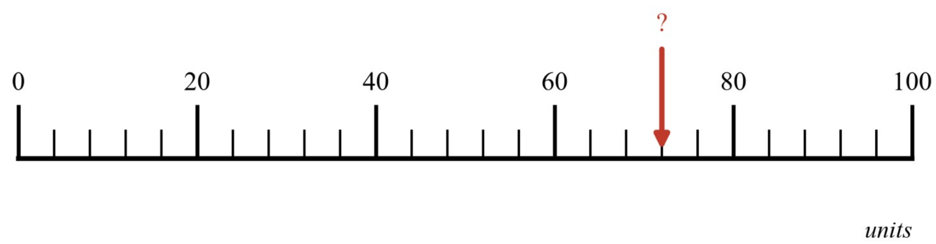
Quantity	Instrument	Common units
Length / distance	ruler, tape measure, trundle wheel, odometer	mm, cm, m, km
Mass	kitchen scale, bathroom scale, balance	g, kg, t
Volume / capacity	measuring jug, measuring cylinder, syringe	mL, L
Temperature	thermometer	°C
Time	clock, stopwatch	s, min, h

A short length is measured in millimetres or centimetres; a road trip in kilometres. Using the wrong unit (weighing a letter on bathroom scales) gives a useless result.

Reading a scale.

Most analogue instruments have a few **labelled** marks with smaller unlabelled marks between them. To read the scale:

1. Find the value of one **small division** $= \dfrac{\text{gap between two labelled marks}}{\text{number of equal spaces between them}}$.
2. Count the small divisions from the nearest labelled mark to the pointer or level.
3. Multiply and add.



On the scale above the labelled marks are 20 apart with 5 equal spaces between them, so each small division is $20 \div 5 = 4$ units. The arrow is 3 divisions past 60, so it reads $60 + 3 \times 4 = 72$ units.

Estimating first.

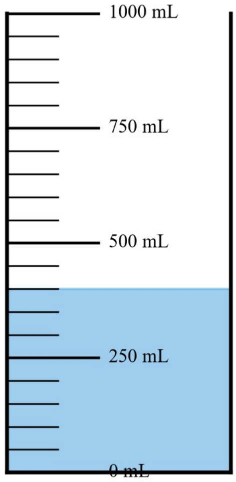
Before you measure, **estimate**. Knowing a doorway is “about 2 m” or a bag of sugar is “about 1 kg” lets you spot a silly reading (a doorway can’t be 20 m).

Accuracy, precision and rounding.

No measurement is exact — an instrument can only be read to its smallest division. A length recorded as 154 cm on a tape marked in whole centimetres really lies somewhere between 153.5 cm and 154.5 cm; that is, it is accurate to ± 0.5 cm (**half a division**). A scale with smaller divisions is more **precise**. Always record a measurement **with its unit**, rounded to the precision the instrument allows.

Worked examples

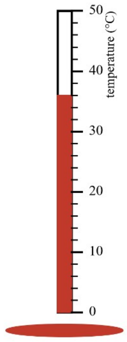
Example 1 — scaffolded



The measuring jug above is used to measure cordial. State the value of one small division, and read the volume shown.

Working	Comment
Labelled marks are 250 mL apart with 5 equal spaces between them.	Count the spaces between 0 and 250.
One small division $= 250 \div 5 = 50$ mL.	Gap ÷ number of spaces.
The level is 3 divisions above 250 mL: $250 + 3 \times 50 = 400$ mL.	Count up from the nearest label.
400 mL $= 0.4$ L.	1000 mL $= 1$ L.

Example 2 — scaffolded



Read the temperature shown on the thermometer, and state the value of one small division.

Working	Comment
Labelled marks are 10°C apart with 5 spaces between them.	Look between 30 and 40.
One small division $= 10 \div 5 = 2$°C.	Gap ÷ number of spaces.
The level is 3 divisions above 30°C: $30 + 3 \times 2 = 36$°C.	Count up from 30.

Example 3 — unscaffolded

A digital kitchen scale shows a bag of flour as 1.865 kg. (a) Round this to the nearest 10 g. (b) An older analogue scale nearby is marked every 100 g; to the nearest division, what would it read? (c) Which scale is more precise?

First convert: 1.865 kg = 1865 g. (a) To the nearest 10 g, 1865 g rounds up to 1870 g = 1.87 kg. (b) To the nearest 100 g, 1865 g rounds up to 1900 g = 1.9 kg. (c) The digital scale reads to the nearest 1 g, a smaller division than 100 g, so it is more precise.

∴ 1.87 kg (digital) versus 1.9 kg (analogue); the digital scale is more precise.

Example 4 — unscaffolded

A carpenter measures a benchtop with a tape marked in millimetres and records 1.842 m. (a) Convert the length to millimetres. (b) Between what two lengths must the true benchtop length lie? (c) Explain why the measurement can never be exact.

(a) $1.842\text{ m} = 1.842 \times 1000 = 1842\text{ mm}$.
(b) The smallest division is 1 mm, so the reading is accurate to ± 0.5 mm: the true length lies between 1841.5 mm and 1842.5 mm.
(c) A tape can only be read to the nearest mark, so every measurement is rounded to the nearest division — there is always a small uncertainty of up to half a division.

∴ 1842 mm, true length between 1841.5 mm and 1842.5 mm; measurement is limited by the smallest division.

Practice questions

Scaffolded

Q1. For each task, name a suitable instrument and a suitable unit. (a) The length of a classroom. (b) The mass of an apple. (c) The temperature of an oven.

Q2. A dial is labelled $0, 10, 20, 30, 40$ with 5 equal spaces between each pair of labels. (a) Find the value of one small division. (b) What does the dial read if the pointer is 3 divisions past 20? (c) What does it read if the pointer is 1 division before 40?

Q3. A measuring jug is marked every 500 mL, with 5 equal spaces between the labels. (a) Find the value of one small division. (b) The liquid level is at the 4th mark above 500 mL. What volume is this? (c) Write your answer to (b) in litres.

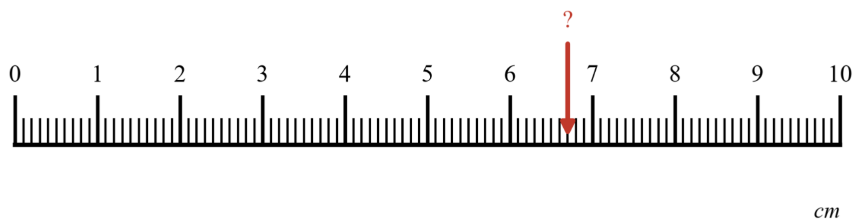
Q4. A digital scale shows 3.457 kg. Round this mass to the nearest (a) 10 g, (b) 100 g, (c) kilogram.

Q5. A tape marked in whole centimetres measures a table as 126 cm. (a) State the smallest division. (b) Between what two lengths must the true length lie? (c) Write the measurement in the form (value) ± (uncertainty).

Q6. Estimate a sensible measurement, and state the unit, for each. (a) The mass of a small car. (b) The capacity of a teaspoon. (c) The length of a soccer pitch.

Unscaffolded

Q7. The object below is measured against a centimetre ruler with millimetre marks. State its length in centimetres and in millimetres.



Q8. A thermometer is labelled every 5°C, with small divisions every 1°C. The mercury sits 3 divisions above the 20 °C mark. State the temperature and the value of one small division.

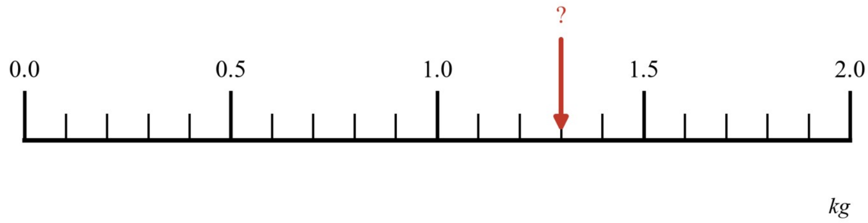
Q9. A measuring cylinder holds 100 mL. It is labelled every 20 mL, with 4 equal spaces between the labels. (a) Find the value of one small division. (b) Read the volume when the level is 3 divisions above 40 mL.

Q10. A person wants to know the mass of a letter to work out the postage. Explain why a bathroom scale is a poor choice, and name a better instrument and unit.

Q11. A baby's mass on a digital scale is 4.687 kg. Round it to the nearest (a) 10 g, (b) 100 g. (c) Which of these is closer to the true mass, and why?

Q12. At the start of a trip a car's odometer reads 48 267 km. At the end it reads 48 519 km. How far was the trip?

Q13. The kitchen scale below is used to weigh potatoes. State the value of one small division, and read the mass shown, in kilograms and in grams.



Q14. One measuring jug is marked to the nearest 50 mL; another is marked to the nearest 10 mL. A cook needs 340 mL of milk. (a) Which jug can measure it more precisely? (b) To the nearest division, what would each jug read?

Q15. A trundle wheel clicks once for each complete turn, and one turn covers exactly 1 m. A student wheels it along a path, counts 47 clicks, and then reaches the end mark 60 cm past the last click. How long is the path, in metres?

Q16. For each quantity, state a sensible unit and estimate a reasonable value. (a) The temperature of a warm bath. (b) The distance of a marathon. (c) A dose of children's liquid medicine.

Q17. A kitchen scale is marked every 100 g. The needle sits exactly halfway between the 700 g and 800 g marks. (a) What is the best reading you can give? (b) To what accuracy (in grams) can you state it?

Q18. A nurse must draw up 0.8 mL of a medicine in a syringe marked every 0.1 mL. (a) How many divisions from zero is the 0.8 mL mark? (b) The smallest division is 0.1 mL, so the dose is accurate to within how many millilitres? (c) Write 0.8 mL as a fraction of 1 mL, in simplest form.

Answers

Q1. (a) tape measure / metre ruler, in m. (b) kitchen scale, in g. (c) (oven) thermometer, in °C. **Q2.** (a) 2. (b) 26. (c) 38. **Q3.** (a) 100 mL. (b) 900 mL. (c) 0.9 L. **Q4.** (a) 3.46 kg (3460 g). (b) 3.5 kg (3500 g). (c) 3 kg. **Q5.** (a) 1 cm. (b) between 125.5 cm and 126.5 cm. (c) 126 ± 0.5 cm. **Q6.** (a) about 1500 kg (≈ 1.5 t). (b) about 5 mL. (c) about 100 m. (Reasonable estimates accepted.) **Q7.** 6.7 cm $= 67$ mm. **Q8.** 23°C; one small division $= 1$°C. **Q9.** (a) 5 mL. (b) 55 mL. **Q10.** A bathroom scale reads to about the nearest 100–500 g, far too coarse for a letter of a few grams; use a kitchen or postal scale reading in grams. **Q11.** (a) 4.69 kg. (b) 4.7 kg. (c) the nearest 10 g (4.69 kg), because a smaller division is more precise. **Q12.** 252 km. **Q13.** one small division $= 0.1$ kg; reading 1.3 kg $= 1300$ g. **Q14.** (a) the 10 mL jug. (b) 50 mL jug reads 350 mL; 10 mL jug reads 340 mL. **Q15.** 47.6 m. **Q16.** (a) about 40°C. (b) about 42 km. (c) about 5 mL. (Reasonable estimates accepted.) **Q17.** (a) 750 g. (b) to within 50 g (half a division). **Q18.** (a) 8 divisions. (b) 0.05 mL. (c) $\frac{4}{5}$.

Fully-worked solutions

Q1. (a) A classroom is metres long, so a tape measure (or metre ruler), reading in **metres**. (b) An apple is light, so a kitchen scale, in **grams**. (c) An oven is hot, so an oven thermometer, in **degrees Celsius**.

Q2. The labels are 10 apart with 5 equal spaces between them. (a) One small division $= 10 \div 5 = 2$. (b) 3 divisions past 20: $20 + 3 \times 2 = 26$. (c) 1 division before 40: $40 - 2 = 38$.

Q3. (a) One small division $= 500 \div 5 = 100$ mL. (b) The 4th mark above 500 mL is $500 + 4 \times 100 = 900$ mL. (c) 900 mL $= 900 \div 1000 = 0.9$ L.

