

# Foundation Mathematics Units 1 & 2

## Chapter 10 Shape, angles and plans

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### Theory

Everything we measure in the world around us — a length of skirting board, the carpet for a floor, the water that fills a tank — is a measurement in **one, two or three dimensions**. Getting the number of dimensions right is the first step to choosing the correct method and, just as importantly, the correct **units**.

#### Point, line, surface, solid.

A **dimension** is a direction in which something has size.

- A **point** marks a position but has no length, width or height. It has **zero dimensions (0D)**.
- A **line** or **edge** has length only — you can move along it in one direction. It is **one-dimensional (1D)**.
- A **flat shape** or **surface** has length *and* width. It is **two-dimensional (2D)**, and the space it covers is its **area**.
- A **solid** object has length, width *and* height. It is **three-dimensional (3D)**, and the space it takes up is its **volume**.

Point (0D)



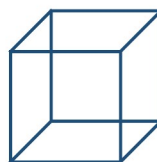
Line — length (1D)



Surface — area (2D)



Solid — volume (3D)



#### What we measure, and the units.

Each dimension has its own kind of measurement and its own units.

| Dimensions | What it is       | What we measure      | Typical units                                                        |
|------------|------------------|----------------------|----------------------------------------------------------------------|
| 1D         | length, distance | length               | mm, cm, m, km                                                        |
| 2D         | a flat region    | area                 | mm <sup>2</sup> , cm <sup>2</sup> , m <sup>2</sup> , km <sup>2</sup> |
| 3D         | a solid region   | volume (or capacity) | mm <sup>3</sup> , cm <sup>3</sup> , m <sup>3</sup> ; mL, L, kL       |

The little raised **2** in cm<sup>2</sup> and **3** in cm<sup>3</sup> are a reminder of the number of dimensions: an area is a length “times” a length (two dimensions), and a volume is a length “times” a length “times” a length (three dimensions).

#### Length, area and volume are not interchangeable.

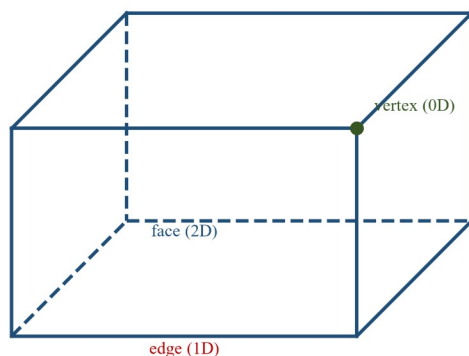
Choosing the right dimension is a real-world decision:

- the **fence** around a paddock, or the **skirting board** around a room, is a **length** (1D) — measured in metres;
- the **turf** for a lawn, or the **paint** for a wall, is an **area** (2D) — measured in square metres;
- the **water** to fill a pool, or the **soil** to fill a garden bed, is a **volume** (3D) — measured in cubic metres (or litres).

If someone orders “12 metres of tiles” for a floor they have used the wrong dimension — a floor is covered by **area**, so it should be about “12 square metres of tiles”.

#### Faces, edges and vertices.

A solid (3D) object is bounded by flat or curved **faces** (2D). In a solid with flat faces, the faces meet at **edges** (1D), which meet at **vertices**, or corners (0D) — so a box-shaped solid contains all the lower dimensions inside it. A solid bounded only by a curved surface, such as a ball, has no edges or vertices at all.



For example, a **rectangular prism** (a box shape) has **6** flat rectangular faces, **12** edges and **8** vertices. A **cube** is a special rectangular prism whose 6 faces are all squares. Some everyday solids have curved surfaces:

- a **cylinder** (a can) has 2 flat circular faces and 1 curved surface, and 0 vertices;
- a **cone** has 1 flat circular face, 1 curved surface and 1 vertex (the apex);
- a **sphere** (a ball) has a single curved surface, with no flat faces, edges or vertices.

## Worked examples

### Example 1 — scaffolded

For a rectangular classroom, decide whether each job needs a **length (1D)**, an **area (2D)** or a **volume (3D)** measurement, and give a sensible unit for each: (a) fitting skirting board around the base of the walls, (b) laying new carpet on the floor, (c) working out how much air the room holds for the air-conditioner.

| Working                                                                                                                                                      | Comment                                                    |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------|
| (a) Skirting board runs <i>along</i> the base of the walls — a single length around the room.<br>So (a) is a <b>length</b> , measured in <b>metres (m)</b> . | Following a line is <b>1D</b> .<br>1D → length units.      |
| (b) Carpet <i>covers</i> the flat floor — length across width.<br>So (b) is an <b>area</b> , measured in <b>square metres (m<sup>2</sup>)</b> .              | Covering a flat region is <b>2D</b> .<br>2D → area units.  |
| (c) Air <i>fills</i> the whole room — length, width and height.<br>So (c) is a <b>volume</b> , measured in <b>cubic metres (m<sup>3</sup>)</b> .             | Filling a solid space is <b>3D</b> .<br>3D → volume units. |

### Example 2 — scaffolded

A paperweight is shaped like a triangular pyramid: it has one triangular base and three triangular faces that rise to meet at a single apex. State how many **faces**, **edges** and **vertices** it has, the shape of its faces, and the dimension of a face, an edge and a vertex.

| Working                                                                  | Comment                                  |
|--------------------------------------------------------------------------|------------------------------------------|
| Faces: the 1 triangular base + 3 sloping triangular sides = 4 faces.     | Every face is a flat surface.            |
| Edges: 3 around the base + 3 rising from the base to the apex = 6 edges. | Lines where two faces meet.              |
| Vertices: 3 corners on the base + 1 apex = 4 vertices.                   | Points where edges meet.                 |
| Each face is a <b>triangle</b> (a 2D shape).                             | A face is a flat surface, so <b>2D</b> . |
| An <b>edge</b> is a line where two faces meet — <b>1D</b> .              | Length only.                             |
| A <b>vertex</b> is a corner point where edges meet — <b>0D</b> .         | A position, no size.                     |

### Example 3 — unscaffolded

A drink can is a cylinder. Describe its surfaces, and state which measurement — length, area or volume — you would use for (a) the height of the can, (b) the amount of paper in its label wrapped once around the side, and (c) the amount of drink it holds.

A cylinder has two flat circular faces (the top and the base) joined by one curved surface; it has no vertices.

- (a) The **height** is a single distance up the side, so it is a **length** (1D), measured in centimetres.
- (b) The label covers the curved surface — a flat region once unrolled — so it is an **area** (2D), measured in square centimetres.
- (c) The drink fills the inside of the can, so it is a **volume** or **capacity** (3D), measured in cubic centimetres or millilitres.

∴ height = length (1D), label = area (2D), drink = volume (3D).

### Example 4 — unscaffolded

A landscaper writes a quick quote: “Edging: 18 m. Lawn turf: 40 m. Garden soil: 3 m<sup>3</sup>.” One of these three units is wrong for what it measures. Identify the mistake and give the correct unit.

Edging runs *around* the garden as a line, so 18 m (a length, 1D) is correct. Lawn turf *covers* the flat ground, which is an **area** (2D), so it should be measured in **square metres**: the “40 m” should be “40 m<sup>2</sup>”. Garden soil *fills* the bed, a volume (3D), so 3 m<sup>3</sup> (a volume in cubic metres) is already correct.

∴ the turf entry is wrong: it should read 40 m<sup>2</sup>, not 40 m.

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## Practice questions

### Scaffolded

**Q1.** State the number of dimensions (0D, 1D, 2D or 3D) of each: (a) a single dot marking a spot on a map; (b) the length of a piece of string; (c) the surface of a tabletop.

**Q2.** For each job, state whether it needs a length, an area or a volume: (a) the timber for a picture frame around a photo; (b) the glass to fill the front of the same picture frame; (c) the water needed to fill a rainwater tank.

**Q3.** Choose a sensible unit (m, m<sup>2</sup> or m<sup>3</sup>) for each measurement: (a) the perimeter of a soccer field; (b) the playing surface of the soccer field; (c) the concrete poured for a solid footpath.

**Q4.** A storage box is shaped like a cube. State the number of dimensions of (a) one of its vertices, (b) one of its edges, and (c) one of its faces.

**Q5.** For a square-based pyramid: (a) name the shape of the base; (b) name the shape of each of the four side faces; (c) state the total number of faces.

**Q6.** Match each measurement to its dimension (1D, 2D or 3D): (a) 250 mm; (b) 18 cm<sup>3</sup>; (c) 4.5 m<sup>2</sup>.

### Unscaffolded

**Q7.** For each item, state the number of dimensions and the quantity measured (length, area or volume): (a) the distance a runner jogs; (b) the amount of wrapping paper to cover one face of a gift box; (c) the sand needed to fill a sandpit.

**Q8.** A heptagonal prism has 2 heptagonal faces with 7 sides each, and 7 rectangular faces. State its total number of (a) faces, (b) edges and (c) vertices.

**Q9.** Explain why area is measured in square units such as cm<sup>2</sup> while volume is measured in cubic units such as cm<sup>3</sup>, referring to the number of dimensions of each.

- Q10.** A council worker records three quantities for a rectangular park: the fencing around it, the grass covering it, and the mulch spread as a layer. State the correct unit (m, m<sup>2</sup> or m<sup>3</sup>) for each.
- Q11.** State how many flat faces, curved surfaces and vertices each solid has: (a) a cylinder; (b) a cone; (c) a sphere.
- Q12.** A tradesperson says a wall needs “15 metres of paint”. Explain why the unit is wrong and state the correct unit for measuring paint coverage.
- Q13.** Give a real-life example of something measured in each of the following, and state its dimension: (a) kilometres; (b) square metres; (c) litres.
- Q14.** A rectangular prism has faces in three matching pairs. If the box is 30 cm long, 20 cm wide and 10 cm high, state the dimensions of (a) the largest pair of faces and (b) the smallest pair of faces (you do **not** need to calculate any areas).
- Q15.** A designer lists what is needed to build an open-top rectangular garden bed: timber to edge the four top sides, lining to cover the four inside walls and the base, and soil to fill it. State the dimension (1D, 2D or 3D) of each of the three quantities.
- Q16.** State whether each statement is true or false, and correct any that are false: (a) an edge of a solid is one-dimensional; (b) a sphere has six faces; (c) a vertex has zero dimensions.
- Q17.** A square-based pyramid and a cube are compared. State which solid has more vertices, and by how many.
- Q18.** A packaging company describes a new box as having 6 faces, 12 edges and 8 vertices. (a) Name a solid it could be. (b) If instead a box had 5 faces, 9 edges and 6 vertices, name that solid.
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## Answers

**Q1.** (a) 0D. (b) 1D. (c) 2D. **Q2.** (a) length. (b) area. (c) volume. **Q3.** (a) m. (b)  $\text{m}^2$ . (c)  $\text{m}^3$ . **Q4.** (a) 0D. (b) 1D. (c) 2D. **Q5.** (a) a square. (b) a triangle. (c) 5. **Q6.** (a) 1D. (b) 3D. (c) 2D. **Q7.** (a) 1D, length. (b) 2D, area. (c) 3D, volume. **Q8.** (a) 9. (b) 21. (c) 14. **Q9.** Area covers a 2D region (length  $\times$  width  $\rightarrow$  two dimensions  $\rightarrow$  square units); volume fills a 3D region (length  $\times$  width  $\times$  height  $\rightarrow$  three dimensions  $\rightarrow$  cubic units). **Q10.** fencing m, grass  $\text{m}^2$ , mulch  $\text{m}^3$ . **Q11.** (a) 2 flat faces, 1 curved surface, 0 vertices. (b) 1 flat face, 1 curved surface, 1 vertex. (c) 0 flat faces, 1 curved surface, 0 vertices. **Q12.** Paint covers a 2D surface, so it is an area; the correct unit is square metres ( $\text{m}^2$ ), not metres. **Q13.** e.g. (a) distance between towns — 1D; (b) floor of a room — 2D; (c) petrol in a tank — 3D (accept any sensible examples). **Q14.** (a) 30 cm by 20 cm. (b) 20 cm by 10 cm. **Q15.** timber 1D, lining 2D, soil 3D. **Q16.** (a) true. (b) false — a sphere has no flat faces (one curved surface). (c) true. **Q17.** The cube has more vertices, by 3 (8 versus 5). **Q18.** (a) a rectangular prism (or cube). (b) a triangular prism.

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## Fully-worked solutions

**Q1.** (a) A dot marks a position only, with no length — **0D**. (b) A length runs in one direction — **1D**. (c) A tabletop is a flat surface with length and width — **2D**.

**Q2.** (a) The frame timber runs *along* the edge of the photo — a length, **1D**. (b) The glass *covers* the flat opening — an area, **2D**. (c) The water *fills* the tank — a volume, **3D**.

**Q3.** (a) A perimeter is a length, so **m**. (b) A playing surface is an area, so  **$\text{m}^2$** . (c) A poured solid footpath is a volume, so  **$\text{m}^3$** .

**Q4.** (a) A vertex is a corner point — it marks a position only, so **0D**. (b) An edge is a line where two faces meet — it has length only, so **1D**. (c) A face is a flat square region with length and width, so **2D**.

**Q5.** A square-based pyramid: (a) the base is a **square**; (b) each of the four sloping side faces is a **triangle**; (c) that is  $1 + 4 = 5$  faces in total.

**Q6.** (a) 250 mm is a length — **1D**. (b)  $18 \text{ cm}^3$  (cubic units) is a volume — **3D**. (c)  $4.5 \text{ m}^2$  (square units) is an area — **2D**.

**Q7.** (a) A jog distance is a length — **1D**. (b) Covering one face uses paper over a flat region — an area, **2D**. (c) Filling a sandpit uses sand throughout a space — a volume, **3D**.

**Q8.** A heptagonal prism has 2 heptagonal faces + 7 rectangular faces = 9 faces. Its edges: 7 around each heptagon ( $2 \times 7 = 14$ ) plus 7 joining the two heptagons = 21 edges. Its vertices: 7 on each heptagon = 14 vertices.

**Q9.** An area measures a two-dimensional region, found by multiplying a length by a length; two lengths multiplied give square units (e.g.  $\text{cm} \times \text{cm} = \text{cm}^2$ ). A volume measures a three-dimensional region, found by multiplying three lengths; three lengths multiplied give cubic units (e.g.  $\text{cm} \times \text{cm} \times \text{cm} = \text{cm}^3$ ). The index ( $^2$  or  $^3$ ) records the number of dimensions.

**Q10.** The fencing runs around the park as a line — a length, **m**. The grass covers the flat ground — an area,  **$\text{m}^2$** . The mulch is spread as a layer filling a thin volume over the ground — a volume,  **$\text{m}^3$** .

**Q11.** (a) A cylinder has 2 flat circular faces (top and base) and 1 curved surface, with 0 vertices. (b) A cone has 1 flat circular face and 1 curved surface, with 1 vertex (the apex). (c) A sphere has 0 flat faces and 1 curved surface, with 0 vertices.

**Q12.** Paint spreads over the flat surface of the wall, which is a two-dimensional region, so paint coverage is an **area**. Area is measured in **square metres ( $\text{m}^2$ )**, not metres, so “15 metres of paint” should be a number of square metres.

**Q13.** Sensible examples: (a) kilometres measure a distance, such as the length of a road — **1D**; (b) square metres measure a surface, such as the floor of a room — **2D**; (c) litres measure a volume/capacity, such as fuel in a tank — **3D**. (Any correct example of the right dimension is acceptable.)

**Q14.** The prism is  $30 \times 20 \times 10$  cm. Its faces come in three pairs, each pair using two of the three measurements. (a) The largest faces use the two largest measurements, 30 cm by 20 cm. (b) The smallest faces use the two smallest measurements, 20 cm by 10 cm.

**Q15.** The edging timber follows the four top edges as a line — **1D**. The lining covers the four walls and the base, which are flat surfaces — **2D**. The soil fills the inside of the bed — **3D**.

**Q16.** (a) **True** — an edge is a line where two faces meet, which is one-dimensional. (b) **False** — a sphere has no flat faces at all; it has a single curved surface. (c) **True** — a vertex is a single corner point, which has zero dimensions.

**Q17.** A cube has 8 vertices and a square-based pyramid has 5 vertices. The **cube** has more, by  $8 - 5 = 3$  vertices.

**Q18.** (a) A solid with 6 faces, 12 edges and 8 vertices is a **rectangular prism** (a cube is an acceptable special case). (b) A solid with 5 faces, 9 edges and 6 vertices is a **triangular prism**.

## Chapter 10 Shape, angles and plans — 10D Angles

### Theory

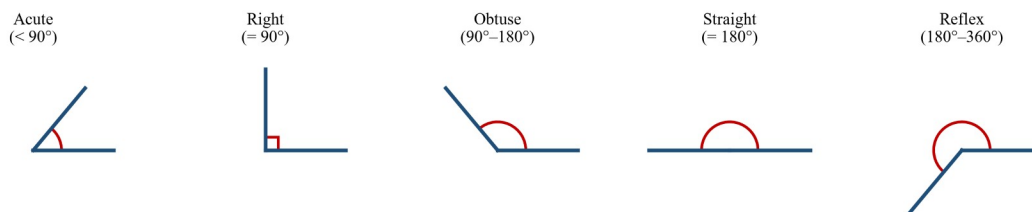
An **angle** measures the amount of **turn** between two rays (arms) that meet at a point called the **vertex**. Angles are measured in **degrees** ( $^{\circ}$ ); a full turn is  $360^{\circ}$ . Angles are everywhere in the built world — roof pitches, road junctions, ramps, the hands of a clock — so being able to name, measure and calculate them is a core practical skill.

### Naming angles.

An angle can be named by three points with the **vertex in the middle**: the angle  $\angle ABC$  has its vertex at  $B$ . A single lower-case letter or a number inside the angle is also common.

### Types of angle.

Angles are classified by size:



| Type       | Size                                              |
|------------|---------------------------------------------------|
| Acute      | between $0^{\circ}$ and $90^{\circ}$              |
| Right      | exactly $90^{\circ}$ (marked with a small square) |
| Obtuse     | between $90^{\circ}$ and $180^{\circ}$            |
| Straight   | exactly $180^{\circ}$ (a straight line)           |
| Reflex     | between $180^{\circ}$ and $360^{\circ}$           |
| Revolution | exactly $360^{\circ}$ (a full turn)               |

### Measuring with a protractor.

Place the protractor's centre on the vertex and the  $0^{\circ}$  line along one arm, then read where the second arm crosses the scale. Choose the scale (inner or outer) that starts at  $0^{\circ}$  on your first arm, and check your reading against the type: an acute angle must read less than  $90^{\circ}$ .

### Complementary and supplementary angles.

Two angles are **complementary** if they add to  $90^{\circ}$ , and **supplementary** if they add to  $180^{\circ}$ . So the complement of an angle  $a$  is  $90^{\circ} - a$ , and its supplement is  $180^{\circ} - a$ .

### Angle facts for finding unknown angles.

- **Angles on a straight line** add to  $180^{\circ}$ .
- **Angles at a point** (around a full turn) add to  $360^{\circ}$ .
- **Vertically opposite angles** (formed where two straight lines cross) are **equal**.
- **The angle sum of a triangle** is  $180^{\circ}$ .

To find an unknown angle, write the correct fact as an equation and solve it — for example, if two angles  $x$  and  $110^{\circ}$  lie on a straight line then  $x + 110 = 180$ , so  $x = 70^{\circ}$ .



## Worked examples

### Example 1 — scaffolded

Classify each angle by type:  $35^\circ$ ,  $90^\circ$ ,  $128^\circ$ ,  $180^\circ$ ,  $250^\circ$ . Then write the complement and the supplement of  $35^\circ$ .

| Working                                                                                                                                                                                       | Comment                                             |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------|
| $35^\circ$ is between $0^\circ$ and $90^\circ \rightarrow$ <b>acute</b> ; $90^\circ \rightarrow$ <b>right</b> ; $128^\circ$ is between $90^\circ$ and $180^\circ \rightarrow$ <b>obtuse</b> . | Compare each size with $90^\circ$ and $180^\circ$ . |
| $180^\circ \rightarrow$ <b>straight</b> ; $250^\circ$ is between $180^\circ$ and $360^\circ \rightarrow$ <b>reflex</b> .                                                                      | A reflex angle is more than a straight line.        |
| Complement of $35^\circ = 90 - 35 = 55^\circ$ .                                                                                                                                               | Complementary angles add to $90^\circ$ .            |
| Supplement of $35^\circ = 180 - 35 = 145^\circ$ .                                                                                                                                             | Supplementary angles add to $180^\circ$ .           |

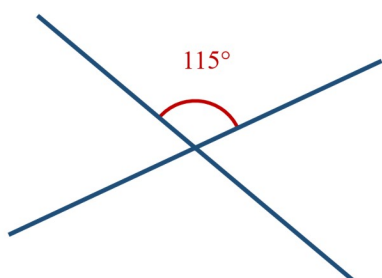
### Example 2 — scaffolded

Three angles  $x^\circ$ ,  $47^\circ$  and  $68^\circ$  sit together on a straight line. Find  $x$ .

| Working                                                              | Comment                                      |
|----------------------------------------------------------------------|----------------------------------------------|
| Angles on a straight line add to $180^\circ$ : $x + 47 + 68 = 180$ . | Write the straight-line fact as an equation. |
| $x + 115 = 180$ .                                                    | Add the known angles: $47 + 68 = 115$ .      |
| $x = 180 - 115 = 65$ .                                               | Subtract to make $x$ the subject.            |
| $\therefore x = 65^\circ$ (an acute angle — a sensible size).        | Check the answer's type is reasonable.       |

### Example 3 — unscaffolded

Two straight roads cross. One of the four angles formed is  $115^\circ$ . Find the sizes of the other three angles.



The angle **vertically opposite** the  $115^\circ$  angle is equal to it, so it is  $115^\circ$ . Each angle **next to** the  $115^\circ$  angle lies with it on a straight line, so

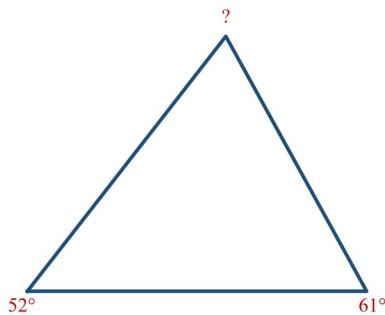
$$180 - 115 = 65^\circ.$$

There are two such angles, each  $65^\circ$ . Check:  $115 + 115 + 65 + 65 = 360^\circ$ , the angles at a point.

$\therefore$  the other three angles are  $115^\circ$ ,  $65^\circ$  and  $65^\circ$ .

### Example 4 — unscaffolded

A triangular garden bed has two corner angles of  $52^\circ$  and  $61^\circ$ . Find the third angle.



The angle sum of a triangle is  $180^\circ$ , so the third angle is

$$180 - 52 - 61 = 67^\circ.$$

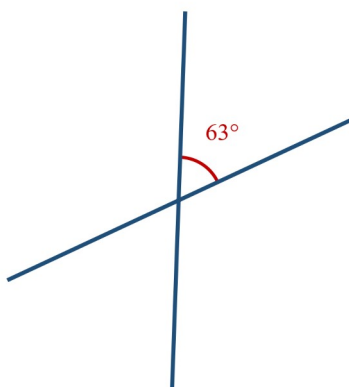
$\therefore$  the third angle is  $67^\circ$  (an acute angle).

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## Practice questions

### Scaffolded

- Q1.** Classify each angle by type. (a)  $72^\circ$ . (b)  $90^\circ$ . (c)  $155^\circ$ .
- Q2.** Find: (a) the complement of  $28^\circ$ ; (b) the supplement of  $28^\circ$ ; (c) the supplement of  $103^\circ$ .
- Q3.** Find the value of  $x$  in each case (angles on a straight line). (a)  $x^\circ$  and  $55^\circ$ . (b)  $x^\circ$ ,  $90^\circ$  and  $40^\circ$ . (c)  $x^\circ$  and  $2x^\circ$ .
- Q4.** Find the value of  $x$  (angles at a point, adding to  $360^\circ$ ). (a)  $x^\circ$ ,  $130^\circ$  and  $100^\circ$ . (b)  $x^\circ$ ,  $90^\circ$ ,  $90^\circ$  and  $45^\circ$ . (c) four equal angles  $x^\circ$ .
- Q5.** Two straight lines cross, forming an angle of  $63^\circ$ .



- (a) State the angle vertically opposite the  $63^\circ$  angle.
- (b) Find one of the angles next to the  $63^\circ$  angle.
- (c) List all four angles.
- Q6.** Find the third angle of a triangle whose other two angles are: (a)  $40^\circ$  and  $60^\circ$ ; (b)  $90^\circ$  and  $35^\circ$ ; (c) two angles of an equilateral triangle.

### Un scaffolded

- Q7.** An angle measures  $210^\circ$ . Classify it, and find the size of the other angle around the same vertex (the angle that completes a full turn).

- Q8.** A loading ramp rises at  $15^\circ$  to the horizontal ground. Find the angle between the ramp and the vertical wall it meets (the complement of  $15^\circ$ ).
- Q9.** Three roads meet at a single point. The angles between them are  $140^\circ$ ,  $95^\circ$  and  $x^\circ$ . Find  $x$ .
- Q10.** A steel brace meets a straight beam, forming two angles on the beam. One angle is  $30^\circ$  larger than the other. Find the two angles.
- Q11.** A roof truss is an isosceles triangle with an apex angle of  $40^\circ$ . The two base angles are equal. Find each base angle.
- Q12.** Two straight lines cross so that one angle is four times the angle next to it on the straight line. Find the sizes of these two angles.
- Q13.** A signpost bracket contains three marked angles of  $40^\circ$ ,  $125^\circ$  and  $90^\circ$ . Classify each one by type.
- Q14.** A right-angled corner of a tiled floor is split by a diagonal line into two angles. One of them is  $34^\circ$ . Find the other.
- Q15.** The three angles of a triangle are in the ratio  $1:2:3$ . Find the three angles, and state what type of triangle it is.
- Q16.** Four garden paths meet at a fountain, dividing the full turn into four angles. Three of them are  $100^\circ$ ,  $85^\circ$  and  $95^\circ$ . Find the fourth.
- Q17.** A triangular sail has angles of  $55^\circ$  and  $75^\circ$  at two corners. (a) Find the third angle. (b) One side at the third corner is extended in a straight line. Find the angle between the extension and the other side at that corner (the supplement of the third angle).
- Q18.** On a clock face the hour hand turns  $30^\circ$  each hour. (a) Find the angle between the hands at 4:00, and classify it. (b) Find the reflex angle between the hands at 4:00.
-

## Answers

**Q1.** (a) acute. (b) right. (c) obtuse. **Q2.** (a)  $62^\circ$ . (b)  $152^\circ$ . (c)  $77^\circ$ . **Q3.** (a)  $125^\circ$ . (b)  $50^\circ$ . (c)  $60^\circ$ . **Q4.** (a)  $130^\circ$ . (b)  $135^\circ$ . (c)  $90^\circ$ . **Q5.** (a)  $63^\circ$ . (b)  $117^\circ$ . (c)  $63^\circ$ ,  $63^\circ$ ,  $117^\circ$ ,  $117^\circ$ . **Q6.** (a)  $80^\circ$ . (b)  $55^\circ$ . (c)  $60^\circ$ . **Q7.** reflex; the other angle is  $150^\circ$ . **Q8.**  $75^\circ$ . **Q9.**  $125^\circ$ . **Q10.**  $75^\circ$  and  $105^\circ$ . **Q11.**  $70^\circ$  each. **Q12.**  $36^\circ$  and  $144^\circ$ . **Q13.**  $40^\circ$  acute,  $125^\circ$  obtuse,  $90^\circ$  right. **Q14.**  $56^\circ$ . **Q15.**  $30^\circ$ ,  $60^\circ$ ,  $90^\circ$ ; a right-angled triangle. **Q16.**  $80^\circ$ . **Q17.** (a)  $50^\circ$ . (b)  $130^\circ$ . **Q18.** (a)  $120^\circ$ , obtuse. (b)  $240^\circ$ .