Q4. 3.457 kg $= 3457$ g. (a) Nearest 10 g: the ones digit is $7 \geq 5$, so $3457 \rightarrow 3460$ g $= 3.46$ kg. (b) Nearest 100 g: the tens digit is $5 \geq 5$, so $3457 \rightarrow 3500$ g $= 3.5$ kg. (c) Nearest kg: 3457 g is less than 3500 g, so it rounds down to 3000 g $= 3$ kg.

Q5. (a) The tape is marked in whole centimetres, so the smallest division is 1 cm. (b) The reading is accurate to half a division, ± 0.5 cm, so the true length lies between $126 - 0.5 = 125.5$ cm and $126 + 0.5 = 126.5$ cm. (c) 126 ± 0.5 cm.

Q6. Sensible estimates: (a) a small car is about 1500 kg (about 1.5 tonnes); (b) a teaspoon holds about 5 mL; (c) a soccer pitch is about 100 m long. Any estimate close to these, with the right unit, is acceptable.

Q7. The object runs from 0 to the 6.7 cm mark (seven millimetre marks past 6 cm). So its length is 6.7 cm, and $6.7 \times 10 = 67$ mm.

Q8. The small divisions are every 1°C, so one division $= 1$°C. The mercury is 3 divisions above 20°C: $20 + 3 \times 1 = 23$ °C.

Q9. The labels are 20 mL apart with 4 equal spaces between them. (a) One small division $= 20 \div 4 = 5$ mL. (b) 3 divisions above 40 mL: $40 + 3 \times 5 = 55$ mL.

Q10. A letter has a mass of only a few grams, but a bathroom scale is designed for tens of kilograms and reads only to about the nearest 100–500 g, so it would show 0 or a very rough value. A kitchen scale or postal scale reading in **grams** is far more suitable.

Q11. 4.687 kg $= 4687$ g. (a) Nearest 10 g: the ones digit $7 \geq 5$, so $4687 \rightarrow 4690$ g $= 4.69$ kg. (b) Nearest 100 g: the tens digit $8 \geq 5$, so $4687 \rightarrow 4700$ g $= 4.7$ kg. (c) Rounding to the nearest 10 g keeps more detail (a smaller division), so 4.69 kg is closer to the true mass.

Q12. Distance = final reading $-$ start reading $= 48\,519 - 48\,267 = 252$ km.

Q13. The kitchen scale is labelled every 0.5 kg with 5 equal spaces between the labels, so one small division $= 0.5 \div 5 = 0.1$ kg. The needle points to 1.3 kg, which is $1.3 \times 1000 = 1300$ g.

Q14. (a) The 10 mL jug has smaller divisions, so it can measure 340 mL more precisely. (b) To the nearest 50 mL, 340 mL rounds to 350 mL (it is 40 above 300 and only 10 below 350). To the nearest 10 mL, 340 mL is already a multiple of 10, so it reads 340 mL.

Q15. Each complete turn is 1 m, so 47 clicks cover 47 m. The extra 60 cm = 0.6 m. Total $= 47 + 0.6 = 47.6$ m.

Q16. Sensible answers: (a) a warm bath is about 40°C; (b) a marathon is about 42 km; (c) a dose of children's medicine is about 5 mL. Any close estimate with the correct unit is acceptable.

Q17. (a) Halfway between 700 g and 800 g is $\frac{700+800}{2} = 750$ g, the best estimate. (b) The divisions are 100 g apart, so a reading between marks is accurate to half a division, ± 50 g.

Q18. (a) Each division is 0.1 mL, so $0.8 \div 0.1 = 8$ divisions from zero. (b) The dose can be set to the nearest division, ± 0.05 mL (half of 0.1 mL). (c) $0.8 = \frac{8}{10} = \frac{4}{5}$, so 0.8 mL $= \frac{4}{5}$ of a millilitre.

Theory

Time is the one measurement everybody uses every day — to catch a train, start a shift, cook a meal or join an online meeting. Working with time is different from ordinary measurement because it is **not** a decimal system: there are 60 seconds in a minute and 60 minutes in an hour, not 100. That single fact is behind almost every mistake people make with time, so treat hours-and-minutes carefully and never just type 8.45 into a calculator when you mean 8 hours and 45 minutes.

12-hour and 24-hour time.

Everyday clocks use **12-hour time** with **am** (midnight to noon) and **pm** (noon to midnight). Timetables, computers and the defence and health industries use **24-hour time**, which runs from $00:00$ to $23:59$ and needs no am/pm. To convert:

- **12-hour to 24-hour:** for a pm time (except 12 pm) add 12 to the hours; 12 am is written $00:$, and 12 pm stays $12:$.
- **24-hour to 12-hour:** for an hour of 13 or more, subtract 12 and add “pm”; $00:$ becomes $12:$ am.

12-hour	9:00 am	12:00 noon	1:30 pm	8:15 pm	12:00 midnight
24-hour	09:00	12:00	13:30	20:15	00:00

Units of time.

The standard conversions are worth knowing by heart:

×	to get
60 seconds	1 minute
60 minutes	1 hour
24 hours	1 day
7 days	1 week
365 days (366 in a leap year)	1 year

To change a bigger unit into a smaller one you **multiply**; to change a smaller unit into a bigger one you **divide**. For example 3 hours $= 3 \times 60 = 180$ minutes, and 150 seconds $= 150 \div 60 = 2$ minutes 30 seconds.

Elapsed time (duration).

Elapsed time is how long something lasts — the time from a start to a finish. The safe method is to **count on** in easy steps, or to subtract using 60 (not 100) when borrowing. To add or subtract times, keep hours and minutes in separate columns and carry 60 minutes as 1 hour. When a period **crosses midnight**, split it at midnight: count the time up to $24:00$, then add the time after midnight.

Dates and the calendar.

Months have 28–31 days (“30 days has September, April, June and November; all the rest have 31, except February with 28, or 29 in a leap year”). To find the number of days from one date to another, count the days remaining in the first month and add the days into the second month.

Time zones in Australia.

Australia has three standard time zones:

Zone	Standard time	Where
Australian Western Standard Time	UTC $+8$	WA

Zone	Standard time	Where
(AWST)		
Australian Central Standard Time (ACST)	UTC $+9:30$	SA, NT
Australian Eastern Standard Time (AEST)	UTC $+10$	Qld, NSW, ACT, Vic, Tas

So **Perth (AWST) is** 2 **hours behind** the eastern states, and **Adelaide (ACST) is** 30 **minutes behind** them. To find another zone's time, add or subtract the difference. (In summer, the southern and eastern states put clocks **forward one hour** for daylight saving, but Queensland, WA and the NT do not — the questions here state the difference to use.)

Timetables and schedules.

A **timetable** or **schedule** is a table of start and finish times. Read across a row to find an event's times, and use elapsed time to find how long it runs or how long you must wait between events.

Worked examples

Example 1 — scaffolded

Write $7:45$ am, $3:20$ pm and $11:55$ pm in 24-hour time, and write $06:30$ and $20:15$ in 12-hour time.

Working	Comment
$7:45$ am $\rightarrow$ **07:45**.	An am time keeps the same hours (with a leading zero).
$3:20$ pm $\rightarrow 3+12=15$, so **15:20**.	For a pm time add 12 to the hours.
$11:55$ pm $\rightarrow 11+12=23$, so **23:55**.	Still a pm time, so add 12.
$06:30 \rightarrow$ **6:30 am**.	Hours under 12 are am.
$20:15 \rightarrow 20-12=8$, so **8:15 pm**.	Hours of 13–23 are pm after subtracting 12.

Example 2 — scaffolded

A nurse's shift runs from $7:15$ am to $3:45$ pm, with a 30-minute unpaid meal break. Find the total time from start to finish, and the time she is paid for.

Working	Comment
Write both in 24-hour time: start $07:15$, finish $15:45$.	Converting removes the am/pm confusion.
Count on: $07:15 \rightarrow 15:15$ is 8 hours, then $15:15 \rightarrow 15:45$ is 30 minutes.	Count the whole hours first, then the extra minutes.
Total time $=8$ hours 30 minutes.	Start to finish.
Paid time $=8$ h 30 min -30 min $=8$ hours.	Subtract the unpaid break.

Example 3 — unscaffolded

A football match in Perth (AWST) starts at $7:00$ pm local time. AWST is 2 hours behind Australian Eastern Standard Time and 1 hour 30 minutes behind Australian Central Standard Time. Find the start time for a viewer in Melbourne (AEST) and for a viewer in Adelaide (ACST).

Melbourne is 2 hours **ahead** of Perth, so add 2 hours:

$$7:00\,\text{pm}+2\,\text{h}=9:00\,\text{pm (AEST)}.$$

Adelaide is 1 hour 30 minutes ahead of Perth, so add 1 h 30 min:

$$7:00\,\text{pm}+1\,\text{h}\,30\,\text{min}=8:30\,\text{pm (ACST)}.$$

∴ the match starts at $9:00$ pm in Melbourne and $8:30$ pm in Adelaide.

Example 4 — unscaffolded

An overnight flight leaves Sydney at $11:40$ pm on 3 May and takes 5 hours 25 minutes to reach Perth. Perth time is 2 hours behind Sydney time. Find (a) the Sydney time and date when it lands, and (b) the local Perth time and date when it lands.

Add the flight time to the departure time. From $11:40$ pm ($23:40$), 5 hours reaches $04:40$ the next morning (4 May), and a further 25 minutes gives

$$23:40+5\,\text{h}\,25\,\text{min}=05:05\,\text{on}\,4\,\text{May (Sydney time)}.$$

Perth is 2 hours behind Sydney, so subtract 2 hours:

$$05:05-2\,\text{h}=03:05\,\text{on}\,4\,\text{May (Perth time)}.$$

∴ the flight lands at $5:05$ am on 4 May Sydney time, which is $3:05$ am on 4 May in Perth.

Practice questions

Scaffolded

Q1. Write each time in 24-hour time. (a) $9:30$ am. (b) $2:15$ pm. (c) $10:50$ pm.

Q2. Write each time in 12-hour time (with am or pm). (a) $08:05$. (b) $16:40$. (c) $00:20$.

Q3. Convert each of the following. (a) 3 hours to minutes. (b) 240 seconds to minutes. (c) 2 days to hours.

Q4. A movie starts at $6:50$ pm and runs for 2 hours 25 minutes. (a) Write the start time in 24-hour time. (b) Find the finish time in 24-hour time. (c) Write the finish time in 12-hour time.

Q5. A worker's shift runs from $8:45$ am to $4:15$ pm with a 45-minute unpaid lunch break. (a) Find the total time from start to finish. (b) Subtract the break to find the paid time. (c) The worker is paid $\$28.40$ per hour. Find the day's pay.

Q6. Perth (AWST) is 2 hours behind Melbourne (AEST), and Adelaide (ACST) is 30 minutes behind Melbourne. (a) When it is $1:00$ pm in Melbourne, what time is it in Perth? (b) When it is $1:00$ pm in Melbourne, what time is it in Adelaide? (c) A friend in Melbourne rings at $9:00$ am Melbourne time. What is the local time in Perth?

Unscaffolded

Q7. Write $11:05$ pm and $12:00$ noon in 24-hour time, and write $00:45$ and $13:30$ in 12-hour time.

Q8. A train leaves at $9:52$ am and arrives at $11:38$ am. How long is the journey?

Q9. A baker puts a first batch of bread in the oven at $5:40$ am; it bakes for 35 minutes. A second batch, needing 40 minutes, goes in as soon as the first comes out. At what time is the second batch ready?

Q10. Convert each of the following. (a) 3 hours 20 minutes to minutes. (b) 200 minutes to hours and minutes. (c) 1.5 hours to minutes.

Q11. A night-shift worker starts at $10:30$ pm and finishes at $6:15$ am the next morning. How long is the shift?

Q12. A parking meter is paid at $11:50$ am for 3 hours 30 minutes. At what time (12-hour time) does the parking expire?

Q13. How many days later is 10 April than 25 March? (March has 31 days.)

Q14. A flight departs Brisbane (AEST) at $3:30$ pm and takes 5 hours 10 minutes to reach Perth. Perth (AWST) is 2 hours behind Brisbane. (a) At what Brisbane time does it land? (b) What is the local Perth time when it lands?

Q15. An evening TV guide is shown below.

Program	Start	End
News	6:00 pm	6:30 pm
Quiz show	6:30 pm	7:30 pm
Movie	7:30 pm	9:45 pm

(a) How long does the quiz show run?