---

## Fully-worked solutions

**Q1.** (a)  $72^\circ$  is between  $0^\circ$  and  $90^\circ$ , so it is **acute**. (b)  $90^\circ$  is exactly a right angle. (c)  $155^\circ$  is between  $90^\circ$  and  $180^\circ$ , so it is **obtuse**.

**Q2.** (a) Complement  $= 90 - 28 = 62^\circ$ . (b) Supplement  $= 180 - 28 = 152^\circ$ . (c) Supplement  $= 180 - 103 = 77^\circ$ .

**Q3.** Angles on a straight line add to  $180^\circ$ . (a)  $x + 55 = 180$ , so  $x = 125^\circ$ . (b)  $x + 90 + 40 = 180$ , so  $x + 130 = 180$  and  $x = 50^\circ$ . (c)  $x + 2x = 180$ , so  $3x = 180$  and  $x = 60^\circ$ .

**Q4.** Angles at a point add to  $360^\circ$ . (a)  $x + 130 + 100 = 360$ , so  $x + 230 = 360$  and  $x = 130^\circ$ . (b)  $x + 90 + 90 + 45 = 360$ , so  $x + 225 = 360$  and  $x = 135^\circ$ . (c)  $4x = 360$ , so  $x = 90^\circ$ .

**Q5.** (a) Vertically opposite angles are equal, so it is  $63^\circ$ . (b) An angle next to the  $63^\circ$  angle lies on a straight line with it:  $180 - 63 = 117^\circ$ . (c) The four angles are  $63^\circ$ ,  $63^\circ$  (vertically opposite pair) and  $117^\circ$ ,  $117^\circ$  (the other pair). Check:  $63 + 63 + 117 + 117 = 360^\circ$ .

**Q6.** The angle sum of a triangle is  $180^\circ$ . (a)  $180 - 40 - 60 = 80^\circ$ . (b)  $180 - 90 - 35 = 55^\circ$ . (c) An equilateral triangle has three equal angles, so each is  $180 \div 3 = 60^\circ$ .

**Q7.**  $210^\circ$  is between  $180^\circ$  and  $360^\circ$ , so it is a **reflex** angle. The two angles around the vertex complete a full turn, so the other is  $360 - 210 = 150^\circ$ .

**Q8.** The ramp and the vertical wall together make a right angle with the ground line, so the two angles are complementary:  $90 - 15 = 75^\circ$ .

**Q9.** The three angles meet at a point, so they add to  $360^\circ$ :  $140 + 95 + x = 360$ , giving  $235 + x = 360$  and  $x = 125^\circ$ .

**Q10.** Let the smaller angle be  $a$ . The two angles lie on the straight beam, so  $a + (a + 30) = 180$ . Then  $2a + 30 = 180$ , so  $2a = 150$  and  $a = 75$ . The angles are  $75^\circ$  and  $75 + 30 = 105^\circ$  (check:  $75 + 105 = 180^\circ$ ).

**Q11.** The three angles add to  $180^\circ$  and the two base angles are equal. Subtract the apex:  $180 - 40 = 140^\circ$  is shared between the two base angles, so each is  $140 \div 2 = 70^\circ$ .

**Q12.** The two angles lie on a straight line, so they add to  $180^\circ$ . Let the smaller be  $x$ ; the larger is  $4x$ . Then  $x + 4x = 180$ , so  $5x = 180$  and  $x = 36$ . The angles are  $36^\circ$  and  $4 \times 36 = 144^\circ$ .

**Q13.**  $40^\circ$  is between  $0^\circ$  and  $90^\circ \rightarrow$  **acute**;  $125^\circ$  is between  $90^\circ$  and  $180^\circ \rightarrow$  **obtuse**;  $90^\circ$  is a **right** angle.

**Q14.** The corner is a right angle ( $90^\circ$ ) split into two parts that must add to  $90^\circ$ , so the other part is  $90 - 34 = 56^\circ$ .

**Q15.** Let the angles be  $x$ ,  $2x$  and  $3x$ . Their sum is  $180^\circ$ :  $x + 2x + 3x = 180$ , so  $6x = 180$  and  $x = 30$ . The angles are  $30^\circ$ ,  $60^\circ$  and  $90^\circ$ . Because one angle is  $90^\circ$ , it is a **right-angled triangle**.

**Q16.** The four angles meet at a point, adding to  $360^\circ$ :  $100 + 85 + 95 + x = 360$ , so  $280 + x = 360$  and  $x = 80^\circ$ .

**Q17.** (a) Angle sum of a triangle:  $180 - 55 - 75 = 50^\circ$ . (b) The extension forms a straight line with the extended side, so the angle between the extension and the other side at that corner is supplementary to the  $50^\circ$  angle:  $180 - 50 = 130^\circ$ .

**Q18.** (a) From 12 to 4 is 4 hours, and the hour hand turns  $30^\circ$  per hour, so the angle is  $4 \times 30 = 120^\circ$ , which is **obtuse**. (b) The reflex angle completes the full turn:  $360 - 120 = 240^\circ$ .

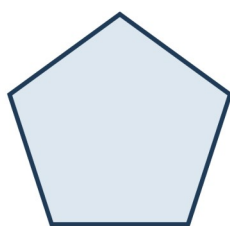
### Theory

Being able to name shapes correctly lets us describe objects clearly and choose the right formulas later for perimeter, area and volume. We name **flat (2D) shapes** by their sides and **solid (3D) objects** by their faces.

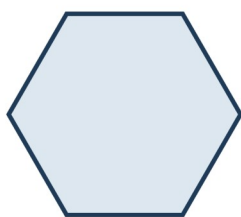
### Polygons.

A **polygon** is a closed 2D shape with straight sides. We name a polygon by its **number of sides** — which is always equal to its number of corners (**vertices**). A polygon is **regular** if all its sides and all its angles are equal (a stop sign is a regular octagon), and **irregular** otherwise.

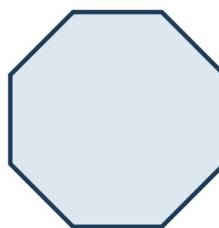
| Sides | Name          | Sides | Name      |
|-------|---------------|-------|-----------|
| 3     | triangle      | 8     | octagon   |
| 4     | quadrilateral | 9     | nonagon   |
| 5     | pentagon      | 10    | decagon   |
| 6     | hexagon       | 12    | dodecagon |
| 7     | heptagon      |       |           |



pentagon  
(5 sides)



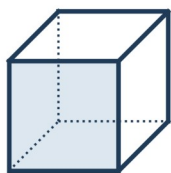
hexagon  
(6 sides)



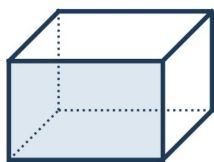
octagon  
(8 sides)

### Solids (3D objects).

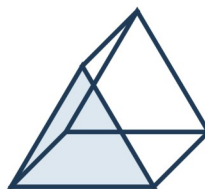
A **solid** is a three-dimensional object. Everyday objects are modelled by common solids: a shoebox is a **rectangular prism**, a die is a **cube**, a tin of food is a **cylinder**, an ice-cream cone is a **cone**, a ball is a **sphere**, and the roof of a tent is often a **triangular prism**.



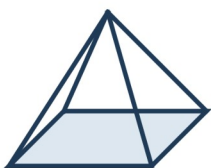
cube



rectangular prism



triangular prism



square pyramid



cylinder



cone

### Prisms and pyramids.

A **prism** has two identical **parallel ends** joined by rectangles; it is named after the shape of those ends (a triangular prism has triangular ends, a hexagonal prism has hexagonal ends). A **pyramid** has one **base** and triangular faces that meet at a single point (the **apex**); it is named after its base (a square pyramid has a square base).

### Faces, edges and vertices.

Solids with flat faces (**polyhedra**) are described by three counts:

- a **face** is a flat surface;
- an **edge** is a line where two faces meet;
- a **vertex** is a corner where edges meet.

For example, a **cube** has 6 faces, 12 edges and 8 vertices. For a prism with an  $n$ -sided end there are  $n + 2$  faces,  $3n$  edges and  $2n$  vertices; for a pyramid with an  $n$ -sided base there are  $n + 1$  faces,  $2n$  edges and  $n + 1$  vertices.

### Euler's rule (a handy check).

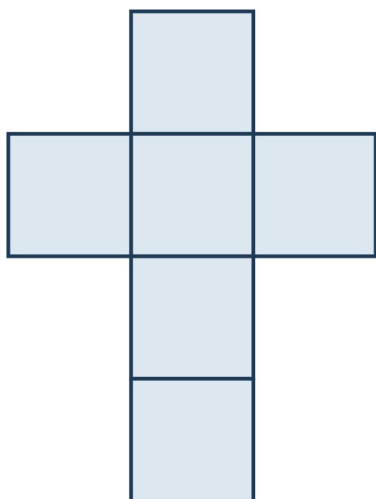
For any solid with flat faces, the three counts are linked by **Euler's rule**:

$$F + V - E = 2,$$

where  $F$ ,  $V$  and  $E$  are the numbers of faces, vertices and edges. If your counts do not satisfy this, you have miscounted. (Curved solids such as the cylinder, cone and sphere are not polyhedra, so Euler's rule does not apply to them.)

### Nets.

A **net** is a flat pattern that folds up to make a solid. The number of shapes in a net matches the number of faces of the solid — a cube's net is made of 6 squares, and a square pyramid's net is a square with a triangle on each side.



## Worked examples

### Example 1 — scaffolded

A café tiles its floor with shapes that each have 6 straight, equal sides and 6 equal angles. Name the shape, state how many vertices it has, and say whether it is regular.

Working

Comment

The shape has 6 sides, so it is a **hexagon**.

Name a polygon by its number of sides.

A polygon has as many vertices as sides, so it has 6 vertices.

Sides = vertices for any polygon.

All 6 sides and angles are equal, so it is a **regular hexagon**.

“Regular” means equal sides and angles.

### Example 2 — scaffolded

A chocolate box is shaped like a **triangular prism**. Count its faces, edges and vertices, and check your counts with Euler’s rule.

Working

Comment

Faces: 2 triangular ends + 3 rectangular sides = 5 faces.

Count the flat surfaces.

Edges: 3 on the front triangle, 3 on the back, 3 joining them = 9 edges.

Lines where faces meet.

Vertices: 3 corners on each triangular end = 6 vertices.

Corners where edges meet.

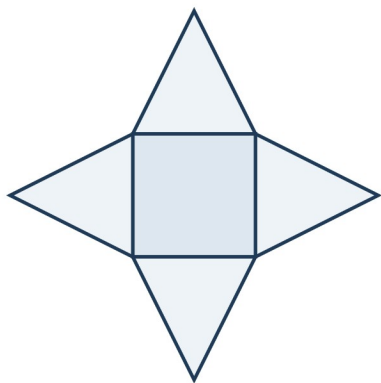
Check:  $F + V - E = 5 + 6 - 9 = 2$ . ✓

Euler’s rule confirms the counts.

### Example 3 — unscaffolded

The net below is cut from card. Name the solid it folds up to make, and state how many faces it has.





The net is a **square** in the middle with a **triangle** attached to each of its four sides. The square becomes the base and the four triangles fold up to meet at a single apex, so the solid is a **square pyramid**. It has the 1 square base plus 4 triangular faces.

∴ the net makes a **square pyramid**, which has 5 faces.

#### **Example 4 — unscaffolded**

A hardware store sells a storage container shaped like a **hexagonal prism**. Find its number of faces, edges and vertices, and confirm the total with Euler's rule.

A hexagonal prism has hexagonal ends (6 sides each). Using the prism pattern with  $n=6$ :

$$F = n + 2 = 8, E = 3n = 18, V = 2n = 12.$$

Check with Euler's rule:  $F + V - E = 8 + 12 - 18 = 2$ . ✓

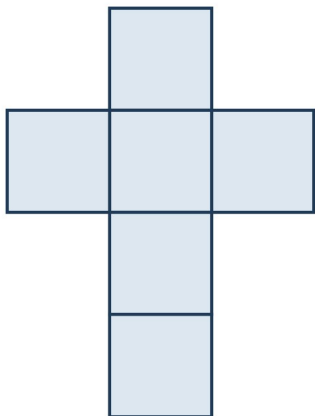
∴ the hexagonal prism has 8 faces, 18 edges and 12 vertices.

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### **Practice questions**

#### **Scaffolded**

- Q1.** Name the polygon with (a) 5 sides, (b) 8 sides, (c) 10 sides.
- Q2.** For a **cube**, state the number of (a) faces, (b) edges, (c) vertices.
- Q3.** For a **triangular prism**, state the number of (a) faces, (b) edges, (c) vertices.
- Q4.** Name the solid that best models each object: (a) a tin of soup, (b) an ice-cream cone, (c) a soccer ball.
- Q5.** For a **square pyramid**, state the number of (a) faces, (b) edges, (c) vertices.
- Q6.** The net shown is made of 6 equal squares.



- (a) Name the solid it folds up to make.
- (b) How many faces does that solid have?
- (c) How many edges does it have?

### Un scaffolded

**Q7.** Name the polygon that has 9 straight sides.

**Q8.** A **pentagonal prism** has pentagonal ends. State its number of faces, edges and vertices.

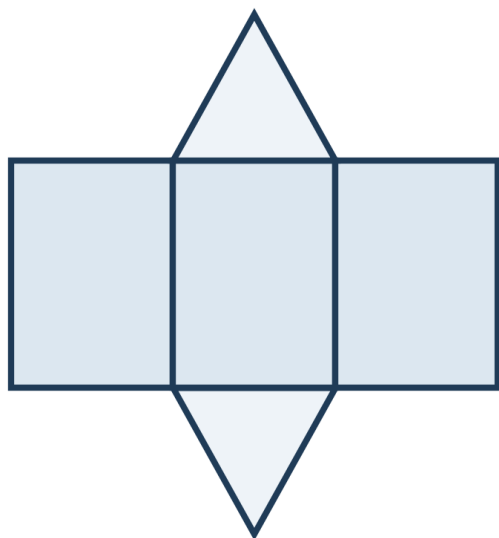
**Q9.** A regular polygon has 12 equal sides. Name it and state how many vertices it has.

**Q10.** Name the solid that best models each object: (a) a Toblerone (triangular-ended) box, (b) a shoebox, (c) a party hat (pointed, round base).

**Q11.** A **pentagonal pyramid** has a pentagon base and triangular sides meeting at an apex. State its number of faces, edges and vertices.

**Q12.** A solid has 6 faces and 8 vertices. Use Euler's rule  $F + V - E = 2$  to find its number of edges.

**Q13.** The net below folds up into a solid.



Name the solid, and state how many faces it has.

**Q14.** At any single vertex of a **cube**, how many faces meet?

**Q15.** Name the solid that has two flat circular faces joined by one curved surface, and state why Euler's rule does **not** apply to it.

**Q16.** A road sign has straight sides and 8 equal angles. Name its shape.

**Q17.** A packing carton is a **rectangular prism**. State its number of faces, edges and vertices, and confirm your answer with Euler's rule.

**Q18.** A marquee's frame is a solid with a hexagon base and 6 triangular faces that meet at one point above the base.

(a) Name the solid. (b) State its number of faces. (c) State its number of edges. (d) State its number of vertices.

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## Answers

**Q1.** (a) pentagon. (b) octagon. (c) decagon. **Q2.** (a) 6. (b) 12. (c) 8. **Q3.** (a) 5. (b) 9. (c) 6. **Q4.** (a) cylinder. (b) cone. (c) sphere. **Q5.** (a) 5. (b) 8. (c) 5. **Q6.** (a) cube. (b) 6. (c) 12. **Q7.** nonagon. **Q8.** 7 faces, 15 edges, 10 vertices. **Q9.** dodecagon; 12 vertices. **Q10.** (a) triangular prism. (b) rectangular prism. (c) cone. **Q11.** 6 faces, 10 edges, 6 vertices. **Q12.**  $E = 12$ . **Q13.** triangular prism; 5 faces. **Q14.** 3 faces. **Q15.** cylinder; it has a curved surface, so it is not a polyhedron and Euler's rule (for flat-faced solids) does not apply. **Q16.** octagon. **Q17.** 6 faces, 12 edges, 8 vertices; check  $6 + 8 - 12 = 2$ . ✓ **Q18.** (a) hexagonal pyramid. (b) 7. (c) 12. (d) 7.

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## Fully-worked solutions

**Q1.** A polygon is named by its number of sides: (a) 5 sides is a **pentagon**; (b) 8 sides is an **octagon**; (c) 10 sides is a **decagon**.

**Q2.** A cube is a box with all square faces: (a) 6 faces (top, bottom and 4 sides); (b) 12 edges; (c) 8 vertices. (Check:  $6 + 8 - 12 = 2$ .)

**Q3.** A triangular prism has 2 triangular ends and 3 rectangular sides: (a)  $2 + 3 = 5$  faces; (b)  $3 + 3 + 3 = 9$  edges; (c)  $3 + 3 = 6$  vertices.

**Q4.** (a) A tin of soup is a **cylinder**; (b) an ice-cream cone is a **cone**; (c) a soccer ball is a **sphere**.

**Q5.** A square pyramid has a square base and 4 triangular faces: (a)  $1 + 4 = 5$  faces; (b) 4 base edges + 4 slant edges = 8 edges; (c) 4 base corners + 1 apex = 5 vertices.

**Q6.** Six squares folding into a solid make a **cube**: (a) cube; (b) 6 faces (one per square); (c) 12 edges.

**Q7.** Nine straight sides make a **nonagon**.

**Q8.** A pentagonal prism has 5-sided ends, so with  $n = 5$ : faces =  $n + 2 = 7$ , edges =  $3n = 15$ , vertices =  $2n = 10$ . (Check:  $7 + 10 - 15 = 2$ .)

**Q9.** Twelve equal sides make a **dodecagon**; a polygon has as many vertices as sides, so it has 12 vertices.

**Q10.** (a) A triangular-ended box is a **triangular prism**; (b) a shoebox is a **rectangular prism**; (c) a pointed hat with a round base is a **cone**.

**Q11.** A pentagonal pyramid has a 5-sided base, so with  $n = 5$ : faces =  $n + 1 = 6$ , edges =  $2n = 10$ , vertices =  $n + 1 = 6$ . (Check:  $6 + 6 - 10 = 2$ .)

**Q12.** Rearranging Euler's rule,  $E = F + V - 2 = 6 + 8 - 2 = 12$  edges. (The solid could be a cube or any rectangular prism; these counts alone do not say which.)

**Q13.** The net has 3 rectangles in a row with a triangle above and below the middle rectangle. The two triangles are the ends and the three rectangles are the sides, so it folds into a **triangular prism**, which has 5 faces.

**Q14.** At each corner of a cube, three faces (for example the top, front and a side) come together, so 3 faces meet.

**Q15.** A **cylinder** has two flat circular faces and one curved surface. Because it has a curved surface it is not a polyhedron, and Euler's rule  $F + V - E = 2$  applies only to flat-faced solids, so it does not apply here.

**Q16.** A polygon has as many sides as angles, so 8 angles mean 8 sides and the shape is an **octagon**. (Equal angles do not guarantee equal sides, so the angles alone do not make it regular.)

**Q17.** A rectangular prism (box) has 6 rectangular faces, 12 edges and 8 vertices. Euler's rule checks out:  $6 + 8 - 12 = 2$ . ✓

**Q18.** A solid with a hexagon base and 6 triangular faces meeting at an apex is a **hexagonal pyramid** ( $n = 6$ ): (a) hexagonal pyramid; (b)  $n + 1 = 7$  faces; (c)  $2n = 12$  edges; (d)  $n + 1 = 7$  vertices. (Check:  $7 + 7 - 12 = 2$ .)

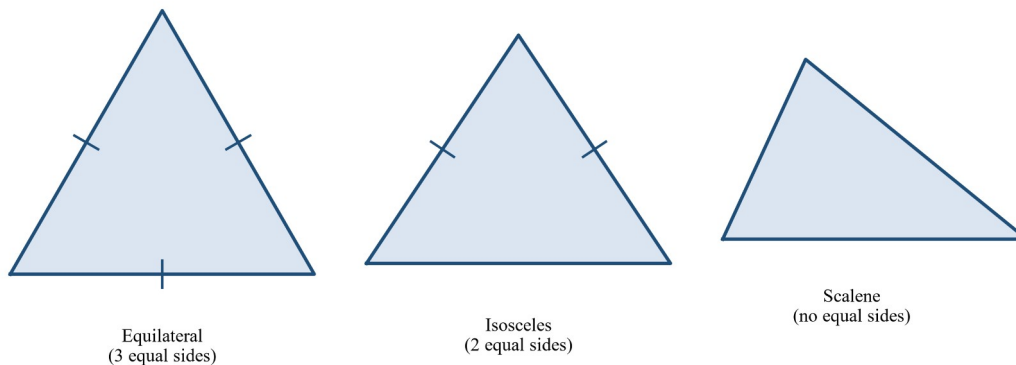


### Theory

Shapes are all around us — in road signs, buildings, packaging and logos. To describe and compare them we **classify** them by their properties: the number of sides, which sides are equal, the size of the angles, and the symmetry they have.

#### Classifying triangles by their sides.

Every triangle has three sides. We name a triangle by how many of those sides are equal. Equal sides are shown on a diagram with matching **tick marks**.



- An **equilateral** triangle has all three sides equal (and all three angles equal to  $60^\circ$ ).
- An **isosceles** triangle has exactly two sides equal (and the two **base angles**, opposite the equal sides, are equal).
- A **scalene** triangle has all three sides different (and all three angles different).

#### Classifying triangles by their angles.

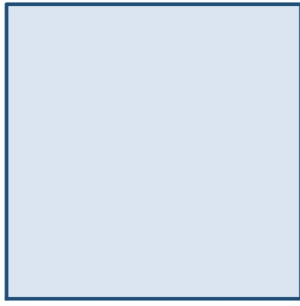
We can also name a triangle by its largest angle:

- an **acute** triangle has all three angles less than  $90^\circ$ ;
- a **right-angled** triangle has one angle of exactly  $90^\circ$ ;
- an **obtuse** triangle has one angle greater than  $90^\circ$ .

The three angles of any triangle always add to  $180^\circ$ , so a triangle can have **at most one** right or obtuse angle.

#### Classifying quadrilaterals.

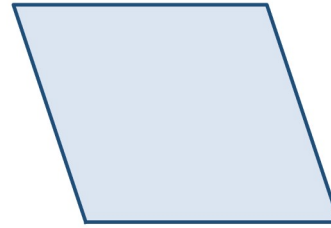
A **quadrilateral** is any four-sided shape; its four angles always add to  $360^\circ$ . The special quadrilaterals are named by their sides and angles.



Square



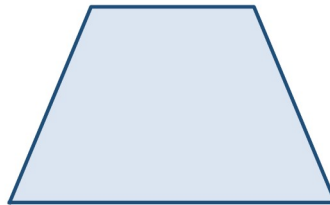
Rectangle



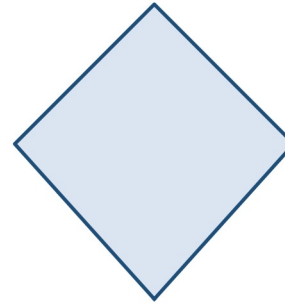
Rhombus



Parallelogram



Trapezium



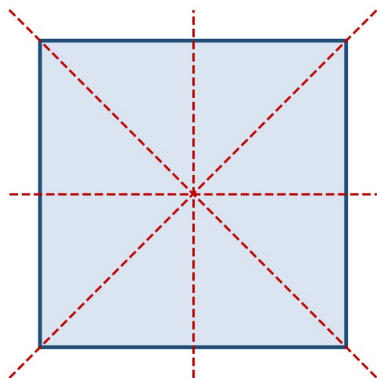
Kite

| Shape                | Key properties                                           |
|----------------------|----------------------------------------------------------|
| <b>Square</b>        | 4 equal sides, 4 right angles                            |
| <b>Rectangle</b>     | opposite sides equal, 4 right angles                     |
| <b>Rhombus</b>       | 4 equal sides, opposite angles equal                     |
| <b>Parallelogram</b> | opposite sides parallel and equal, opposite angles equal |
| <b>Trapezium</b>     | exactly one pair of parallel sides                       |
| <b>Kite</b>          | two pairs of equal <i>adjacent</i> sides                 |

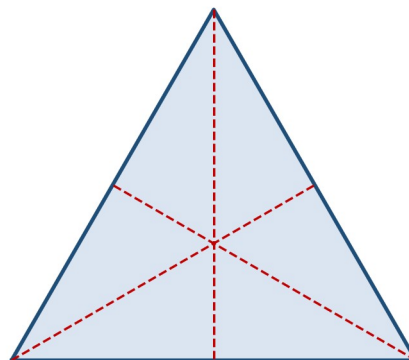
A square is a special rectangle (and a special rhombus): it fits their properties **and more**.

### Symmetry.

A shape has a **line of symmetry** (an axis of reflection) if a straight line divides it into two halves that are mirror images. A shape has **rotational symmetry** if it can be turned about its centre by less than a full turn and look exactly the same; the **order** of rotational symmetry is the number of times it matches itself in one full turn (every shape has order at least 1).