(b) How long does the movie run?

(c) A viewer records from the start of the news to the end of the movie. How long is the recording?

Q16. A recipe says to slow-cook a roast for 4 hours 45 minutes. To have it ready at $7:00$ pm, at what time must it go into the oven?

Q17. A webinar is scheduled for $2:00$ pm AEST (Sydney). (a) A viewer in Perth (AWST), 2 hours behind Sydney, joins at the scheduled time. What is the local Perth time? (b) A viewer in Adelaide (ACST), 30 minutes behind Sydney — what is the local time there? (c) The webinar runs for 90 minutes. At what local Perth time does it finish?

Q18. A worker records the week's hours as Monday 7 h 30 min, Tuesday 6 h 45 min, Wednesday 8 h 15 min, Thursday 7 h and Friday 5 h 30 min. (a) Find the total time worked, in hours and minutes. (b) Write the total as a number of hours (a decimal). (c) At $\$32.60$ per hour, find the week's pay.

Answers

Q1. (a) $09:30$. (b) $14:15$. (c) $22:50$. **Q2.** (a) $8:05$ am. (b) $4:40$ pm. (c) $12:20$ am. **Q3.** (a) 180 minutes. (b) 4 minutes. (c) 48 hours. **Q4.** (a) $18:50$. (b) $21:15$. (c) $9:15$ pm. **Q5.** (a) 7 h 30 min. (b) 6 h 45 min. (c) $\$191.70$. **Q6.** (a) $11:00$ am. (b) $12:30$ pm. (c) $7:00$ am. **Q7.** $23:05$; $12:00$; $12:45$ am; $1:30$ pm. **Q8.** 1 h 46 min. **Q9.** $6:55$ am. **Q10.** (a) 200 minutes. (b) 3 h 20 min. (c) 90 minutes. **Q11.** 7 h 45 min. **Q12.** $3:20$ pm. **Q13.** 16 days. **Q14.** (a) $8:40$ pm. (b) $6:40$ pm. **Q15.** (a) 1 hour. (b) 2 h 15 min. (c) 3 h 45 min. **Q16.** $2:15$ pm. **Q17.** (a) $12:00$ noon. (b) $1:30$ pm. (c) $1:30$ pm. **Q18.** (a) 35 h 0 min. (b) 35 hours. (c) $\$1141.00$.

Fully-worked solutions

Q1. (a) $9:30$ am is an am time, so it stays $09:30$. (b) $2:15$ pm: add 12 to the hours, $2+12=14$, giving $14:15$. (c) $10:50$ pm: $10+12=22$, giving $22:50$.

Q2. (a) $08:05$ has hours under 12, so it is $8:05$ am. (b) $16:40$: subtract 12, $16-12=4$, giving $4:40$ pm. (c) $00:20$ is just after midnight, written $12:20$ am.

Q3. (a) Hours to minutes, multiply by 60: $3\times60=180$ minutes. (b) Seconds to minutes, divide by 60: $240\div60=4$ minutes. (c) Days to hours, multiply by 24: $2\times24=48$ hours.

Q4. (a) $6:50$ pm $\rightarrow 6+12=18$, so $18:50$. (b) Add 2 h 25 min: $18:50+2\,\text{h}=20:50$, then $+25$ min $=21:15$. (c) $21:15\rightarrow 21-12=9$, so $9:15$ pm.

Q5. (a) Start $08:45$, finish $16:15$. From $08:45$ to $16:45$ would be 8 hours, but the finish is $16:15$, which is 30 minutes earlier, so the total is 8 h -30 min $=7$ h 30 min. (Check by counting on: $08:45\rightarrow15:45$ is 7 hours, then $15:45\rightarrow16:15$ is 30 minutes.) (b) Paid time $=7$ h 30 min -45 min $=6$ h 45 min. (c) 6 h 45 min $=6.75$ hours, so the pay is $6.75\times\$28.40=\$191.70$.

Q6. (a) Perth is 2 hours behind Melbourne: $1:00$ pm -2 h $=11:00$ am. (b) Adelaide is 30 minutes behind Melbourne: $1:00$ pm -30 min $=12:30$ pm. (c) $9:00$ am Melbourne -2 h $=7:00$ am in Perth.

Q7. $11:05$ pm $\rightarrow 11+12=23$, so $23:05$. $12:00$ noon is $12:00$ in 24-hour time. $00:45$ is $12:45$ am. $13:30\rightarrow13-12=1$, so $1:30$ pm.

Q8. From $9:52$ am, counting on: $9:52\rightarrow10:52$ is 1 hour, then $10:52\rightarrow11:38$ is 46 minutes. The journey takes 1 hour 46 minutes.

Q9. First batch: $5:40$ am $+35$ min $=6:15$ am. The second batch goes straight in and needs 40 minutes: $6:15$ am $+40$ min $=6:55$ am.

Q10. (a) 3 h 20 min $=3\times60+20=180+20=200$ minutes. (b) $200\div60=3$ remainder 20, so 200 minutes $=3$ h 20 min. (c) $1.5\times60=90$ minutes.

Q11. Split at midnight. From $10:30$ pm to midnight is 1 h 30 min. From midnight to $6:15$ am is 6 h 15 min. Total $=1$ h 30 min $+6$ h 15 min $=7$ h 45 min.

Q12. $11:50$ am $+3$ h 30 min: $11:50+3$ h $=14:50$ ($2:50$ pm), then $+30$ min $=15:20$, which is $3:20$ pm.

Q13. From 25 March to the end of March is $31-25=6$ days, then 10 more days into April, giving $6+10=16$ days.

Q14. (a) $3:30$ pm $=15:30$; add 5 h 10 min: $15:30+5$ h $=20:30$, then $+10$ min $=20:40$, i.e. $8:40$ pm Brisbane time. (b) Perth is 2 hours behind Brisbane: $8:40$ pm -2 h $=6:40$ pm.

Q15. (a) Quiz show: $6:30$ pm to $7:30$ pm is 1 hour. (b) Movie: $7:30$ pm to $9:45$ pm is 2 hours (to $9:30$) plus 15 minutes $=2$ h 15 min. (c) News start ($6:00$ pm) to movie end ($9:45$ pm): $6:00\rightarrow9:00$ is 3 hours, then $9:00\rightarrow9:45$ is 45 minutes, so 3 h 45 min.

Q16. Work backwards from $7:00$ pm: $7:00$ pm $=19:00$; subtract 4 h 45 min. $19:00-4$ h $=15:00$, then -45 min $=14:15$, which is $2:15$ pm.

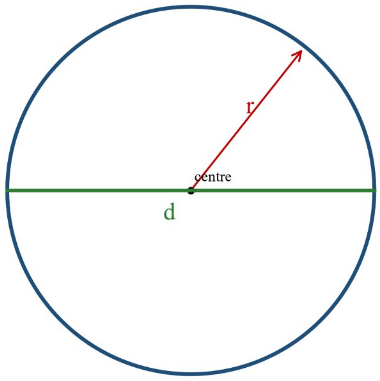
Q17. (a) Perth is 2 hours behind Sydney: $2:00$ pm -2 h $=12:00$ noon. (b) Adelaide is 30 minutes behind Sydney: $2:00$ pm -30 min $=1:30$ pm. (c) In Perth the webinar starts at $12:00$ noon and runs 90 min $=1$ h 30 min, so it finishes at $12:00+1$ h 30 min $=1:30$ pm.

Q18. (a) Add the minutes: $30+45+15+0+30=120$ min $=2$ h. Add the hours: $7+6+8+7+5=33$ h. Total $=33$ h $+2$ h $=35$ h 0 min. (b) 35 h 0 min $=35$ hours exactly. (c) $35\times\$32.60=\$1141.00$.

Theory

Many everyday objects are round — wheels, plates, pipes, tanks, garden beds. To measure and build with them we need the mathematics of the **circle**.

Parts of a circle.



- The **centre** is the middle point.
- The **radius** r is the distance from the centre to the edge.
- The **diameter** d is the distance straight across through the centre. It is twice the radius:

$$d = 2r, r = \frac{d}{2}.$$

- The **circumference** C is the distance all the way around the edge (the "perimeter" of the circle).

The special number π.

If you measure the circumference of *any* circle and divide it by its diameter, you always get the same number, about $3.14159\ldots$ This constant is called **pi**, written π. So $\frac{C}{d} = \pi$, which rearranges to the circumference rule.

Circumference of a circle.

$$C = \pi d \text{ or, since } d = 2r, C = 2\pi r.$$

Use the π key on your scientific calculator for accurate work; the approximation $\pi \approx 3.14$ is fine for a quick estimate or a check. Always round **only at the end**, and keep the units.

Working backwards.

If you know the circumference you can find the diameter or radius by dividing:

$$d = \frac{C}{\pi}, r = \frac{C}{2\pi}.$$

Part-circles and composite shapes.

A **semicircle** is half a circle, so its curved edge is half the circumference, $\frac{\pi d}{2}$ (which equals πr). The distance **around** a composite shape is found by adding the straight edges to the curved (arc) parts — a wheel rolling one full turn travels one whole circumference along the ground.

Worked examples

Example 1 — scaffolded

A round dinner plate has a radius of 12 cm. Find its circumference, correct to two decimal places.

Working	Comment
Radius $r = 12$ cm, so use $C = 2\pi r$.	The radius is given, so the $2\pi r$ form is easiest.
$C = 2 \times \pi \times 12 = 24\pi$.	Leave it as 24π until the last step.
$C = 75.398\ldots \approx 75.40$ cm.	Use the π key, then round to 2 d.p.

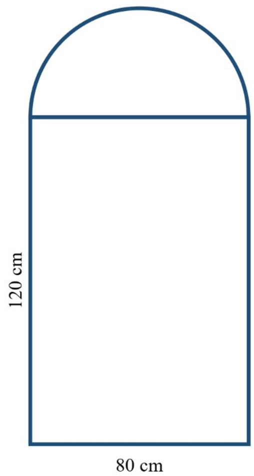
Example 2 — scaffolded

The circumference of a circular tabletop is 250 cm. Find its diameter, to the nearest centimetre.

Working	Comment
Use $C = \pi d$, so $d = \dfrac{C}{\pi}$.	Rearrange the circumference rule for d.
$d = \dfrac{250}{\pi} = 79.577\ldots$ cm.	Divide the circumference by π.
$d \approx 80$ cm.	Round to the nearest centimetre.

Example 3 — unscaffolded

A window is shaped like a rectangle 80 cm wide and 120 cm tall, with a semicircular top whose diameter is the 80 cm width. Find the total distance around the outside of the window, correct to two decimal places.



The outline is made of the bottom edge, the two straight sides, and the semicircular arc across the top.

Straight parts: bottom $= 80$ cm and two sides $= 2 \times 120 = 240$ cm.

Semicircular arc (diameter 80 cm): $\dfrac{\pi d}{2} = \dfrac{\pi \times 80}{2} = 40\pi = 125.66\ldots$ cm.

Total $= 80 + 240 + 125.66\ldots = 445.66$ cm.

$\therefore$ the distance around the window is about 445.66 cm.

Example 4 — unscaffolded

A bicycle wheel has a diameter of 68 cm. (a) How far does the bike travel in one complete turn of the wheel? (b) Approximately how many complete turns does the wheel make over 1 km ($100\,000$ cm)?

(a) One turn moves the bike forward by one circumference:

$$C = \pi d = \pi \times 68 = 213.628\ldots \approx 213.63\,\text{cm}\,(\approx 2.14\,\text{m}).$$

(b) Number of turns $= \dfrac{\text{total distance}}{\text{one circumference}} = \dfrac{100\,000}{213.628\ldots} = 468.1\ldots$

∴ each turn covers about 213.63 cm, and the wheel makes about 468 complete turns per kilometre.

Practice questions

Scaffolded

Q1. A circle has a radius of 9 cm. (a) State its diameter. (b) Find its circumference using $C = 2\pi r$, correct to two decimal places. (c) Find the circumference again using the approximation $\pi \approx 3.14$, and compare.

Q2. A circle has a diameter of 25 cm. (a) State its radius. (b) Find its circumference, correct to two decimal places. (c) Would 78 cm or 79 cm be the better whole-number answer?

Q3. A circular athletics track has a circumference of 400 m. (a) Find its diameter, correct to two decimal places. (b) Find its radius, correct to two decimal places. (c) Explain why you divide by π (not multiply) to get the diameter.