Square: 4 lines of symmetry



Equilateral triangle:  
3 lines of symmetry

| Shape                | Lines of symmetry | Order of rotational symmetry |
|----------------------|-------------------|------------------------------|
| Equilateral triangle | 3                 | 3                            |
| Square               | 4                 | 4                            |
| Rectangle            | 2                 | 2                            |
| Rhombus              | 2                 | 2                            |
| Parallelogram        | 0                 | 2                            |
| Kite                 | 1                 | 1                            |

### Regular polygons.

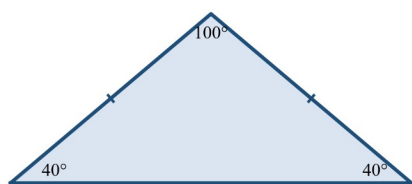
A polygon is **regular** if **all** its sides are equal **and** all its angles are equal (for example a square, or an equilateral triangle); otherwise it is **irregular**. A regular polygon with  $n$  sides has  $n$  lines of symmetry and rotational symmetry of order  $n$ , and each interior angle is

$$\text{interior angle} = \frac{(n - 2) \times 180^\circ}{n}.$$

### Worked examples

#### Example 1 — scaffolded

Classify the triangle shown below by its sides and by its angles, and state its number of lines of symmetry and its order of rotational symmetry.



| Working                                                                                           | Comment                                                    |
|---------------------------------------------------------------------------------------------------|------------------------------------------------------------|
| The tick marks show two equal sides, so the triangle is <b>isosceles</b> .                        | Two equal sides $\Rightarrow$ isosceles.                   |
| The angles are $40^\circ, 40^\circ, 100^\circ$ ; check $40 + 40 + 100 = 180^\circ$ .              | The angle sum of a triangle is $180^\circ$ .               |
| The largest angle, $100^\circ$ , is greater than $90^\circ$ , so it is an <b>obtuse</b> triangle. | Named by its largest angle.                                |
| An isosceles triangle has <b>1</b> line of symmetry and rotational symmetry of order <b>1</b> .   | The axis runs through the apex and the middle of the base. |

#### Example 2 — scaffolded

An isosceles triangle has an apex angle of  $36^\circ$ . Find the size of each of the two equal base angles.

| Working                                                                  | Comment                            |
|--------------------------------------------------------------------------|------------------------------------|
| The three angles add to $180^\circ$ , and the two base angles are equal. | Property of an isosceles triangle. |
| Base angles together $= 180^\circ - 36^\circ = 144^\circ$ .              | Subtract the apex angle.           |
| Each base angle $= 144^\circ \div 2 = 72^\circ$ .                        | Share equally between the two.     |



### Example 3 — unscaffolded

A quadrilateral has all four sides equal in length but no right angles. Its angles are  $110^\circ$ ,  $70^\circ$ ,  $110^\circ$  and  $70^\circ$ . Name the shape, and state its number of lines of symmetry and its order of rotational symmetry.

Four equal sides with opposite angles equal (and no right angles) is a **rhombus**. Checking the angle sum,  $110^\circ + 70^\circ + 110^\circ + 70^\circ = 360^\circ$ , as required for a quadrilateral. A rhombus has two lines of symmetry (along its diagonals) and rotational symmetry of order 2.

$\therefore$  the shape is a rhombus, with 2 lines of symmetry and rotational symmetry of order 2.

### Example 4 — unscaffolded

A regular hexagon is used as the shape of a paving stone. Find the size of each interior angle, and state the number of lines of symmetry and the order of rotational symmetry.

A hexagon has  $n = 6$  sides, so each interior angle is

$$\frac{(6 - 2) \times 180^\circ}{6} = \frac{4 \times 180^\circ}{6} = \frac{720^\circ}{6} = 120^\circ.$$

Because the hexagon is regular, it has 6 lines of symmetry and rotational symmetry of order 6.

$\therefore$  each interior angle is  $120^\circ$ , and the hexagon has 6 lines of symmetry and order 6 rotational symmetry.

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## Practice questions

### Scaffolded

**Q1.** Classify each triangle by its sides as equilateral, isosceles or scalene. (a) sides 5 cm, 5 cm, 5 cm. (b) sides 7 cm, 7 cm, 4 cm. (c) sides 4 cm, 6 cm, 9 cm.

**Q2.** Classify each triangle by its angles as acute, right-angled or obtuse. (a) angles  $60^\circ$ ,  $60^\circ$ ,  $60^\circ$ . (b) angles  $90^\circ$ ,  $45^\circ$ ,  $45^\circ$ . (c) angles  $30^\circ$ ,  $50^\circ$ ,  $100^\circ$ .

**Q3.** Find the missing angle(s) in each isosceles triangle. (a) the apex angle is  $50^\circ$ ; find each base angle. (b) each base angle is  $70^\circ$ ; find the apex angle. (c) the apex angle is  $90^\circ$ ; find each base angle.

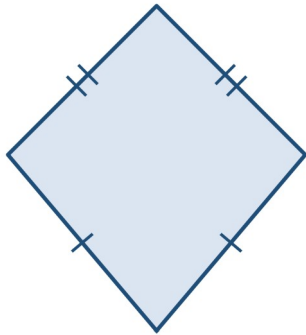
**Q4.** Find the missing angle in each quadrilateral (the four angles add to  $360^\circ$ ). (a)  $90^\circ$ ,  $90^\circ$ ,  $90^\circ$ ,  $?$ . (b)  $100^\circ$ ,  $80^\circ$ ,  $100^\circ$ ,  $?$ . (c)  $120^\circ$ ,  $60^\circ$ ,  $110^\circ$ ,  $?$ .

**Q5.** State the number of lines of symmetry of (a) a square, (b) a rectangle, (c) an equilateral triangle.

**Q6.** State the order of rotational symmetry of (a) a square, (b) a parallelogram, (c) a kite.

### Unscaffolded

**Q7.** Name the quadrilateral shown below (note the tick marks), and state its number of lines of symmetry and its order of rotational symmetry.



- Q8.** A triangle has three sides each of length 8 cm. Classify it by its sides and by its angles, state the size of each angle, and give its number of lines of symmetry and order of rotational symmetry.
- Q9.** A quadrilateral has four equal sides and four right angles. Name it, and state its number of lines of symmetry and its order of rotational symmetry.
- Q10.** A quadrilateral has both pairs of opposite sides parallel and equal, with opposite angles of  $70^\circ$  and  $110^\circ$ , and no right angles. Name the shape and state its order of rotational symmetry and number of lines of symmetry.
- Q11.** An isosceles triangle has an apex angle of  $116^\circ$ . Find each base angle, and classify the triangle by its angles.
- Q12.** A regular pentagon is used on a road sign. Find the size of each interior angle, and state its number of lines of symmetry and its order of rotational symmetry.
- Q13.** Find the size of each interior angle of a regular octagon (an 8-sided polygon).
- Q14.** A “give way” sign is an equilateral triangle. State its number of lines of symmetry and its order of rotational symmetry.
- Q15.** A rhombus has angles of  $65^\circ$  and  $115^\circ$ . (a) Show that its four angles add to  $360^\circ$ . (b) State its number of lines of symmetry and its order of rotational symmetry.
- Q16.** An isosceles trapezium has two base angles of  $72^\circ$ . Find the size of each of the other two angles, and state how many lines of symmetry the shape has.
- Q17.** A triangle has angles of  $90^\circ$ ,  $45^\circ$  and  $45^\circ$ . Classify it by its angles **and** by its sides, and state its number of lines of symmetry.
- Q18.** A regular polygon has an interior angle of  $140^\circ$ . Using  $\frac{(n-2) \times 180^\circ}{n} = 140^\circ$ , find the number of sides  $n$ , and name the polygon.
-

## Answers

**Q1.** (a) equilateral. (b) isosceles. (c) scalene. **Q2.** (a) acute. (b) right-angled. (c) obtuse. **Q3.** (a)  $65^\circ$  each. (b)  $40^\circ$ . (c)  $45^\circ$  each. **Q4.** (a)  $90^\circ$ . (b)  $80^\circ$ . (c)  $70^\circ$ . **Q5.** (a) 4. (b) 2. (c) 3. **Q6.** (a) 4. (b) 2. (c) 1. **Q7.** kite; 1 line of symmetry; order 1. **Q8.** equilateral and acute; each angle  $60^\circ$ ; 3 lines of symmetry; order 3. **Q9.** square; 4 lines of symmetry; order 4. **Q10.** parallelogram; 0 lines of symmetry; order 2. **Q11.**  $32^\circ$  each; obtuse. **Q12.**  $108^\circ$ ; 5 lines of symmetry; order 5. **Q13.**  $135^\circ$ . **Q14.** 3 lines of symmetry; order 3. **Q15.** (a)  $65 + 115 + 65 + 115 = 360^\circ$ . (b) 2 lines of symmetry; order 2. **Q16.**  $108^\circ$  each; 1 line of symmetry. **Q17.** right-angled and isosceles; 1 line of symmetry. **Q18.**  $n = 9$ ; a (regular) nonagon.

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## Fully-worked solutions

**Q1.** Count the equal sides. (a) all three equal  $\Rightarrow$  **equilateral**. (b) exactly two equal (7, 7)  $\Rightarrow$  **isosceles**. (c) all different  $\Rightarrow$  **scalene**.

**Q2.** Look at the largest angle. (a) all angles  $60^\circ < 90^\circ \Rightarrow$  **acute**. (b) one angle is exactly  $90^\circ \Rightarrow$  **right-angled**. (c)  $100^\circ > 90^\circ \Rightarrow$  **obtuse**.

**Q3.** The angles add to  $180^\circ$  and the two base angles are equal. (a) base angles together  $= 180^\circ - 50^\circ = 130^\circ$ , so each  $= 130^\circ \div 2 = 65^\circ$ . (b) apex  $= 180^\circ - 2 \times 70^\circ = 180^\circ - 140^\circ = 40^\circ$ . (c) base angles together  $= 180^\circ - 90^\circ = 90^\circ$ , so each  $= 45^\circ$ .

**Q4.** The four angles add to  $360^\circ$ , so the missing angle is  $360^\circ$  minus the other three. (a)  $360^\circ - (90^\circ + 90^\circ + 90^\circ) = 360^\circ - 270^\circ = 90^\circ$ . (b)  $360^\circ - (100^\circ + 80^\circ + 100^\circ) = 360^\circ - 280^\circ = 80^\circ$ . (c)  $360^\circ - (120^\circ + 60^\circ + 110^\circ) = 360^\circ - 290^\circ = 70^\circ$ .

**Q5.** (a) A square has 4 lines of symmetry (two through opposite side-midpoints, two along the diagonals). (b) A rectangle has 2 (through the midpoints of opposite sides only). (c) An equilateral triangle has 3 (one from each vertex to the middle of the opposite side).

**Q6.** (a) A square looks the same after a quarter, half and three-quarter turn, so order 4. (b) A parallelogram matches itself only after a half turn, so order 2. (c) A kite matches only after a full turn, so order 1.

**Q7.** The tick marks show two pairs of equal **adjacent** sides, which is a **kite**. A kite has one line of symmetry (its main diagonal) and rotational symmetry of order 1.

**Q8.** All three sides equal (8 cm)  $\Rightarrow$  **equilateral**; every angle is  $180^\circ \div 3 = 60^\circ$ , all less than  $90^\circ$ , so it is also **acute**. An equilateral triangle has 3 lines of symmetry and rotational symmetry of order 3.

**Q9.** Four equal sides **and** four right angles is a **square**. It has 4 lines of symmetry and rotational symmetry of order 4.

**Q10.** Opposite sides parallel and equal with opposite angles equal, but no right angles and not all sides equal, is a **parallelogram** (angles  $70^\circ, 110^\circ, 70^\circ, 110^\circ$  add to  $360^\circ$ ). A parallelogram has **no** lines of symmetry but rotational symmetry of order 2.

**Q11.** Base angles together  $= 180^\circ - 116^\circ = 64^\circ$ , so each base angle  $= 64^\circ \div 2 = 32^\circ$ . The largest angle,  $116^\circ$ , is greater than  $90^\circ$ , so the triangle is **obtuse**.

**Q12.** A pentagon has  $n = 5$  sides, so each interior angle is

$$\frac{(5 - 2) \times 180^\circ}{5} = \frac{540^\circ}{5} = 108^\circ.$$

Being regular, it has 5 lines of symmetry and rotational symmetry of order 5.

**Q13.** An octagon has  $n = 8$  sides, so each interior angle is



$$\frac{(8-2) \times 180^\circ}{8} = \frac{1080^\circ}{8} = 135^\circ.$$

**Q14.** An equilateral triangle has 3 lines of symmetry and rotational symmetry of order 3.

**Q15.** (a) The four angles are  $65^\circ$ ,  $115^\circ$ ,  $65^\circ$ ,  $115^\circ$ , and  $65 + 115 + 65 + 115 = 360^\circ$ , the correct angle sum for a quadrilateral. (b) A rhombus has 2 lines of symmetry (along its diagonals) and rotational symmetry of order 2.

**Q16.** In an isosceles trapezium the parallel sides make the base and top angles co-interior, so each top angle  $= 180^\circ - 72^\circ = 108^\circ$ . The four angles are  $72^\circ$ ,  $72^\circ$ ,  $108^\circ$ ,  $108^\circ$  (which add to  $360^\circ$ ). An isosceles trapezium has 1 line of symmetry.

**Q17.** The  $90^\circ$  angle makes it **right-angled**; the two equal  $45^\circ$  angles mean two sides are equal, so it is also **isosceles**. A right isosceles triangle has 1 line of symmetry (through the right angle and the middle of the opposite side).

**Q18.** Solve  $\frac{(n-2) \times 180}{n} = 140$ . Multiply both sides by  $n$ :  $180n - 360 = 140n$ . Then  $180n - 140n = 360$ , so  $40n = 360$  and  $n = 9$ . A regular 9-sided polygon is a **nonagon**.

## Chapter 10 Shape, angles and plans — 10G Plans, views and measurements

### Theory

Builders, engineers and designers need to show a **three-dimensional** object on a flat, **two-dimensional** page. The standard way is to draw the object as it looks from three directions at once. These are called **orthographic views** — you look straight at the object from each direction, so no perspective is added and the measurements stay true.

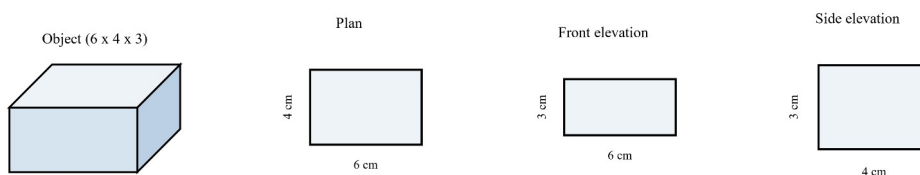
**The three standard views.**

- The **plan** is the view looking **straight down from above** (the “top view”).
- The **front elevation** is the view looking **horizontally from the front**.
- The **side elevation** is the view looking **horizontally from one side** (usually the left or right).