Q4. A semicircular rug has a straight edge (its diameter) of 1.4 m. (a) Find the length of the curved edge, correct to two decimal places. (b) Find the total distance around the rug (curved edge plus straight edge).

Q5. A car tyre has a diameter of 60 cm. (a) Find the circumference of the tyre, correct to two decimal places. (b) How far, in metres, does the car travel in 10 complete turns of the tyre?

Q6. A circular garden bed has a radius of 2.5 m. Edging strip is sold by the whole metre at $\$12$ per metre. (a) Find the circumference of the bed, correct to two decimal places. (b) How many whole metres of edging must be bought to go right around? (c) Find the total cost.

Unscaffolded

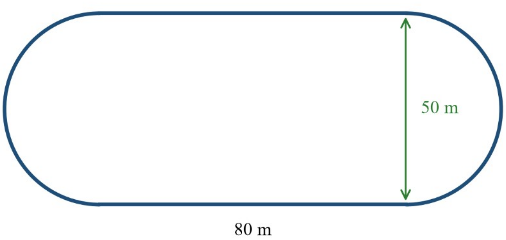
Q7. Find the circumference of a circle with radius 15 mm, correct to two decimal places.

Q8. A dinner plate has a circumference of 88 cm. Find its diameter, correct to two decimal places.

Q9. A bicycle wheel has a diameter of 70 cm. How far, in metres, does the bike travel in 500 complete turns of the wheel? Give your answer correct to two decimal places.

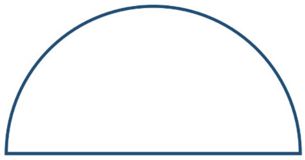
Q10. A circular window has a radius of 35 cm. A rubber seal is fitted right around the edge. Find the length of seal needed, to the nearest centimetre.

Q11. A sports field is shaped like a rectangle 80 m long and 50 m wide with a semicircle on each of the two shorter ends. Find the distance around the outside of the field, correct to two decimal places.



Q12. A pizza has a circumference of 100 cm. (a) Find its diameter, correct to two decimal places. (b) Find its radius, correct to two decimal places.

Q13. A semicircular protractor has a straight edge (its diameter) of 12 cm. Find the total distance around the protractor, correct to two decimal places.



Q14. A circular pond of radius 3 m is surrounded by a path 1.5 m wide. Find the circumference of the **outer** edge of the path, correct to two decimal places.

Q15. A roller has a diameter of 55 cm. How far, in metres, does it travel in 200 complete turns? Give your answer correct to two decimal places.

Q16. A circular swimming pool has a radius of 3.5 m. A fence is to be built right around it. Fencing is sold by the whole metre at $\$18$ per metre. (a) How many whole metres of fencing must be bought? (b) Find the total cost.

Q17. The minute hand of a clock is 15 cm long. (a) How far does the tip of the hand travel in one hour (one full turn), correct to two decimal places? (b) How far does the tip travel in 15 minutes?

Q18. A student calculates the circumference of a circle of radius 10 cm and writes 628 cm. Explain what has most likely gone wrong, and give the correct circumference.

Answers

Q1. (a) 18 cm. (b) 56.55 cm. (c) 56.52 cm — almost the same; 3.14 is a slightly rounded value of π. **Q2.** (a) 12.5 cm. (b) 78.54 cm. (c) 79 cm. **Q3.** (a) 127.32 m. (b) 63.66 m. (c) see solution. **Q4.** (a) 2.20 m. (b) 3.60 m. **Q5.** (a) 188.50 cm. (b) 18.85 m. **Q6.** (a) 15.71 m. (b) 16 m. (c) $\$192$. **Q7.** 94.25 mm. **Q8.** 28.01 cm. **Q9.** 1099.56 m. **Q10.** 220 cm. **Q11.** 317.08 m. **Q12.** (a) 31.83 cm. (b) 15.92 cm. **Q13.** 30.85 cm. **Q14.** 28.27 m. **Q15.** 345.58 m. **Q16.** (a) 22 m. (b) $\$396$. **Q17.** (a) 94.25 cm. (b) 23.56 cm. **Q18.** The radius was squared (or a decimal misplaced): $2 \times 3.14 \times 100 = 628$; circumference uses r, not r^2. Correct value 62.83 cm.

Fully-worked solutions

Q1. (a) $d = 2r = 2 \times 9 = 18$ cm. (b) $C = 2\pi r = 2\pi \times 9 = 18\pi = 56.548\ldots \approx 56.55$ cm. (c) With $\pi \approx 3.14$: $C \approx 18 \times 3.14 = 56.52$ cm. The two answers differ by only about 0.03 cm because 3.14 is π rounded to two decimal places.

Q2. (a) $r = \frac{d}{2} = \frac{25}{2} = 12.5$ cm. (b) $C = \pi d = \pi \times 25 = 25\pi = 78.539\ldots \approx 78.54$ cm. (c) 78.54 rounds to 79 cm, so 79 cm is the better whole-number value.

Q3. (a) $d = \frac{C}{\pi} = \frac{400}{\pi} = 127.323\ldots \approx 127.32$ m. (b) $r = \frac{d}{2} = \frac{127.32\ldots}{2} = 63.66$ m (or $r = \frac{C}{2\pi}$). (c) Because $C = \pi d$, the circumference is π times the diameter, so to undo that and recover the diameter you divide the circumference by π.

Q4. (a) Curved edge $= \frac{\pi d}{2} = \frac{\pi \times 1.4}{2} = 0.7\pi = 2.199\ldots \approx 2.20$ m. (b) Total = curved + straight $= 2.199\ldots + 1.4 = 3.60$ m.

Q5. (a) $C = \pi d = \pi \times 60 = 188.495\ldots \approx 188.50$ cm. (b) Ten turns $= 10 \times 188.495\ldots = 1884.95\ldots$ cm $= 18.85$ m.

Q6. (a) $C = 2\pi r = 2\pi \times 2.5 = 5\pi = 15.707\ldots \approx 15.71$ m. (b) You need at least 15.71 m, and edging is sold by the whole metre, so buy 16 m. (c) Cost $= 16 \times \$12 = \$192$.

Q7. $C = 2\pi r = 2\pi \times 15 = 30\pi = 94.247\ldots \approx 94.25$ mm.

Q8. $d = \frac{C}{\pi} = \frac{88}{\pi} = 28.011\ldots \approx 28.01$ cm.

Q9. One turn $= \pi d = \pi \times 70 = 219.911\ldots$ cm. In 500 turns: $500 \times 219.911\ldots = 109\,955.7\ldots$ cm $= 1099.56$ m.

Q10. $C = 2\pi r = 2\pi \times 35 = 70\pi = 219.911\ldots$ cm, which is 220 cm to the nearest centimetre.

Q11. The two long sides are straight: $2 \times 80 = 160$ m. The two semicircular ends have diameter 50 m; together they make one full circle, of circumference $\pi \times 50 = 157.079\ldots$ m. Total $= 160 + 157.079\ldots = 317.08$ m.

Q12. (a) $d = \frac{C}{\pi} = \frac{100}{\pi} = 31.830\ldots \approx 31.83$ cm. (b) $r = \frac{d}{2} = \frac{31.83\ldots}{2} = 15.92$ cm (or $r = \frac{C}{2\pi}$).

Q13. Curved edge $= \frac{\pi d}{2} = \frac{\pi \times 12}{2} = 6\pi = 18.849\ldots$ cm. Add the straight edge: $18.849\ldots + 12 = 30.85$ cm.

Q14. The path adds 1.5 m to the radius, so the outer radius is $3 + 1.5 = 4.5$ m. Outer circumference $= 2\pi r = 2\pi \times 4.5 = 9\pi = 28.274\ldots \approx 28.27$ m.

Q15. One turn $= \pi d = \pi \times 55 = 172.787\ldots$ cm. In 200 turns: $200 \times 172.787\ldots = 34\,557.5\ldots$ cm $= 345.58$ m.

Q16. $C = 2\pi r = 2\pi \times 3.5 = 7\pi = 21.991\ldots$ m. (a) Buy the next whole metre up: 22 m. (b) Cost $= 22 \times \$18 = \$396$.

Q17. The tip moves around a circle of radius 15 cm. (a) One full turn $= 2\pi r = 2\pi \times 15 = 30\pi = 94.247\ldots \approx 94.25$ cm. (b) 15 minutes is a quarter of an hour, so the tip travels $\frac{94.247\ldots}{4} = 23.56$ cm.

Q18. The correct circumference is $C = 2\pi r = 2\pi \times 10 = 20\pi = 62.83$ cm. The student's 628 cm is what you get from $2 \times 3.14 \times 100$ — that is, by using 100 in place of 10 with $\pi \approx 3.14$ — most likely the radius was squared ($10^2 = 100$). Circumference depends on r to the first power, not r^2, so the answer should be about 62.83 cm.

Theory

Area is the amount of flat surface a shape covers. It is measured in **square units** — square millimetres (mm^2), square centimetres (cm^2), square metres (m^2) and square kilometres (km^2) — because area is a length multiplied by a length. Working out area is one of the most common everyday measurement jobs: paint for a wall, turf for a lawn, tiles for a floor, or the size of a block of land.

Land: the hectare.

Large areas of land are measured in **hectares (ha)**. One hectare is the area of a square $100\,\mathrm{m}$ by $100\,\mathrm{m}$:

$$1\,\mathrm{ha} = 100 \times 100 = 10\,000\,\mathrm{m}^2.$$

To change square metres to hectares, divide by $10\,000$.

Area formulas.

For the standard shapes, use these formulas (they are on the Formula Sheet). In each, take the **perpendicular height** — the straight-line distance across the shape, at right angles to the base.

Shape	Formula
Rectangle	$A = l\,w$ (length × width)
Square	$A = s^2$
Triangle	$A = \frac{1}{2} b\,h$
Parallelogram	$A = b\,h$
Trapezium	$A = \frac{1}{2}(a+b)h$
Circle	$A = \pi r^2$

For a circle, r is the **radius**; if you are given the diameter, halve it first. Use the π key on your calculator and round only at the end.

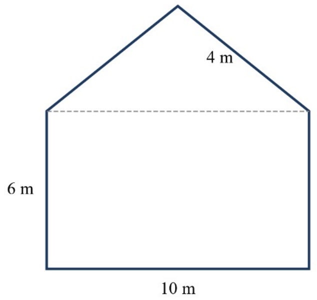
Composite shapes.

Many real shapes are made from these basic ones joined together. To find the area of a **composite shape**, split it into shapes you know, find each area, then **add** them (or **subtract** a piece that has been cut out, such as a pond in a courtyard).

Worked examples

Example 1 — scaffolded

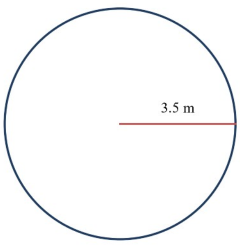
A garden bed is shaped like a house outline: a rectangle $10\,\mathrm{m}$ wide and $6\,\mathrm{m}$ tall with a triangular top of height $4\,\mathrm{m}$. Find its area.



Working	Comment
Split into a rectangle and a triangle.	A composite shape — do each part.
Rectangle: $A = l\,w = 10 \times 6 = 60\,\mathrm{m}^2$.	Length × width.
Triangle: $A = 1/2\,b\,h = 1/2 \times 10 \times 4 = 20\,\mathrm{m}^2$.	Base 10, perpendicular height 4.
Total $= 60 + 20 = 80\,\mathrm{m}^2$.	Add the two parts.

Example 2 — scaffolded

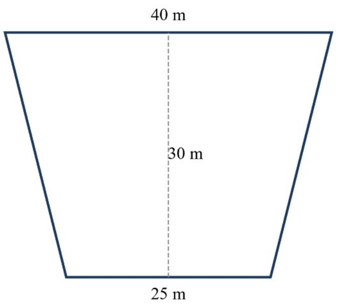
A circular vegetable bed has a radius of $3.5\,\mathrm{m}$. One bag of mulch covers $5\,\mathrm{m}^2$. Find the area of the bed and the number of bags needed.



Working	Comment
Area $= \pi\,r^2 = \pi \times 3.5^2$.	Use the radius.
$= \pi \times 12.25 = 38.48\,\mathrm{m}^2$ (2 d.p.).	Keep full precision on the calculator.
Bags $= 38.48 \div 5 = 7.7$.	Area divided by coverage per bag.
Round **up** to 8 bags.	A part-bag must be bought whole.

Example 3 — unscaffolded

A block of land is a trapezium. Its two parallel boundaries are $40\,\mathrm{m}$ and $25\,\mathrm{m}$ long, and the distance between them is $30\,\mathrm{m}$. Find the area of the block, in square metres and in hectares.



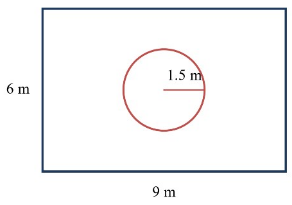
$$A = 1/2\left(a + b\right)h = 1/2\left(40 + 25\right) \times 30 = 1/2 \times 65 \times 30 = 975\,\mathrm{m}^2.$$

Converting to hectares: $975 \div 10\,000 = 0.0975\,\mathrm{ha}$.

$\therefore$ the block has an area of $975\,\mathrm{m}^2$, or $0.0975\,\mathrm{ha}$.

Example 4 — unscaffolded

A rectangular courtyard is $9\,\mathrm{m}$ by $6\,\mathrm{m}$. A circular pond of radius $1.5\,\mathrm{m}$ sits in the middle. The rest of the courtyard is to be paved. Find the area to be paved.



$$\text{courtyard} = 9 \times 6 = 54\,\mathrm{m}^2, \text{pond} = \pi \times 1.5^2 = 7.07\,\mathrm{m}^2.$$

Paved area = courtyard − pond $= 54 - 7.07 = 46.93\,\mathrm{m}^2$ (2 d.p.).

∴ about $46.93\,\mathrm{m}^2$ must be paved.

Practice questions

Scaffolded

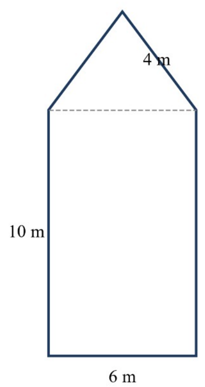
Q1. Find the area of each rectangle or square. (a) A rectangle $15\,\mathrm{m}$ long and $4\,\mathrm{m}$ wide. (b) A square of side $9\,\mathrm{cm}$. (c) A rectangle $2.5\,\mathrm{m}$ by $1.2\,\mathrm{m}$.

Q2. Find the area of each triangle. (a) Base $10\,\mathrm{cm}$, height $6\,\mathrm{cm}$. (b) Base $7\,\mathrm{m}$, height $4\,\mathrm{m}$. (c) A right-angled triangle with the two short sides $12\,\mathrm{mm}$ and $5\,\mathrm{mm}$.

Q3. Find the area of each shape. (a) A parallelogram with base $8\,\mathrm{m}$ and height $5\,\mathrm{m}$. (b) A trapezium with parallel sides $6\,\mathrm{m}$ and $10\,\mathrm{m}$, height $4\,\mathrm{m}$. (c) A trapezium with parallel sides $12\,\mathrm{m}$ and $8\,\mathrm{m}$, height $5\,\mathrm{m}$.

Q4. Find the area of each circle, correct to two decimal places. (a) Radius $4\,\mathrm{m}$. (b) Diameter $10\,\mathrm{cm}$. (c) Radius $2.5\,\mathrm{m}$.

Q5. A sign is shaped like a house outline: a rectangle $6\,\mathrm{m}$ wide and $10\,\mathrm{m}$ tall, with a triangular top of height $4\,\mathrm{m}$.



(a) Find the area of the rectangle.
(b) Find the area of the triangle.
(c) Find the total area of the sign.

Q6. A rectangular room is $5\,\mathrm{m}$ long and $4\,\mathrm{m}$ wide. (a) Find the floor area. (b) Carpet costs $\$45$ per square metre. Find the cost of carpeting the room. (c) Underlay comes in rolls that each cover $2\,\mathrm{m}^2$. How many rolls are needed?

Unscaffolded

Q7. A triangular garden bed has a base of $9\,\mathrm{m}$ and a perpendicular height of $6\,\mathrm{m}$. Find its area.

Q8. A field is a trapezium with parallel sides $30\,\mathrm{m}$ and $50\,\mathrm{m}$ that are $20\,\mathrm{m}$ apart. Find its area.

Q9. A circular swimming pool has a radius of $4\,\mathrm{m}$. A pool cover costs $\$18$ per square metre. Find the area of the pool (to 2 d.p.) and the cost of a cover.

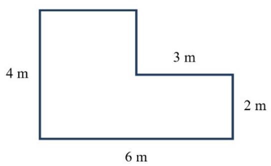
Q10. A rectangular lawn is $20\,\mathrm{m}$ by $15\,\mathrm{m}$. A rectangular garden bed $6\,\mathrm{m}$ by $4\,\mathrm{m}$ is dug out of it. Find the remaining area of lawn.

Q11. A rectangular paddock is $200\,\mathrm{m}$ long and $150\,\mathrm{m}$ wide. Find its area in square metres and in hectares.

Q12. A wall is $4\,\mathrm{m}$ long and $2.5\,\mathrm{m}$ high and is to be given **two** coats of paint. One litre of paint covers $12\,\mathrm{m}^2$. (a) Find the total area to be painted (two coats). (b) How many whole litres of paint must be bought?

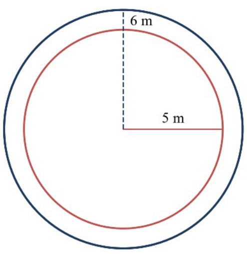
Q13. A backyard measuring $12\,\mathrm{m}$ by $8\,\mathrm{m}$ is to be turfed. Turf costs $\$9.50$ per square metre. Find the area and the total cost.

Q14. A room is L-shaped, as shown. Find its floor area.



Q15. (a) A rectangular room has an area of $24\,\mathrm{m}^2$ and a length of $6\,\mathrm{m}$. Find its width. (b) A triangle has an area of $20\,\mathrm{cm}^2$ and a base of $8\,\mathrm{cm}$. Find its perpendicular height.

Q16. A circular garden of radius $5\,\mathrm{m}$ is surrounded by a path, making the outer radius $6\,\mathrm{m}$.



Find the area of the **path** only, correct to two decimal places.

Q17. A flower bed is a trapezium with parallel sides $3\,\mathrm{m}$ and $5\,\mathrm{m}$, $2\,\mathrm{m}$ apart. Soil to fill it costs $\$6$ per square metre of bed. Find the area of the bed and the cost of the soil.

Q18. A rectangular house block is $25\,\mathrm{m}$ by $18\,\mathrm{m}$. The house on it covers a rectangle $12\,\mathrm{m}$ by $9\,\mathrm{m}$; the rest is lawn. (a) Find the area of the block. (b) Find the area covered by the house. (c) Find the area of lawn. (d) Turf for the lawn costs $\$8.50$ per square metre. Find the cost of turfing the lawn.

Answers

Q1. (a) $60\,\mathrm{m}^2$. (b) $81\,\mathrm{cm}^2$. (c) $3\,\mathrm{m}^2$. **Q2.** (a) $30\,\mathrm{cm}^2$. (b) $14\,\mathrm{m}^2$. (c) $30\,\mathrm{mm}^2$. **Q3.** (a) $40\,\mathrm{m}^2$. (b) $32\,\mathrm{m}^2$. (c) $50\,\mathrm{m}^2$. **Q4.** (a) $50.27\,\mathrm{m}^2$. (b) $78.54\,\mathrm{cm}^2$. (c) $19.63\,\mathrm{m}^2$. **Q5.** (a) $60\,\mathrm{m}^2$. (b) $12\,\mathrm{m}^2$. (c) $72\,\mathrm{m}^2$. **Q6.** (a) $20\,\mathrm{m}^2$. (b) $\$900$. (c) 10 rolls. **Q7.** $27\,\mathrm{m}^2$. **Q8.** $800\,\mathrm{m}^2$. **Q9.** $50.27\,\mathrm{m}^2$; $\$904.78$. **Q10.** $276\,\mathrm{m}^2$. **Q11.** $30\,000\,\mathrm{m}^2 = 3\,\mathrm{ha}$. **Q12.** (a) $20\,\mathrm{m}^2$. (b) 2 litres. **Q13.** $96\,\mathrm{m}^2$; $\$912$. **Q14.** $18\,\mathrm{m}^2$. **Q15.** (a) $4\,\mathrm{m}$. (b) $5\,\mathrm{cm}$. **Q16.** $34.56\,\mathrm{m}^2$. **Q17.** $8\,\mathrm{m}^2$; $\$48$. **Q18.** (a) $450\,\mathrm{m}^2$. (b) $108\,\mathrm{m}^2$. (c) $342\,\mathrm{m}^2$. (d) $\$2907$.

Fully-worked solutions

Q1. (a) $A = l\,w = 15 \times 4 = 60\,\mathrm{m}^2$. (b) $A = s^2 = 9^2 = 81\,\mathrm{cm}^2$. (c) $A = 2.5 \times 1.2 = 3\,\mathrm{m}^2$.

Q2. (a) $A = 1/2\,b\,h = 1/2 \times 10 \times 6 = 30\,\mathrm{cm}^2$. (b) $A = 1/2 \times 7 \times 4 = 14\,\mathrm{m}^2$. (c) The two short sides of a right-angled triangle are the base and height: $A = 1/2 \times 12 \times 5 = 30\,\mathrm{mm}^2$.

Q3. (a) Parallelogram $A = b\,h = 8 \times 5 = 40\,\mathrm{m}^2$. (b) Trapezium $A = 1/2(6+10) \times 4 = 1/2 \times 16 \times 4 = 32\,\mathrm{m}^2$. (c) $A = 1/2(12+8) \times 5 = 1/2 \times 20 \times 5 = 50\,\mathrm{m}^2$.

Q4. (a) $A = \pi r^2 = \pi \times 4^2 = 16\pi = 50.27\,\mathrm{m}^2$. (b) Diameter $10\,\mathrm{cm}$ gives radius $5\,\mathrm{cm}$: $A = \pi \times 5^2 = 25\pi = 78.54\,\mathrm{cm}^2$. (c) $A = \pi \times 2.5^2 = 6.25\pi = 19.63\,\mathrm{m}^2$.

Q5. (a) Rectangle $A = 6 \times 10 = 60\,\mathrm{m}^2$. (b) Triangle $A = 1/2 \times 6 \times 4 = 12\,\mathrm{m}^2$ (base 6, the width; height 4). (c) Total $= 60 + 12 = 72\,\mathrm{m}^2$.

Q6. (a) Floor area $= 5 \times 4 = 20\,\mathrm{m}^2$. (b) Cost $= 20 \times \$45 = \$900$. (c) Rolls $= 20 \div 2 = 10$ rolls.

Q7. $A = 1/2\,b\,h = 1/2 \times 9 \times 6 = 27\,\mathrm{m}^2$.

Q8. $A = 1/2(30+50) \times 20 = 1/2 \times 80 \times 20 = 800\,\mathrm{m}^2$.

Q9. Area $= \pi \times 4^2 = 16\pi = 50.27\,\mathrm{m}^2$ (2 d.p.). Using the full-precision area, the cover costs $16\pi \times \$18 = \$904.78$.

Q10. Lawn area $= 20 \times 15 - 6 \times 4 = 300 - 24 = 276\,\mathrm{m}^2$.

Q11. $A = 200 \times 150 = 30\,000\,\mathrm{m}^2$. In hectares, $30\,000 \div 10\,000 = 3\,\mathrm{ha}$.

Q12. (a) One coat covers $4 \times 2.5 = 10\,\mathrm{m}^2$; two coats cover $10 \times 2 = 20\,\mathrm{m}^2$. (b) Paint needed $= 20 \div 12 = 1.67\,\mathrm{L}$, so 2 whole litres must be bought (round up).

Q13. Area $= 12 \times 8 = 96\,\mathrm{m}^2$; cost $= 96 \times \$9.50 = \$912$.

Q14. Split the L into a full rectangle $6 \times 4 = 24\,\mathrm{m}^2$ and subtract the cut-out corner $3 \times 2 = 6\,\mathrm{m}^2$: $24 - 6 = 18\,\mathrm{m}^2$.