Each view is a flat shape. Because a solid has three measurements — its **length**, its **depth** (front-to-back) and its **height** — each view shows exactly **two** of them:

| View            | You are looking | Shows                  |
|-----------------|-----------------|------------------------|
| Plan            | down from above | length $\times$ depth  |
| Front elevation | from the front  | length $\times$ height |
| Side elevation  | from the side   | depth $\times$ height  |

**A worked model.** A rectangular box 6 cm long, 4 cm deep and 3 cm high has these three views:



The plan is a  $6 \times 4$  rectangle, the front elevation a  $6 \times 3$  rectangle, and the side elevation a  $4 \times 3$  rectangle. Notice that the two views sharing a measurement line up: the front elevation sits directly **below** the plan (both are 6 wide), and the side elevation is drawn beside the front (both are 3 high).

**Conventions.**

- Views are lined up so that matching measurements are directly across from one another.
- An edge you could see is drawn as a **solid line**; an edge hidden behind the solid is drawn as a **dashed line**.
- A **scale** is often used so a large object fits on the page (for example 1 cm on the drawing representing 1 m in real life) — scales are covered in 10H.

**Reading measurements.** Because each view keeps true measurements, you can read a real length straight off the correct view: read a **height** from an elevation, a **depth** from the plan or the side elevation, and a **length** from the plan or the front elevation.

### Worked examples

#### Example 1 — scaffolded

A shipping carton is a rectangular box 8 cm long, 5 cm deep and 4 cm high. State the shape and dimensions of its plan, front elevation and side elevation.

Working

Comment

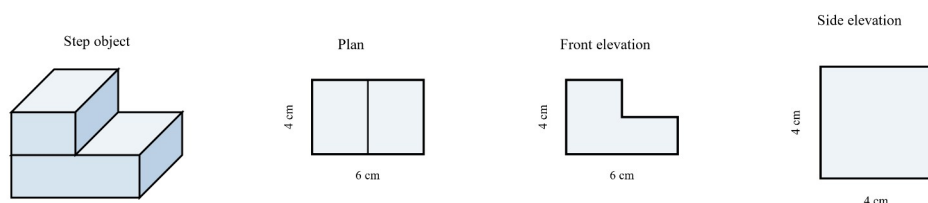
The plan looks down from above, so it shows length  $\times$  depth.

Top view.

| Working                                                                      | Comment                 |
|------------------------------------------------------------------------------|-------------------------|
| Plan: an $8 \times 5$ rectangle.                                             | Length 8, depth 5.      |
| The front elevation shows length $\times$ height: an $8 \times 4$ rectangle. | Looking from the front. |
| The side elevation shows depth $\times$ height: a $5 \times 4$ rectangle.    | Looking from the side.  |

### Example 2 — scaffolded

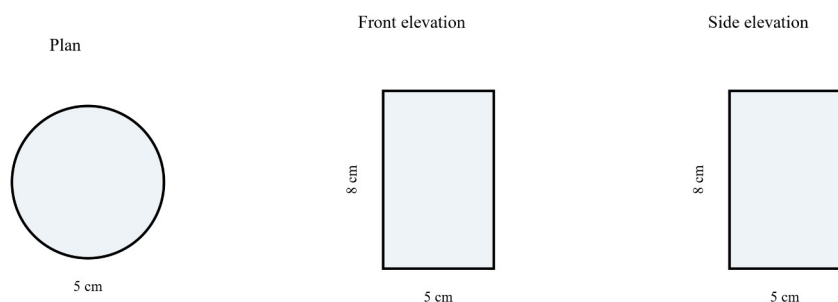
A step is made from a lower block 6 cm long, 4 cm deep and 2 cm high, with a second block 3 cm long, 4 cm deep and 2 cm high sitting on the **left** half of the top. Describe its three views.



| Working                                                                                                                         | Comment                                         |
|---------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------|
| Height at the left is $2 + 2 = 4$ cm; at the right it is 2 cm.                                                                  | The upper block doubles the height on the left. |
| Front elevation: an L-shaped step — 4 cm high for the left 3 cm, then 2 cm high for the right 3 cm.                             | Length $\times$ height, seen from the front.    |
| Plan: a $6 \times 4$ rectangle with a line across it at 3 cm, marking the front edge of the upper block.                        | Length $\times$ depth, seen from above.         |
| Side elevation: a $4 \times 4$ square (the tall left block runs the full depth, so the side silhouette is depth 4 by height 4). | Depth $\times$ height.                          |

### Example 3 — unscaffolded

A solid has the three views shown: a **circle** of diameter 5 cm for the plan, and a  $5 \text{ cm} \times 8 \text{ cm}$  **rectangle** for both the front and the side elevation. Name the solid and state its dimensions.



A circular top view with rectangular front and side views is a **cylinder** standing upright. The circular plan gives the base: diameter 5 cm, so radius 2.5 cm. The rectangles are 8 cm tall, which is the height, and 5 cm wide, matching the diameter.

$\therefore$  the solid is a cylinder of diameter 5 cm (radius 2.5 cm) and height 8 cm.

### Example 4 — unscaffolded

A small cube of side 2 cm sits centred on top of a rectangular block 5 cm long, 5 cm deep and 3 cm high. Find the total height of the object and describe its front elevation.

The two solids are stacked, so the total height is the sum of the two heights:

$$3 + 2 = 5 \text{ cm.}$$

Seen from the front, the block gives a 5 cm wide, 3 cm high rectangle, with the 2 cm by 2 cm cube centred on top, reaching up to 5 cm.

$\therefore$  the total height is 5 cm; the front elevation is a  $5 \times 3$  rectangle with a  $2 \times 2$  square centred on top.

## Practice questions

### Scaffolded

**Q1.** (a) Name the three standard orthographic views of a solid. (b) Which view do you see when you look straight down on the object from above? (c) Which two measurements does a front elevation show?

**Q2.** A rectangular box is 10 cm long, 6 cm deep and 4 cm high. (a) State the dimensions of its plan. (b) State the dimensions of its front elevation. (c) State the dimensions of its side elevation.

**Q3.** A solid has a circular plan and a rectangular front elevation and side elevation. (a) Name the solid. (b) If the rectangle is 6 cm wide and 9 cm tall, state the height of the solid. (c) State the diameter of its circular base.

**Q4.** A cube has a side length of 5 cm. (a) What shape is its plan? (b) State the dimensions of the plan. (c) State the dimensions of the front elevation.

**Q5.** A solid has a plan that is a  $4 \text{ cm} \times 4 \text{ cm}$  square with both diagonals drawn, and a front elevation and side elevation that are each triangles. (a) Name the solid. (b) If each triangle is 4 cm wide at the base and 6 cm tall, state the height of the solid.

**Q6.** The front elevation of a step object is 8 cm wide overall. Its left 5 cm is 6 cm tall and its right 3 cm is 2 cm tall. (a) State the greatest height of the object. (b) State the height of the lower step.

### Un scaffolded

**Q7.** A cone stands on its circular base. Describe the shape of its plan, its front elevation and its side elevation.

**Q8.** A sphere of diameter 7 cm is viewed from above, from the front and from the side. Describe all three views and state their key measurement.

**Q9.** A square-based pyramid has a base 6 cm by 6 cm and a height of 5 cm. Describe its plan and its front elevation, giving the base and height of the front elevation.

**Q10.** A solid has a plan, front elevation and side elevation that are **all** identical  $4 \text{ cm} \times 4 \text{ cm}$  squares. Name the solid and state its dimensions.

**Q11.** The three views of a rectangular box are: plan  $7 \text{ cm} \times 3 \text{ cm}$ , front elevation  $7 \text{ cm} \times 2 \text{ cm}$ , side elevation  $3 \text{ cm} \times 2 \text{ cm}$ . (a) State the length, depth and height of the box. (b) How many 1 cm cubes would exactly fill it?

**Q12.** A triangular prism lies with its rectangular face down and its triangular ends facing left and right. Its length is 10 cm, and each triangular end is 4 cm wide and 3 cm tall. (a) What shape is the plan? (b) What shape is the side elevation (looking at a triangular end)? (c) State the dimensions of the front elevation.

**Q13.** A solid is built from 1 cm cubes. Seen from above, the number of cubes stacked on each square of the base is:

|                  | Left | Middle | Right |
|------------------|------|--------|-------|
| <b>Back row</b>  | 3    | 2      | 1     |
| <b>Front row</b> | 2    | 2      | 1     |

(a) How many cubes are used altogether?

(b) State the heights (left to right) shown in the front elevation.

(c) State the heights (back to front) shown in the side elevation seen from the right.

**Q14.** Another solid built from 1 cm cubes has this stacking plan:

|                  | Left | Right |
|------------------|------|-------|
| <b>Back row</b>  | 2    | 2     |
| <b>Front row</b> | 1    | 3     |

- (a) How many cubes are used?  
(b) State the greatest height of the solid.

**Q15.** A wardrobe is a rectangular box 1.2 m long, 0.6 m deep and 2.1 m high. (a) State the dimensions of its front elevation. (b) State the dimensions of its plan. (c) A carpenter wants to check the object will fit through a doorway 0.8 m wide and 2.0 m high, carried upright. Explain, using the elevations, whether it fits.

**Q16.** A cylindrical drink can lies on its side on a shelf, with its circular ends facing left and right. Describe its plan, its front elevation and its side elevation.

**Q17.** A “T”-shaped solid is made of two rectangular blocks: a base block 2 cm high, and an upright block on top that adds a further 6 cm. (a) State the total height of the solid. (b) If the base is 8 cm long and the upright block is 2 cm long and centred, describe the front elevation.

**Q18.** A step object is made of a lower block 9 cm long, 4 cm deep and 3 cm high, with an upper block 4 cm long, 4 cm deep and 3 cm high on the right half of the top. (a) State the greatest height of the object. (b) Describe the front elevation (widths and heights). (c) State the dimensions of the side elevation.

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## Answers

**Q1.** (a) plan (top view), front elevation, side elevation. (b) the plan. (c) length and height. **Q2.** (a)  $10 \times 6$ . (b)  $10 \times 4$ . (c)  $6 \times 4$ . **Q3.** (a) cylinder. (b) 9 cm. (c) 6 cm. **Q4.** (a) a square. (b)  $5 \times 5$ . (c)  $5 \times 5$ . **Q5.** (a) a square-based pyramid. (b) 6 cm. **Q6.** (a) 6 cm. (b) 2 cm. **Q7.** plan: a circle; front elevation: a triangle; side elevation: a triangle. **Q8.** all three views are circles of diameter 7 cm. **Q9.** plan: a  $6 \times 6$  square with the diagonals drawn to the centre; front elevation: a triangle 6 cm wide and 5 cm tall. **Q10.** a cube,  $4 \text{ cm} \times 4 \text{ cm} \times 4 \text{ cm}$ . **Q11.** (a) length 7, depth 3, height 2 (cm). (b) 42 cubes. **Q12.** (a) a rectangle ( $10 \times 4$ ). (b) a triangle (4 wide, 3 tall). (c) a  $10 \times 3$  rectangle. **Q13.** (a) 11 cubes. (b) 3, 2, 1. (c) 3, 2. **Q14.** (a) 8 cubes. (b) 3 cm. **Q15.** (a)  $1.2 \text{ m} \times 2.1 \text{ m}$ . (b)  $1.2 \text{ m} \times 0.6 \text{ m}$ . (c) no — it is 2.1 m tall, which is more than the 2.0 m door height. **Q16.** plan: a rectangle; front elevation: a rectangle; side elevation: a circle. **Q17.** (a) 8 cm. (b) an  $8 \times 2$  rectangle with a  $2 \times 6$  rectangle centred on top. **Q18.** (a) 6 cm. (b) left 5 cm is 3 cm tall, right 4 cm is 6 cm tall. (c) a  $4 \times 6$  rectangle.

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## Fully-worked solutions

**Q1.** (a) The three standard views are the **plan** (top view), the **front elevation** and the **side elevation**. (b) Looking straight down gives the **plan**. (c) A front elevation shows the object's **length and height**.

**Q2.** Plan shows  $\text{length} \times \text{depth} = 10 \times 6$ ; front elevation shows  $\text{length} \times \text{height} = 10 \times 4$ ; side elevation shows  $\text{depth} \times \text{height} = 6 \times 4$ .

**Q3.** (a) A circular plan with rectangular elevations is a **cylinder**. (b) The rectangle's height is the solid's height, 9 cm. (c) The rectangle's width equals the diameter, 6 cm.

**Q4.** Every face of a cube is a square, so (a) the plan is a **square**, (b) it is  $5 \times 5$ , and (c) the front elevation is also  $5 \times 5$ .

**Q5.** (a) A square plan with the diagonals meeting at a point, with triangular elevations, is a **square-based pyramid** (the diagonals are the sloping edges seen from above meeting at the apex). (b) The triangle's height is the pyramid's height, 6 cm.

**Q6.** The overall front elevation is 8 cm wide. (a) The greatest height is the taller step, 6 cm. (b) The lower step is 2 cm tall.

**Q7.** Looking down on a cone you see its circular base, so the **plan is a circle**. From the front and from the side, a cone has a straight-sided outline rising to the apex, so each **elevation is a triangle**.

**Q8.** A sphere looks the same from every direction, so all three views are **circles**, each of diameter 7 cm (the diameter of the sphere).

**Q9.** From above, a square-based pyramid is a  $6 \times 6$  **square** with the four sloping edges appearing as the diagonals meeting at the centre (the apex). From the front it is an isosceles **triangle** 6 cm wide at the base and 5 cm tall (the height of the pyramid).

**Q10.** If all three views are identical  $4 \times 4$  squares, the solid is a **cube** of side 4 cm, i.e.  $4 \times 4 \times 4$ .

**Q11.** (a) The plan  $7 \times 3$  gives length 7 and depth 3; the front elevation  $7 \times 2$  confirms length 7 and gives height 2; the side elevation  $3 \times 2$  confirms depth 3 and height 2. So length 7 cm, depth 3 cm, height 2 cm. (b) Number of 1 cm cubes  $= 7 \times 3 \times 2 = 42$ .

**Q12.** With the prism lying on a rectangular face and its triangular ends to the sides: (a) from above you see a  $10 \text{ cm} \times 4 \text{ cm}$  **rectangle**; (b) looking at a triangular end (the side elevation) you see the **triangle**, 4 cm wide and 3 cm tall; (c) from the front you see the long sloping roof as a 10 cm wide, 3 cm tall **rectangle**.

**Q13.** (a) Add the stacks: back row  $3 + 2 + 1 = 6$ , front row  $2 + 2 + 1 = 5$ , total  $6 + 5 = 11$  cubes. (b) The front elevation shows, for each column, the **tallest** stack in that column: left  $\max(3, 2) = 3$ , middle  $\max(2, 2) = 2$ , right  $\max(1, 1) = 1$ , so heights 3, 2, 1. (c) The side elevation (from the right) shows the tallest stack in each row: back  $\max(3, 2, 1) = 3$ , front  $\max(2, 2, 1) = 2$ , so heights 3, 2.

**Q14.** (a)  $2 + 2 + 1 + 3 = 8$  cubes. (b) The tallest stack has 3 cubes, so the greatest height is 3 cm.

**Q15.** (a) Front elevation shows  $\text{length} \times \text{height} = 1.2 \text{ m} \times 2.1 \text{ m}$ . (b) Plan shows  $\text{length} \times \text{depth} = 1.2 \text{ m} \times 0.6 \text{ m}$ . (c) Carried upright, the wardrobe's height is 2.1 m, which is greater than the 2.0 m door height, so it will **not** fit through upright (even though its 0.6 m depth is less than the 0.8 m door width).

**Q16.** With the can on its side and circular ends facing left and right: from above you see the curved body as a **rectangle** (plan); from the front you also see a **rectangle** (front elevation); looking at a circular end (the side elevation) you see a **circle**.

**Q17.** (a) Total height  $= 2 + 6 = 8$  cm. (b) The base gives an 8 cm wide, 2 cm high rectangle; the upright block adds a 2 cm wide, 6 cm high rectangle centred on top — a “T” outline.

**Q18.** (a) Greatest height  $= 3 + 3 = 6$  cm (the upper block doubles the height on the right). (b) From the front: the left 5 cm is 3 cm tall (lower block only) and the right 4 cm is 6 cm tall (both blocks). (c) The upper block runs the full 4 cm depth and reaches 6 cm high, so the side elevation is a  $4 \text{ cm} \times 6 \text{ cm}$  rectangle.

## Chapter 10 Shape, angles and plans — 10H Plans and scales

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### Theory

A **plan** is an accurate drawing of a real object — a room, a house, a block of land — shrunk down so it fits on paper. To make the drawing useful, every length is reduced by the same amount. The **scale** tells us that amount.

#### Writing a scale.

A scale is written as a ratio **drawing length : real length**, using the *same units on both sides*. A scale of 1 : 100 means

1 unit on the plan represents 100 of the same units in real life.

So 1 cm on the plan represents 100 cm — that is, 1 m — in real life. The number after the colon is the **scale factor**.

#### From the plan to real life.

To turn a length measured on the plan into a real length, **multiply by the scale factor**:

$$\text{real length} = \text{plan length} \times \text{scale factor}.$$

The answer comes out in the *same unit you measured in* (usually centimetres), so you often convert to metres at the end (100 cm = 1 m).

#### From real life to the plan.

To find how long something should be drawn on the plan, **divide by the scale factor** (after making both lengths the same unit):

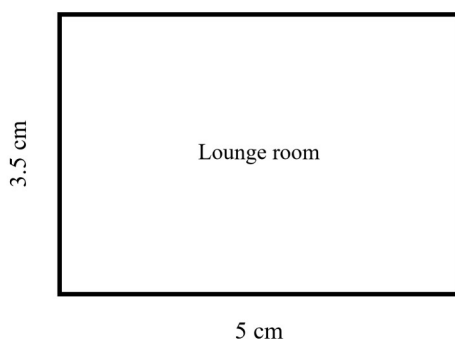
$$\text{plan length} = \frac{\text{real length}}{\text{scale factor}}.$$

#### Areas on a plan.

Lengths shrink by the scale factor, but **area shrinks by the square of the scale factor**. The safe method is always to convert the plan lengths to real lengths *first*, then multiply the real length by the real width. (A handy shortcut: on a 1 : 100 plan, an area of 1 cm<sup>2</sup> represents 1 m<sup>2</sup>, because 100 cm × 100 cm = 10 000 cm<sup>2</sup> = 1 m<sup>2</sup>.)

#### Reading a floor plan.

Floor plan (scale 1 : 100)



On the floor plan above, the lounge room is drawn 5 cm by 3.5 cm at a scale of 1 : 100, so in real life it is 5 m by 3.5 m.

---

## Worked examples

### Example 1 — scaffolded

A house plan is drawn to a scale of 1 : 100. A wall measures 4.5 cm on the plan. Find the real length of the wall, in metres.

| Working                                                          | Comment                                       |
|------------------------------------------------------------------|-----------------------------------------------|
| Scale factor is 100, so real length = $4.5 \times 100 = 450$ cm. | Multiply the plan length by the scale factor. |
| Convert to metres: $450 \div 100 = 4.5$ m.                       | 100 cm = 1 m.                                 |
| The wall is 4.5 m long.                                          | State the answer with its unit.               |

### Example 2 — scaffolded

A room is 6 m long in real life. On a plan drawn to a scale of 1 : 50, how long should the room be drawn, in centimetres?

| Working                                                         | Comment                                               |
|-----------------------------------------------------------------|-------------------------------------------------------|
| Put both lengths in the same unit: 6 m = 600 cm.                | Convert real length to centimetres first.             |
| Plan length = real length $\div$ scale factor = $600 \div 50$ . | Going <i>to</i> the plan, divide by the scale factor. |
| $600 \div 50 = 12$ cm.                                          | Draw the room 12 cm long.                             |

### Example 3 — unscaffolded

A rectangular room on a 1 : 100 plan measures 5 cm by 3.5 cm. Find the real length and width of the room, its real perimeter, and its real floor area.

Real length =  $5 \times 100 = 500$  cm = 5 m. Real width =  $3.5 \times 100 = 350$  cm = 3.5 m. Real perimeter =  $2 \times (5 + 3.5) = 2 \times 8.5 = 17$  m. Real area =  $5 \times 3.5 = 17.5$  m<sup>2</sup>.

$\therefore$  the room is 5 m by 3.5 m, with a perimeter of 17 m and a floor area of 17.5 m<sup>2</sup>.

### Example 4 — unscaffolded

A courtyard is drawn as a rectangle 6 cm by 4 cm on a plan with scale 1 : 100. Find the real area of the courtyard, and check your answer using the area shortcut.

Real length =  $6 \times 100 = 600$  cm = 6 m; real width =  $4 \times 100 = 400$  cm = 4 m. Real area =  $6 \times 4 = 24$  m<sup>2</sup>. Check: the plan area is  $6 \times 4 = 24$  cm<sup>2</sup>, and on a 1 : 100 plan each 1 cm<sup>2</sup> represents 1 m<sup>2</sup>, so 24 cm<sup>2</sup> represents 24 m<sup>2</sup>. ✓

$\therefore$  the courtyard has a real area of 24 m<sup>2</sup>.

## Practice questions

### Scaffolded

**Q1.** A plan is drawn to a scale of 1 : 100. (a) A wall is 7 cm long on the plan. Find its real length in metres. (b) A wall is 3.2 cm long on the plan. Find its real length in metres. (c) A fence is 6 m long in real life. How long is it drawn on the plan, in centimetres?

**Q2.** A plan is drawn to a scale of 1 : 50. (a) A bench is 4 cm long on the plan. Find its real length in metres. (b) A wall is 9 cm long on the plan. Find its real length in metres. (c) A cupboard is 3 m wide in real life. How wide is it drawn on the plan, in centimetres?

**Q3.** A rectangular bedroom on a 1 : 100 plan measures 4 cm by 2.5 cm. (a) Find the real length and width, in metres. (b) Find the real perimeter, in metres. (c) Find the real floor area, in square metres.

**Q4.** A site plan uses a scale of 1 : 200. (a) A driveway is 5.5 cm on the plan. Find its real length in metres. (b) A fence is 24 m long in real life. How long is it on the plan, in centimetres? (c) Complete this sentence: on this plan, 1 cm represents \_\_\_\_ m.

**Q5.** Use the lounge-room floor plan in the Theory section (drawn 5 cm by 3.5 cm at 1 : 100). (a) State the real length and width of the lounge room, in metres. (b) Find the real floor area, in square metres. (c) Carpet costs \$ 45 per square metre. Find the cost of carpeting the lounge room.

**Q6.** A map has a scale of 1 : 25 000. (a) What real distance, in metres, does 1 cm on the map represent? (b) Two towns are 8 cm apart on the map. Find the real distance between them, in kilometres. (c) A road is 5 km long. How long is it drawn on the map, in centimetres?

### **Un scaffolded**

**Q7.** On a 1 : 100 plan a room wall measures 6.2 cm. Find the real length of the wall in metres.

**Q8.** A garden is 15 m long. On a plan drawn to a scale of 1 : 150, how long is the garden drawn, in centimetres?

**Q9.** A rectangular kitchen on a 1 : 100 plan measures 3.5 cm by 3 cm. Find the real perimeter and the real floor area of the kitchen.

**Q10.** A plan is drawn to a scale of 1 : 50. A room is drawn 8 cm by 5 cm. (a) Find the real length and width, in metres. (b) Find the real floor area, in square metres.

**Q11.** A plan states its scale as “1 cm represents 2 m”. (a) Write this scale as a ratio 1 :  $n$ . (b) A line on the plan is 7 cm long. Find the real length it represents, in metres.

**Q12.** The same wall, 5 m long in real life, is drawn on two plans: Plan A at 1 : 100 and Plan B at 1 : 200. (a) Find the length of the wall on each plan, in centimetres. (b) Which plan shows the house at the larger size?

**Q13.** A site plan at scale 1 : 500 shows a rectangular block of land 6 cm by 4 cm. Find the real dimensions of the block, in metres, and its real area, in square metres.

**Q14.** A room measuring 4.8 m by 3.6 m is to be drawn on a plan at a scale of 1 : 120. Find the length and width the room should be drawn, in centimetres.

**Q15.** On a 1 : 100 plan, a rectangular deck measures 2.5 cm by 2 cm. Decking costs \$ 65 per square metre. Find the real area of the deck and the cost to cover it.

**Q16.** A courtyard is drawn on a 1 : 100 plan with a plan area of  $30\text{ cm}^2$ . Find the real area of the courtyard, in square metres.

**Q17.** A walking track is 3.5 km long. On a map with scale 1 : 25 000, how long is the track drawn, in centimetres?

**Q18.** A rectangular paddock is drawn 7 cm by 5 cm on a plan with scale 1 : 2000. (a) Find the real length and width of the paddock, in metres. (b) Find the real perimeter, in metres. (c) Find the real area, in hectares (1 hectare =  $10\,000\text{ m}^2$ ).

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## Answers

**Q1.** (a) 7 m. (b) 3.2 m. (c) 6 cm. **Q2.** (a) 2 m. (b) 4.5 m. (c) 6 cm. **Q3.** (a) 4 m by 2.5 m. (b) 13 m. (c)  $10\text{ m}^2$ . **Q4.** (a) 11 m. (b) 12 cm. (c) 2 m. **Q5.** (a) 5 m by 3.5 m. (b)  $17.5\text{ m}^2$ . (c) \$787.50. **Q6.** (a) 250 m. (b) 2 km. (c) 20 cm. **Q7.** 6.2 m. **Q8.** 10 cm. **Q9.** perimeter 13 m; area  $10.5\text{ m}^2$ . **Q10.** (a) 4 m by 2.5 m. (b)  $10\text{ m}^2$ . **Q11.** (a) 1:200. (b) 14 m. **Q12.** (a) Plan A 5 cm, Plan B 2.5 cm. (b) Plan A. **Q13.** 30 m by 20 m; area  $600\text{ m}^2$ . **Q14.** 4 cm by 3 cm. **Q15.** area  $5\text{ m}^2$ ; cost \$325. **Q16.**  $30\text{ m}^2$ . **Q17.** 14 cm. **Q18.** (a) 140 m by 100 m. (b) 480 m. (c) 1.4 ha.

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## Fully-worked solutions

**Q1.** Scale factor 100. (a)  $7 \times 100 = 700\text{ cm} = 7\text{ m}$ . (b)  $3.2 \times 100 = 320\text{ cm} = 3.2\text{ m}$ . (c)  $6\text{ m} = 600\text{ cm}$ ;  $600 \div 100 = 6\text{ cm}$ .

**Q2.** Scale factor 50. (a)  $4 \times 50 = 200\text{ cm} = 2\text{ m}$ . (b)  $9 \times 50 = 450\text{ cm} = 4.5\text{ m}$ . (c)  $3\text{ m} = 300\text{ cm}$ ;  $300 \div 50 = 6\text{ cm}$ .

**Q3.** Scale factor 100. (a)  $4 \times 100 = 400\text{ cm} = 4\text{ m}$ ;  $2.5 \times 100 = 250\text{ cm} = 2.5\text{ m}$ . (b) Perimeter  $= 2 \times (4 + 2.5) = 2 \times 6.5 = 13\text{ m}$ . (c) Area  $= 4 \times 2.5 = 10\text{ m}^2$ .

**Q4.** Scale factor 200. (a)  $5.5 \times 200 = 1100\text{ cm} = 11\text{ m}$ . (b)  $24\text{ m} = 2400\text{ cm}$ ;  $2400 \div 200 = 12\text{ cm}$ . (c) 1 cm represents  $200\text{ cm} = 2\text{ m}$ .