Q15. (a) $A = l\,w$, so $w = A \div l = 24 \div 6 = 4\,\mathrm{m}$. (b) $A = 1/2\,b\,h$, so $h = \dfrac{2A}{b} = \dfrac{2 \times 20}{8} = 5\,\mathrm{cm}$.

Q16. Path area = outer circle − inner circle $= \pi \times 6^2 - \pi \times 5^2 = 36\pi - 25\pi = 11\pi = 34.56\,\mathrm{m}^2$ (2 d.p.).

Q17. Bed area $= 1/2(3+5) \times 2 = 1/2 \times 8 \times 2 = 8\,\mathrm{m}^2$; soil cost $= 8 \times \$6 = \$48$.

Q18. (a) Block $= 25 \times 18 = 450\,\mathrm{m}^2$. (b) House $= 12 \times 9 = 108\,\mathrm{m}^2$. (c) Lawn $= 450 - 108 = 342\,\mathrm{m}^2$. (d) Turf cost $= 342 \times \$8.50 = \$2907$.

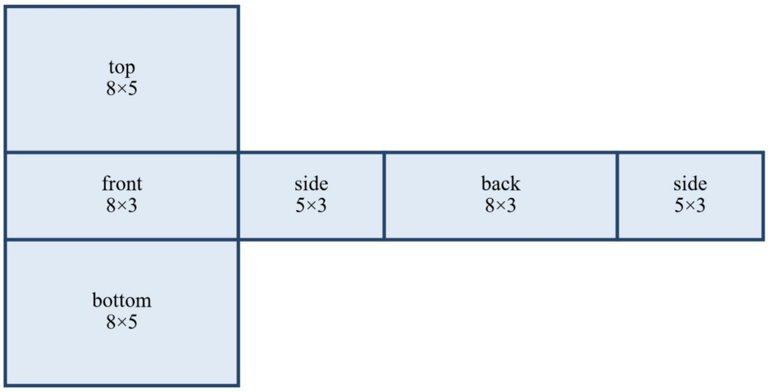
Theory

The **surface area** of a solid is the total area of all of its outside faces added together. Because it is an area, it is measured in **square units** — square centimetres (cm^2), square metres (m^2), and so on. Surface area tells you how much material covers the outside of an object: the cardboard in a box, the paint on a tank, the paper around a gift, or the sheet metal in a duct.

Using a net.

The easiest way to find a surface area is to imagine the solid **unfolded flat** into its **net**. The net shows every face as a flat shape. You find the area of each face, then **add them all up** — that total is the surface area.

net of a rectangular prism (8 × 5 × 3)



Cube.

A cube has 6 identical square faces. If the side length is s, each face has area s^2, so

$$\text{surface area of a cube} = 6s^2.$$

Rectangular prism.

A rectangular prism (a box) with length l, width w and height h has 6 rectangular faces in **three matching pairs**: top and bottom ($l \times w$), front and back ($l \times h$), and the two ends ($w \times h$). So

$$\text{surface area} = 2(lw + lh + wh).$$

For a box $8\,\text{cm} \times 5\,\text{cm} \times 3\,\text{cm}$: surface area $= 2(8 \times 5 + 8 \times 3 + 5 \times 3) = 2(40 + 24 + 15) = 2 \times 79 = 158\,\text{cm}^2$.

Triangular prism.

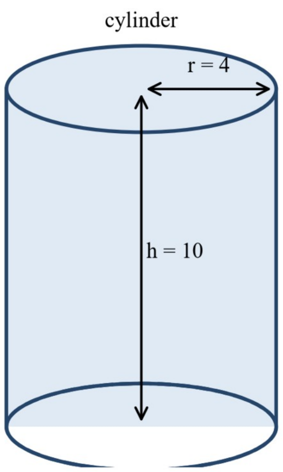
A triangular prism has **two identical triangular ends** and **three rectangles** joining them. Add the two triangles and the three rectangles:

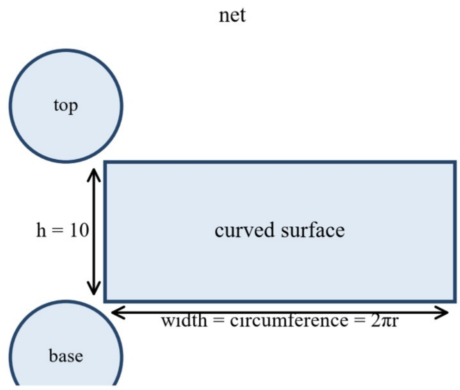
$$\text{surface area} = 2 \times (\text{area of triangle}) + (\text{the three rectangles}).$$

The three rectangles each have the prism's length as one side, so together their area is the **perimeter of the triangle** × the **length** of the prism.

Cylinder.

Unroll a cylinder and its net is **two circles** (the top and the base) plus **one rectangle** (the curved surface). The rectangle's height is the cylinder's height h, and its width is the distance around the circle — the **circumference**, $2\pi r$.





So the two circles give $2\pi r^2$ and the curved rectangle gives $2\pi r \times h$:

$$\text{surface area of a cylinder} = 2\pi r^2 + 2\pi r h.$$

Open solids.

If a solid is **open** — a box with no lid, a cup, an open tank — simply leave out the missing face. An open-top box, for example, has a base and four walls, but no top.

Always give a surface area in **square units**, and keep full calculator precision through the working, rounding only the final answer.

Worked examples

Example 1 — scaffolded

A closed cardboard carton measures $12\,\text{cm}$ long, $8\,\text{cm}$ wide and $6\,\text{cm}$ high. Find the area of card needed to make it (its surface area).

Working	Comment
The three different faces are $12 \times 8 = 96$, $12 \times 6 = 72$ and $8 \times 6 = 48\,\text{cm}^2$.	Length×width, length×height, width×height.
Each occurs twice (matching pairs).	A box has 3 pairs of equal faces.
Surface area $= 2(96 + 72 + 48) = 2 \times 216$.	Add the three, then double.
$= 432\,\text{cm}^2$.	Area, so square centimetres.

Example 2 — scaffolded

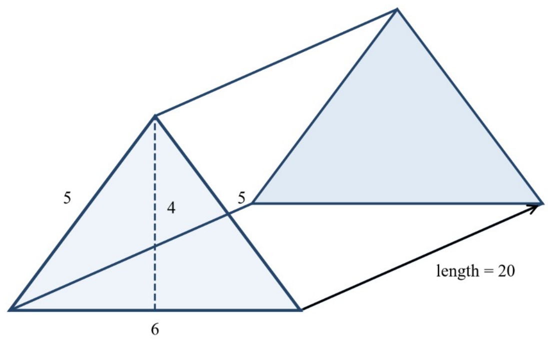
A closed cylindrical can has radius $4\,\text{cm}$ and height $10\,\text{cm}$. Find its surface area, correct to two decimal places.

Working	Comment
Two circles: $2\pi r^2 = 2 \times \pi \times 4^2 = 32\pi\,\text{cm}^2$.	Top and base together.
Curved surface: $2\pi r h = 2 \times \pi \times 4 \times 10 = 80\pi\,\text{cm}^2$.	Width $2\pi r$, height 10.
Surface area $= 32\pi + 80\pi = 112\pi$.	Keep π until the end.
$= 351.86\,\text{cm}^2$ (2 d.p.).	Now evaluate on the calculator.

Example 3 — unscaffolded

A chocolate box is a triangular prism $20\,\text{cm}$ long. Its triangular end has base $6\,\text{cm}$, perpendicular height $4\,\text{cm}$, and the two equal sloping sides are each $5\,\text{cm}$. Find the surface area of the box.

The two triangular ends each have area $\frac{1}{2}\times 6\times 4=12\,\text{cm}^2$, giving $2\times 12=24\,\text{cm}^2$.



The three rectangles all have length $20\,\text{cm}$; their other sides are the triangle's three sides 5, 5 and 6, a perimeter of $16\,\text{cm}$. Their total area is

$$16\times 20=320\,\text{cm}^2.$$

Surface area $=24+320=344\,\text{cm}^2$.

$\therefore$ the box has a surface area of $344\,\text{cm}^2$.

Example 4 — unscaffolded

A sealed cylindrical water tank has radius $2\,\text{m}$ and height $3\,\text{m}$ and is to be painted all over. One litre of paint covers $10\,\text{m}^2$. How many whole litres must be bought?

Surface area $=2\pi r^2+2\pi r h=2\pi(2)^2+2\pi(2)(3)=8\pi+12\pi=20\pi\,\text{m}^2$.

Evaluating, $20\pi=62.83\,\text{m}^2$ (2 d.p.). At $10\,\text{m}^2$ per litre this needs $62.83\div 10=6.28$ litres, and paint must be bought in whole litres, so round **up**.

$\therefore$ 7 litres of paint must be bought.

Practice questions

Scaffolded

Q1. A wooden cube has side length $5\,\text{cm}$. (a) Find the area of one face. (b) How many faces does a cube have? (c) Find the total surface area.

Q2. A box measures $10\,\text{cm}\times 6\,\text{cm}\times 4\,\text{cm}$. (a) Find the area of each of the three different faces. (b) Add these three areas. (c) Hence find the surface area of the box.

Q3. A closed cylinder has radius $3\,\text{cm}$ and height $8\,\text{cm}$. Give each answer to two decimal places. (a) Find the area of the two circular ends, $2\pi r^2$. (b) Find the area of the curved surface, $2\pi r h$. (c) Find the total surface area.

Q4. A triangular prism is $12\,\text{cm}$ long. Its triangular end is right-angled with the two shorter sides $3\,\text{cm}$ and $4\,\text{cm}$ and hypotenuse $5\,\text{cm}$. (a) Find the total area of the two triangular ends. (b) Find the total area of the three rectangles. (c) Find the surface area.

Q5. An open-top storage box (no lid) is $20\,\text{cm}$ long, $15\,\text{cm}$ wide and $10\,\text{cm}$ high. (a) Find the area of the base. (b) Find the total area of the four walls. (c) Find the surface area of the open box.

Q6. A gift box measures $25\,\text{cm}\times 18\,\text{cm}\times 8\,\text{cm}$. (a) Find the surface area of the box. (b) Allowing $10\,\%$ extra wrapping paper for overlaps, how much paper is needed? (c) A sheet of paper is $50\,\text{cm}\times 40\,\text{cm}$. Is one sheet enough?

Unscaffolded

Q7. Find the surface area of a cube of side $9\,\mathrm{cm}$.

Q8. A shipping carton is $1.2\,\mathrm{m}$ long, $0.8\,\mathrm{m}$ wide and $0.5\,\mathrm{m}$ high. Find its surface area in square metres.

Q9. A closed tin has radius $3.5\,\mathrm{cm}$ and height $12\,\mathrm{cm}$. Find its surface area, correct to two decimal places.

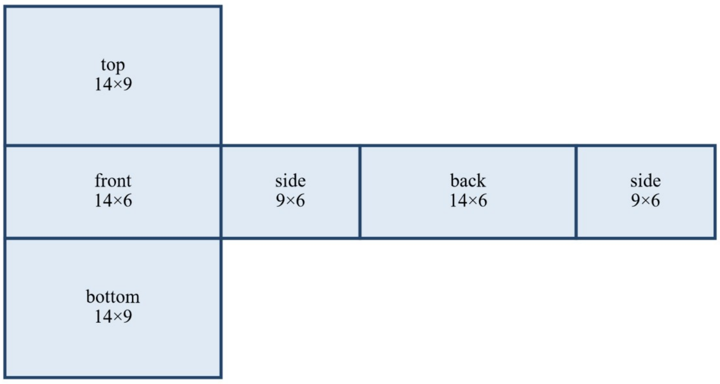
Q10. A sealed cylindrical tank has radius $1\,\mathrm{m}$ and height $2.5\,\mathrm{m}$ and is painted all over. Paint covers $6\,\mathrm{m}^2$ per litre. How many whole litres are needed?

Q11. A ramp is a triangular prism $15\,\mathrm{cm}$ long. Its triangular end is right-angled with shorter sides $6\,\mathrm{cm}$ and $8\,\mathrm{cm}$ and hypotenuse $10\,\mathrm{cm}$. Find the surface area of the prism.

Q12. An open cylindrical pot has no lid, with radius $5\,\mathrm{cm}$ and height $14\,\mathrm{cm}$. Find the outside area of its base plus its curved surface, correct to two decimal places.

Q13. The net below is for a rectangular box. Find its surface area.

net — box 14 cm × 9 cm × 6 cm



Q14. Box A is a cube of side $10\,\mathrm{cm}$. Box B measures $20\,\mathrm{cm} \times 10\,\mathrm{cm} \times 5\,\mathrm{cm}$. (a) Show that both boxes have the same volume. (b) Find the surface area of each box. (c) Which box uses less cardboard, and by how much?

Q15. A closed steel tank is a rectangular prism $2\,\mathrm{m} \times 1.5\,\mathrm{m} \times 1\,\mathrm{m}$. Steel sheet costs $\$18$ per square metre. (a) Find the surface area of the tank. (b) Find the cost of the steel.

Q16. A paper label wraps around the **curved surface only** of a can of radius $4\,\mathrm{cm}$ and height $13\,\mathrm{cm}$. (a) Find the area of the label, correct to two decimal places. (b) The label is printed $5\,\%$ larger so it can overlap. Find the printed area.

Q17. Show that a cube of side $10\,\mathrm{cm}$ has a surface area of $600\,\mathrm{cm}^2$.

Q18. A cube has a surface area of $96\,\mathrm{cm}^2$. Find the length of one edge.

Answers

Q1. (a) $25\,\text{cm}^2$. (b) 6. (c) $150\,\text{cm}^2$. **Q2.** (a) 60, 40, $24\,\text{cm}^2$. (b) $124\,\text{cm}^2$. (c) $248\,\text{cm}^2$. **Q3.** (a) $56.55\,\text{cm}^2$. (b) $150.80\,\text{cm}^2$. (c) $207.35\,\text{cm}^2$. **Q4.** (a) $12\,\text{cm}^2$. (b) $144\,\text{cm}^2$. (c) $156\,\text{cm}^2$. **Q5.** (a) $300\,\text{cm}^2$. (b) $700\,\text{cm}^2$. (c) $1000\,\text{cm}^2$. **Q6.** (a) $1588\,\text{cm}^2$. (b) $1746.8\,\text{cm}^2$. (c) Yes — a 50×40 sheet is $2000\,\text{cm}^2 > 1746.8\,\text{cm}^2$. **Q7.** $486\,\text{cm}^2$. **Q8.** $3.92\,\text{m}^2$. **Q9.** $340.86\,\text{cm}^2$. **Q10.** $7\pi = 21.99\,\text{m}^2$, so 4 litres. **Q11.** $408\,\text{cm}^2$. **Q12.** $518.36\,\text{cm}^2$. **Q13.** $528\,\text{cm}^2$. **Q14.** (a) both $1000\,\text{cm}^3$. (b) A $600\,\text{cm}^2$, B $700\,\text{cm}^2$. (c) Box A (the cube), by $100\,\text{cm}^2$. **Q15.** (a) $13\,\text{m}^2$. (b) $\$234$. **Q16.** (a) $326.73\,\text{cm}^2$. (b) $343.06\,\text{cm}^2$. **Q17.** $6 \times 10^2 = 600\,\text{cm}^2$. **Q18.** $4\,\text{cm}$.

Fully-worked solutions

Q1. (a) One face is a square of side 5: area $5 \times 5 = 25\,\text{cm}^2$. (b) A cube has 6 faces. (c) Surface area $= 6 \times 25 = 150\,\text{cm}^2$.

Q2. (a) The three different faces are $10 \times 6 = 60$, $10 \times 4 = 40$ and $6 \times 4 = 24\,\text{cm}^2$. (b) $60 + 40 + 24 = 124\,\text{cm}^2$. (c) Each face occurs twice, so surface area $= 2 \times 124 = 248\,\text{cm}^2$.

Q3. (a) Two circles: $2\pi r^2 = 2 \times \pi \times 3^2 = 18\pi = 56.55\,\text{cm}^2$. (b) Curved surface: $2\pi r h = 2 \times \pi \times 3 \times 8 = 48\pi = 150.80\,\text{cm}^2$. (c) Total $= 18\pi + 48\pi = 66\pi = 207.35\,\text{cm}^2$.

Q4. (a) Each triangle has area $\frac{1}{2} \times 3 \times 4 = 6\,\text{cm}^2$; two ends give $12\,\text{cm}^2$. (b) The three rectangles are $3 \times 12 = 36$, $4 \times 12 = 48$ and $5 \times 12 = 60\,\text{cm}^2$, a total of $144\,\text{cm}^2$. (c) Surface area $= 12 + 144 = 156\,\text{cm}^2$.

Q5. (a) Base $= 20 \times 15 = 300\,\text{cm}^2$. (b) The four walls are two of $20 \times 10 = 200$ and two of $15 \times 10 = 150$: $2 \times 200 + 2 \times 150 = 400 + 300 = 700\,\text{cm}^2$. (c) With no top, surface area $= 300 + 700 = 1000\,\text{cm}^2$.

Q6. (a) Surface area $= 2(25 \times 18 + 25 \times 8 + 18 \times 8) = 2(450 + 200 + 144) = 2 \times 794 = 1588\,\text{cm}^2$. (b) 10% extra: $1588 \times 1.1 = 1746.8\,\text{cm}^2$. (c) A $50\,\text{cm} \times 40\,\text{cm}$ sheet has area $2000\,\text{cm}^2$, and $2000 > 1746.8$, so one sheet is enough.

Q7. Surface area $= 6s^2 = 6 \times 9^2 = 6 \times 81 = 486\,\text{cm}^2$.

Q8. Surface area $= 2(1.2 \times 0.8 + 1.2 \times 0.5 + 0.8 \times 0.5) = 2(0.96 + 0.6 + 0.4) = 2 \times 1.96 = 3.92\,\text{m}^2$.

Q9. $2\pi r^2 = 2\pi(3.5)^2 = 24.5\pi$ and $2\pi r h = 2\pi(3.5)(12) = 84\pi$. Total $= 108.5\pi = 340.86\,\text{cm}^2$ (2 d.p.).

Q10. Surface area $= 2\pi(1)^2 + 2\pi(1)(2.5) = 2\pi + 5\pi = 7\pi = 21.99\,\text{m}^2$. Litres $= 21.99 \div 6 = 3.66$, and paint is bought whole, so round up to 4 litres.

Q11. Two triangular ends: $2 \times \frac{1}{2} \times 6 \times 8 = 48\,\text{cm}^2$. Three rectangles: perimeter $6 + 8 + 10 = 24$, times length 15, gives $24 \times 15 = 360\,\text{cm}^2$. Surface area $= 48 + 360 = 408\,\text{cm}^2$.

Q12. Open pot (no top): base $\pi r^2 = \pi(5)^2 = 25\pi$; curved surface $2\pi r h = 2\pi(5)(14) = 140\pi$. Total $= 25\pi + 140\pi = 165\pi = 518.36\,\text{cm}^2$ (2 d.p.).

Q13. The net folds into a $14 \times 9 \times 6$ box, so surface area $= 2(14 \times 9 + 14 \times 6 + 9 \times 6) = 2(126 + 84 + 54) = 2 \times 264 = 528\,\text{cm}^2$.

Q14. (a) Box A volume $= 10^3 = 1000\,\text{cm}^3$; box B volume $= 20 \times 10 \times 5 = 1000\,\text{cm}^3$; equal. (b) A: $6 \times 10^2 = 600\,\text{cm}^2$. B: $2(20 \times 10 + 20 \times 5 + 10 \times 5) = 2(200 + 100 + 50) = 700\,\text{cm}^2$. (c) Box A uses less cardboard, by $700 - 600 = 100\,\text{cm}^2$ — the cube is the more efficient shape.

Q15. (a) Surface area $= 2(2 \times 1.5 + 2 \times 1 + 1.5 \times 1) = 2(3 + 2 + 1.5) = 2 \times 6.5 = 13\,\text{m}^2$. (b) Cost $= 13 \times \$18 = \$234$.

Q16. (a) The label is the curved surface: $2\pi r h = 2\pi(4)(13) = 104\pi = 326.73\,\text{cm}^2$ (2 d.p.). (b) 5% larger, carrying full precision: $104\pi \times 1.05 = 343.06\,\text{cm}^2$.

Q17. A cube has 6 equal square faces: surface area $= 6 \times s^2 = 6 \times 10^2 = 6 \times 100 = 600\,\text{cm}^2$, as required.

Q18. A cube's surface area is $6s^2$, so $6s^2 = 96$, giving $s^2 = 96 \div 6 = 16$ and $s = \sqrt{16} = 4\,\text{cm}$.

Theory

Volume is the amount of space a solid object takes up. It is measured in **cubic units**: a cube $1\,\mathrm{cm}$ along each edge has a volume of 1 **cubic centimetre** ($1\,\mathrm{cm}^3$), and a cube $1\,\mathrm{m}$ along each edge has a volume of 1 **cubic metre** ($1\,\mathrm{m}^3$). Because a cubic measure is a length multiplied by a length multiplied by a length, its unit always carries a small 3: cm^3, m^3.

Volume and capacity.

Capacity is the amount a container *holds*, usually measured in millilitres (mL) and litres (L). Volume and capacity are linked by these conversions:

- $1\,\mathrm{cm}^3 = 1\,\mathrm{mL}$
- $1000\,\mathrm{cm}^3 = 1\,\mathrm{L}$
- $1\,\mathrm{m}^3 = 1000\,\mathrm{L}$

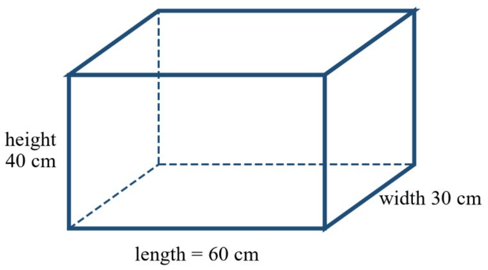
So a shape's volume in cubic centimetres is also its capacity in millilitres — a very useful shortcut.

Volume of a prism.

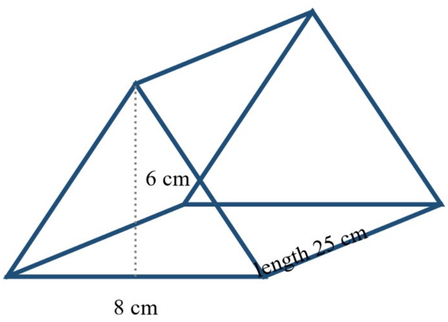
A **prism** is a solid with the same cross-section all the way along its length (a rectangular box, a triangular prism). For **every** prism,

$$\text{volume} = \text{area of cross-section} \times \text{length (or height)}.$$

- **Rectangular prism** (a box): the cross-section is a rectangle, so
$$V = l \times w \times h.$$



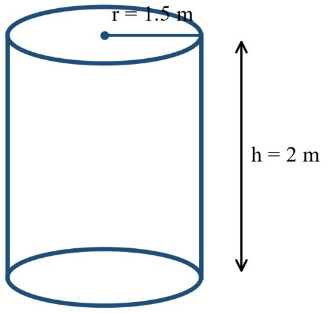
- **Triangular prism**: the cross-section is a triangle of area $1/2\,b\,h$, so
$$V = 1/2\,b\,h \times \text{length}.$$



Volume of a cylinder.

A **cylinder** is not a prism (its surface is curved), but its volume is found the same way: its cross-section is a circle of area πr^2, so

$$V = \pi r^2 h.$$



Working tips. Always use the **same unit** for every length before multiplying (convert first if they differ). Keep the full calculator value through the working and round only the final answer. A quick model: a box $20\,\text{cm} \times 15\,\text{cm} \times 10\,\text{cm}$ has $V = 3000\,\text{cm}^3 = 3\,\text{L}$.

Worked examples

Example 1 — scaffolded

A fish tank is a rectangular prism $60\,\text{cm}$ long, $30\,\text{cm}$ wide and $40\,\text{cm}$ high. Find its volume in cubic centimetres and its capacity in litres.

Working	Comment
$V = l \times w \times h = 60 \times 30 \times 40.$	Volume of a rectangular prism.
$V = 72\,000\,\text{cm}^3.$	Multiply the three lengths.
Capacity $= 72\,000 \div 1000 = 72\,\text{L}.$	$1000\,\text{cm}^3 = 1\,\text{L}.$

Example 2 — scaffolded

A chocolate bar is a triangular prism. Its triangular cross-section has base $8\,\text{cm}$ and height $6\,\text{cm}$, and the bar is $25\,\text{cm}$ long. Find its volume.

Working	Comment
Area of triangle $= 1/2\,b\,h = 1/2 \times 8 \times 6 = 24\,\text{cm}^2.$	Cross-section area first.
$V = \text{area} \times \text{length} = 24 \times 25.$	Volume of a prism.
$V = 600\,\text{cm}^3\,(= 600\,\text{mL} = 0.6\,\text{L}).$	$1\,\text{cm}^3 = 1\,\text{mL}.$

Example 3 — unscaffolded

A cylindrical rainwater tank has radius $1.5\,\text{m}$ and height $2\,\text{m}$. Find its volume, correct to two decimal places, and its capacity in litres.

$$V = \pi r^2 h = \pi \times 1.5^2 \times 2 = \pi \times 2.25 \times 2 = 4.5\pi.$$

$$V = 14.1372\ldots \approx 14.14\,\text{m}^3.$$

Capacity $= 14.1372\ldots \times 1000 \approx 14\,137\,\text{L}$ (since $1\,\text{m}^3 = 1000\,\text{L}$).

$\therefore$ the tank's volume is about $14.14\,\text{m}^3$, holding about $14\,137\,\text{L}$.

Example 4 — unscaffolded

A concrete slab is a rectangular prism $5\,\mathrm{m}$ long, $3\,\mathrm{m}$ wide and $0.1\,\mathrm{m}$ thick. Find its volume, and the cost of the concrete at $\$180$ per cubic metre.

$$V = l \times w \times h = 5 \times 3 \times 0.1 = 1.5\,\mathrm{m}^3.$$

$$\text{Cost} = 1.5 \times \$180 = \$270.$$

$\therefore$ the slab contains $1.5\,\mathrm{m}^3$ of concrete, costing $\$270$.

Practice questions

Scaffolded

Q1. A rectangular carton is $12\,\mathrm{cm} \times 8\,\mathrm{cm} \times 5\,\mathrm{cm}$. (a) Find its volume in cubic centimetres. (b) State its capacity in millilitres. (c) State its capacity in litres.

Q2. A storage box is a rectangular prism $1.2\,\mathrm{m}$ long, $0.8\,\mathrm{m}$ wide and $0.5\,\mathrm{m}$ high. (a) Find its volume in cubic metres. (b) State its capacity in litres.

Q3. A triangular prism has a triangular cross-section of base $10\,\mathrm{cm}$ and height $7\,\mathrm{cm}$, and length $20\,\mathrm{cm}$. (a) Find the area of the triangular cross-section. (b) Find the volume of the prism.

Q4. A cylinder has radius $5\,\mathrm{cm}$ and height $12\,\mathrm{cm}$. (a) Find the area of the circular base, correct to two decimal places. (b) Find the volume, correct to two decimal places.

Q5. A wooden cube has edges of length $6\,\mathrm{cm}$. (a) Find its volume. (b) State its capacity in millilitres.

Q6. A rectangular swimming pool is $8\,\mathrm{m}$ long, $4\,\mathrm{m}$ wide and $1.5\,\mathrm{m}$ deep. (a) Find the volume of the pool in cubic metres. (b) State how many litres of water it holds when full. (c) Find the volume of water when the pool is filled to a depth of only $1.2\,\mathrm{m}$.

Unscaffolded

Q7. A shipping carton measures $40\,\mathrm{cm} \times 30\,\mathrm{cm} \times 25\,\mathrm{cm}$. Find its volume in cubic centimetres and in litres.

Q8. A camping tent is a triangular prism. Its triangular cross-section has base $2.4\,\mathrm{m}$ and height $1.8\,\mathrm{m}$, and the tent is $3\,\mathrm{m}$ long. Find the volume of air inside the tent.

Q9. A cylindrical water tank has radius $0.9\,\mathrm{m}$ and height $1.6\,\mathrm{m}$. Find its volume correct to two decimal places, and its capacity to the nearest litre.

Q10. A juice box is a rectangular prism $4\,\mathrm{cm} \times 6\,\mathrm{cm} \times 10.5\,\mathrm{cm}$. (a) Find its capacity in millilitres. (b) The label says it holds $250\,\mathrm{mL}$. Is this possible? Explain.

Q11. A concrete path is $15\,\mathrm{m}$ long, $1.2\,\mathrm{m}$ wide and $0.1\,\mathrm{m}$ thick. (a) Find the volume of concrete needed. (b) Find the cost at $\$190$ per cubic metre.

Q12. A rectangular prism has a volume of $360\,\mathrm{cm}^3$ and a base measuring $12\,\mathrm{cm} \times 6\,\mathrm{cm}$. Find its height.

Q13. A cylindrical drum has radius $0.3\,\mathrm{m}$ and height $0.9\,\mathrm{m}$. Find its volume correct to two decimal places, and its capacity to the nearest litre.

Q14. A fish tank is $50\,\mathrm{cm} \times 25\,\mathrm{cm} \times 30\,\mathrm{cm}$. It is filled with water to $4\,\mathrm{cm}$ below the top. Find the volume of water, in litres.

Q15. A metal wedge is a triangular prism whose cross-section is a right-angled triangle with the two shorter sides $1.5\,\mathrm{m}$ and $2\,\mathrm{m}$. The prism is $6\,\mathrm{m}$ long. Find its volume.

Q16. Which holds more: a cube of side $10\,\mathrm{cm}$, or a cylinder of radius $6\,\mathrm{cm}$ and height $10\,\mathrm{cm}$? Justify your answer with a calculation (round the cylinder to two decimal places).

Q17. A cylindrical tank has radius $1.2\,\mathrm{m}$ and height $2.5\,\mathrm{m}$. (a) Find its volume correct to two decimal places. (b) State its capacity to the nearest litre. (c) Water is drawn off at $150\,\mathrm{L}$ per day. For how many full days will a full tank last?

Q18. A rectangular water tank must hold $2000\,\mathrm{L}$. Its base measures $1.25\,\mathrm{m} \times 1.0\,\mathrm{m}$. Find the height the tank must be.

Answers

Q1. (a) $480\,\text{cm}^3$. (b) $480\,\text{mL}$. (c) $0.48\,\text{L}$. **Q2.** (a) $0.48\,\text{m}^3$. (b) $480\,\text{L}$. **Q3.** (a) $35\,\text{cm}^2$. (b) $700\,\text{cm}^3$. **Q4.** (a) $78.54\,\text{cm}^2$. (b) $942.48\,\text{cm}^3$. **Q5.** (a) $216\,\text{cm}^3$. (b) $216\,\text{mL}$. **Q6.** (a) $48\,\text{m}^3$. (b) $48\,000\,\text{L}$. (c) $38.4\,\text{m}^3$ ($=38\,400\,\text{L}$). **Q7.** $30\,000\,\text{cm}^3 = 30\,\text{L}$. **Q8.** $6.48\,\text{m}^3$. **Q9.** $4.07\,\text{m}^3$; about $4072\,\text{L}$. **Q10.** (a) $252\,\text{mL}$. (b) Yes — its capacity is $252\,\text{mL}$, which is more than $250\,\text{mL}$. **Q11.** (a) $1.8\,\text{m}^3$. (b) $\$342$. **Q12.** $5\,\text{cm}$. **Q13.** $0.25\,\text{m}^3$; about $254\,\text{L}$. **Q14.** $32.5\,\text{L}$. **Q15.** $9\,\text{m}^3$. **Q16.** Cube $1000\,\text{cm}^3$, cylinder $1130.97\,\text{cm}^3$ — the cylinder holds more. **Q17.** (a) $11.31\,\text{m}^3$. (b) about $11\,310\,\text{L}$. (c) 75 full days. **Q18.** $1.6\,\text{m}$.

Fully-worked solutions

Q1. $V = 12 \times 8 \times 5 = 480\,\text{cm}^3$. (a) $480\,\text{cm}^3$. (b) Since $1\,\text{cm}^3 = 1\,\text{mL}$, this is $480\,\text{mL}$. (c) $480 \div 1000 = 0.48\,\text{L}$.

Q2. (a) $V = 1.2 \times 0.8 \times 0.5 = 0.48\,\text{m}^3$. (b) $1\,\text{m}^3 = 1000\,\text{L}$, so $0.48 \times 1000 = 480\,\text{L}$.

Q3. (a) Area $= 1/2 \times 10 \times 7 = 35\,\text{cm}^2$. (b) $V = 35 \times 20 = 700\,\text{cm}^3$.

Q4. (a) Base area $= \pi r^2 = \pi \times 5^2 = 25\pi = 78.54\,\text{cm}^2$ (2 d.p.). (b) $V = \pi r^2 h = 25\pi \times 12 = 300\pi = 942.48\,\text{cm}^3$ (2 d.p.).

Q5. (a) $V = 6^3 = 6 \times 6 \times 6 = 216\,\text{cm}^3$. (b) $216\,\text{cm}^3 = 216\,\text{mL}$.

Q6. (a) $V = 8 \times 4 \times 1.5 = 48\,\text{m}^3$. (b) $48 \times 1000 = 48\,000\,\text{L}$. (c) With depth $1.2\,\text{m}$:
$V = 8 \times 4 \times 1.2 = 38.4\,\text{m}^3 = 38\,400\,\text{L}$.

Q7. $V = 40 \times 30 \times 25 = 30\,000\,\text{cm}^3$. Dividing by 1000, that is $30\,\text{L}$.

Q8. Cross-section area $= 1/2 \times 2.4 \times 1.8 = 2.16\,\text{m}^2$. Volume $= 2.16 \times 3 = 6.48\,\text{m}^3$.

Q9. $V = \pi \times 0.9^2 \times 1.6 = \pi \times 0.81 \times 1.6 = 1.296\pi = 4.0715\ldots \approx 4.07\,\text{m}^3$. Capacity $= 4.0715\ldots \times 1000 = 4071.5\ldots \approx 4072\,\text{L}$.

Q10. (a) $V = 4 \times 6 \times 10.5 = 252\,\text{cm}^3 = 252\,\text{mL}$. (b) Yes: the box's capacity is $252\,\text{mL}$, which is greater than the $250\,\text{mL}$ of juice, so it fits (with a little room to spare).

Q11. (a) $V = 15 \times 1.2 \times 0.1 = 1.8\,\text{m}^3$. (b) Cost $= 1.8 \times \$190 = \$342$.

Q12. For a rectangular prism, $V = \text{base area} \times \text{height}$. Base area $= 12 \times 6 = 72\,\text{cm}^2$, so height $= 360 \div 72 = 5\,\text{cm}$.

Q13. $V = \pi \times 0.3^2 \times 0.9 = \pi \times 0.09 \times 0.9 = 0.081\pi = 0.2544\ldots \approx 0.25\,\text{m}^3$. Capacity $= 0.2544\ldots \times 1000 = 254.5\ldots \approx 254\,\text{L}$.

Q14. Water depth $= 30 - 4 = 26\,\text{cm}$. Volume of water $= 50 \times 25 \times 26 = 32\,500\,\text{cm}^3 = 32.5\,\text{L}$.

Q15. The right-angled triangular cross-section has area $1/2 \times 1.5 \times 2 = 1.5\,\text{m}^2$. Volume $= 1.5 \times 6 = 9\,\text{m}^3$.

Q16. Cube: $V = 10^3 = 1000\,\text{cm}^3$. Cylinder: $V = \pi \times 6^2 \times 10 = 360\pi = 1130.97\,\text{cm}^3$ (2 d.p.). Since $1130.97 > 1000$, the **cylinder** holds more (by about $131\,\text{cm}^3$).

Q17. (a) $V = \pi \times 1.2^2 \times 2.5 = \pi \times 1.44 \times 2.5 = 3.6\pi = 11.3097\ldots \approx 11.31\,\text{m}^3$. (b) Capacity $= 11.3097\ldots \times 1000 = 11\,309.7\ldots \approx 11\,310\,\text{L}$. (c) $11\,309.7 \div 150 = 75.4\ldots$, so the tank lasts 75 **full** days (the part-day is not a full day's supply).

Q18. $2000\,\text{L} = 2\,\text{m}^3$. Base area $= 1.25 \times 1.0 = 1.25\,\text{m}^2$, so height $= 2 \div 1.25 = 1.6\,\text{m}$.