**Q5.** The lounge is 5 cm by 3.5 cm at 1:100. (a)  $5 \times 100 = 500\text{ cm} = 5\text{ m}$ ;  $3.5 \times 100 = 350\text{ cm} = 3.5\text{ m}$ . (b) Area  $= 5 \times 3.5 = 17.5\text{ m}^2$ . (c) Cost  $= 17.5 \times \$45 = \$787.50$ .

**Q6.** Scale factor 25 000. (a)  $1 \times 25\,000 = 25\,000\text{ cm} = 250\text{ m}$ . (b) 8 cm represents  $8 \times 250 = 2000\text{ m} = 2\text{ km}$ . (c)  $5\text{ km} = 5000\text{ m} = 500\,000\text{ cm}$ ;  $500\,000 \div 25\,000 = 20\text{ cm}$ .

**Q7.**  $6.2 \times 100 = 620\text{ cm} = 6.2\text{ m}$ .

**Q8.**  $15\text{ m} = 1500\text{ cm}$ ;  $1500 \div 150 = 10\text{ cm}$ .

**Q9.** Real dimensions  $3.5 \times 100 = 350\text{ cm} = 3.5\text{ m}$  and  $3 \times 100 = 300\text{ cm} = 3\text{ m}$ . Perimeter  $= 2 \times (3.5 + 3) = 2 \times 6.5 = 13\text{ m}$ ; area  $= 3.5 \times 3 = 10.5\text{ m}^2$ .

**Q10.** Scale factor 50. (a)  $8 \times 50 = 400\text{ cm} = 4\text{ m}$ ;  $5 \times 50 = 250\text{ cm} = 2.5\text{ m}$ . (b) Area  $= 4 \times 2.5 = 10\text{ m}^2$ .

**Q11.** (a)  $2\text{ m} = 200\text{ cm}$ , so the scale is 1:200. (b)  $7 \times 200 = 1400\text{ cm} = 14\text{ m}$ .

**Q12.** The wall is  $5\text{ m} = 500\text{ cm}$ . (a) Plan A:  $500 \div 100 = 5\text{ cm}$ . Plan B:  $500 \div 200 = 2.5\text{ cm}$ . (b) Plan A draws the wall longer (5 cm vs 2.5 cm), so Plan A shows the house at the larger size (a smaller scale factor means a bigger drawing).

**Q13.** Scale factor 500. Length  $6 \times 500 = 3000\text{ cm} = 30\text{ m}$ ; width  $4 \times 500 = 2000\text{ cm} = 20\text{ m}$ . Area  $= 30 \times 20 = 600\text{ m}^2$ .

**Q14.** Scale factor 120.  $4.8\text{ m} = 480\text{ cm}$ , drawn  $480 \div 120 = 4\text{ cm}$ ;  $3.6\text{ m} = 360\text{ cm}$ , drawn  $360 \div 120 = 3\text{ cm}$ .

**Q15.** Real dimensions  $2.5 \times 100 = 250\text{ cm} = 2.5\text{ m}$  and  $2 \times 100 = 200\text{ cm} = 2\text{ m}$ . Area  $= 2.5 \times 2 = 5\text{ m}^2$ ; cost  $= 5 \times \$65 = \$325$ .

**Q16.** On a 1:100 plan each  $1\text{ cm}^2$  represents  $1\text{ m}^2$  (since  $100\text{ cm} \times 100\text{ cm} = 10\,000\text{ cm}^2 = 1\text{ m}^2$ ). So  $30\text{ cm}^2$  represents  $30\text{ m}^2$ .

**Q17.**  $3.5\text{ km} = 3500\text{ m} = 350\,000\text{ cm}$ ;  $350\,000 \div 25\,000 = 14\text{ cm}$ .

**Q18.** Scale factor 2000. (a)  $7 \times 2000 = 14\,000\text{ cm} = 140\text{ m}$ ;  $5 \times 2000 = 10\,000\text{ cm} = 100\text{ m}$ . (b) Perimeter  $= 2 \times (140 + 100) = 2 \times 240 = 480\text{ m}$ . (c) Area  $= 140 \times 100 = 14\,000\text{ m}^2 = 14\,000 \div 10\,000 = 1.4\text{ ha}$ .

## Chapter 10 Shape, angles and plans — 10I Drawing plans

### Theory

A **scale plan** is an accurate drawing of a real object — a room, a garden, a building block — made small enough to fit on paper, with every length reduced by the same amount. Because everything is reduced in the same ratio, the plan keeps the correct *shape* and lets us measure real lengths and areas from it.

#### Scale.

The **scale** tells you how a length on the plan compares to the real length. It is written as a ratio with no units, such as 1 : 100, meaning **1 unit on the plan represents 100 of the same units in reality**. At a scale of 1 : 100, a length of 1 cm on the plan stands for 100 cm = 1 m in real life. A scale is sometimes written in words instead, such as “1 cm represents 2 m”; converting the two lengths to the same unit turns this into a ratio (1 cm : 200 cm = 1 : 200).

The number after the colon is the **scale factor**. The bigger the scale factor, the more the real object is shrunk to fit the page.

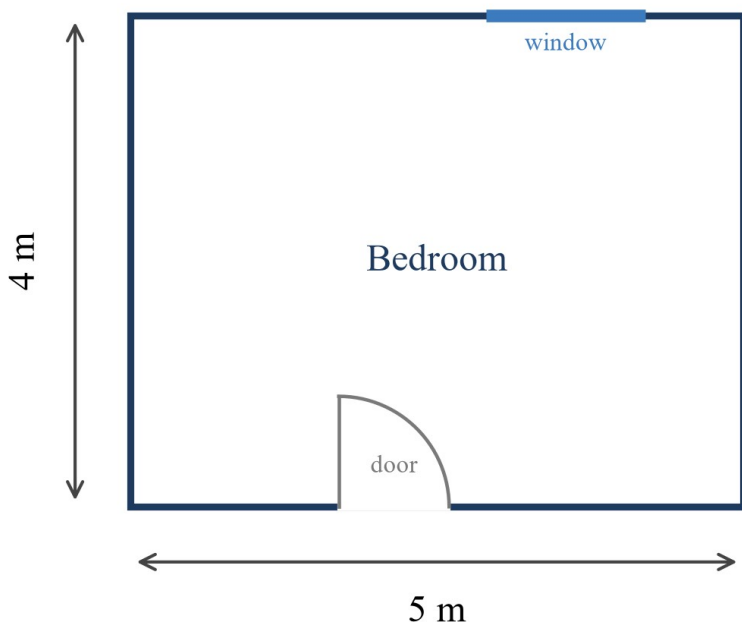
#### Converting between real and plan lengths.

Keep both lengths in the **same unit** (centimetres are easiest for plans), then:

- to find a **plan** length, **divide** the real length by the scale factor;
- to find a **real** length, **multiply** the plan length by the scale factor.

For example, at 1 : 100 a real wall of 5 m = 500 cm is drawn  $500 \div 100 = 5$  cm long; and a 4 m = 400 cm wall is drawn 4 cm long.

Scale 1 : 100 (on the plan, 1 cm represents 1 m)



#### Drawing a plan.

To draw an object to scale:

1. **Choose a scale** so the whole object fits the space on the page. Divide the longest real length by the space available to find the smallest scale factor that will fit, then round *up* to a convenient scale (1 : 50, 1 : 100, 1 : 200, 1 : 500, ...).
2. **Convert every real length** to a plan length by dividing by the scale factor.

3. **Draw** each plan length accurately with a ruler, keeping right angles square.
4. **Label** the drawing with the *real* dimensions and write the scale beside it.

### Reading measurements back from a plan.

To find a real length, measure the plan length and **multiply by the scale factor**. To find a real **area**, first find the real length and real width, then multiply them —  $\text{area} = \text{length} \times \text{width}$ . (Always convert to real lengths first; do not multiply a plan area by the scale factor.)

### Checking a plan.

A plan is only useful if it is consistent: every length must use the *same* scale. A quick check is to pick one labelled length, convert it with the stated scale, and confirm it matches the drawing.

## Worked examples

### Example 1 — scaffolded

A living room is 6 m long and 4.5 m wide. It is to be drawn at a scale of 1 : 100. Find the length and width of the room on the plan, in centimetres.

| Working                                                                 | Comment                                     |
|-------------------------------------------------------------------------|---------------------------------------------|
| Scale factor is 100; work in centimetres. 6 m = 600 cm, 4.5 m = 450 cm. | Convert real lengths to cm first.           |
| Plan length = $600 \div 100 = 6$ cm.                                    | Divide the real length by the scale factor. |
| Plan width = $450 \div 100 = 4.5$ cm.                                   | Same scale for every length.                |

### Example 2 — scaffolded

A backyard is 20 m wide. It must be drawn to fit across a space 10 cm wide on the page. Choose a suitable scale, then find the plan width of the yard and the plan length of a 15 m side fence.

| Working                                                                                  | Comment                                |
|------------------------------------------------------------------------------------------|----------------------------------------|
| 20 m = 2000 cm; to fit in 10 cm the scale factor must be at least $2000 \div 10 = 200$ . | Longest length $\div$ space available. |
| Choose the convenient scale 1 : 200.                                                     | Round up to a standard scale.          |
| Plan width = $2000 \div 200 = 10$ cm.                                                    | The yard just fits.                    |
| Fence: 15 m = 1500 cm, plan = $1500 \div 200 = 7.5$ cm.                                  | Same scale factor.                     |

### Example 3 — unscaffolded

A rectangular room is drawn at a scale of 1 : 50. On the plan it measures 9 cm long and 7 cm wide. Find the real length and width of the room, in metres, and its real floor area.

Multiply each plan length by the scale factor 50:

$$\text{length} = 9 \times 50 = 450 \text{ cm} = 4.5 \text{ m},$$

$$\text{width} = 7 \times 50 = 350 \text{ cm} = 3.5 \text{ m}.$$

$$\text{Real floor area} = 4.5 \times 3.5 = 15.75 \text{ m}^2.$$

$\therefore$  the room is 4.5 m by 3.5 m, with a floor area of  $15.75 \text{ m}^2$ .

### Example 4 — unscaffolded

A rectangular garden 12 m by 8 m is to be drawn at 1 : 200. It contains a square garden shed 4 m by 4 m. Find the plan dimensions of the garden and of the shed, and the real area of garden **not** covered by the shed.



Convert each real length (in cm) by dividing by 200:

$$12\text{ m} = 1200\text{ cm} \rightarrow 1200 \div 200 = 6\text{ cm}, 8\text{ m} = 800\text{ cm} \rightarrow 800 \div 200 = 4\text{ cm}.$$

$$\text{shed: } 4\text{ m} = 400\text{ cm} \rightarrow 400 \div 200 = 2\text{ cm}.$$

So the garden is drawn 6 cm by 4 cm and the shed 2 cm by 2 cm. Working with real areas: garden =  $12 \times 8 = 96\text{ m}^2$ , shed =  $4 \times 4 = 16\text{ m}^2$ , so uncovered garden =  $96 - 16 = 80\text{ m}^2$ .

$\therefore$  the garden is 6 cm  $\times$  4 cm and the shed 2 cm  $\times$  2 cm on the plan, and  $80\text{ m}^2$  of garden is uncovered.

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## Practice questions

### Scaffolded

**Q1.** A plan is drawn at a scale of 1 : 100. Find the plan length (in cm) of each real length. (a) a wall 3 m long. (b) a room 5.5 m long. (c) a house frontage 24 m long.

**Q2.** A plan is drawn at a scale of 1 : 50. Find the real length (in m) of each plan length. (a) 4 cm. (b) 6.5 cm. (c) 12 cm.

**Q3.** A room measuring 8 m by 6 m is drawn at a scale of 1 : 200. (a) Find the plan length. (b) Find the plan width. (c) Find the real floor area of the room.

**Q4.** A paddock is 40 m wide and must be drawn to fit in a space 20 cm wide. (a) Write the real width in centimetres. (b) Which scale fits the space: 1 : 100 or 1 : 200? (c) Find the plan width of the paddock at the scale you chose.

**Q5.** A plan drawn at 1 : 100 shows a rectangular office measuring 7 cm by 5 cm. (a) Find the real length. (b) Find the real width. (c) Find the real floor area.

**Q6.** A plan uses the scale “1 cm represents 2 m”. (a) Write this scale as a ratio. (b) Find the plan length of a fence that is 16 m long. (c) Find the real length of a line drawn 9 cm long on the plan.

### Un scaffolded

**Q7.** A kitchen bench is 3.6 m long. Find its length on a plan drawn at a scale of 1 : 50.

**Q8.** On a plan drawn at 1 : 100, a hallway measures 11.5 cm long. Find the real length of the hallway in metres.

**Q9.** A rectangular classroom is 9 m by 7 m. Find its plan dimensions at a scale of 1 : 150, giving each answer in centimetres to one decimal place.

**Q10.** A site plan is drawn at a scale of 1 : 500. A boundary line is drawn 8.4 cm long. Find the real length of the boundary in metres.

**Q11.** A garden bed 6 m by 2.5 m is drawn at 1 : 50. Find its plan dimensions in centimetres and the real area of the bed.

**Q12.** A house block is 30 m long and must fit on paper 15 cm wide. From the scales 1 : 100, 1 : 200 and 1 : 500, choose the one that fits **and** gives the largest possible drawing, and state the plan length at that scale.

**Q13.** A rectangular room is drawn at 1 : 50 and measures 8 cm by 6 cm on the plan. (a) Find the real dimensions in metres. (b) Find the real floor area. (c) Find the cost of carpeting the floor at \$ 45 per square metre.

**Q14.** A shed is 5 m by 3 m and is drawn at a scale of 1 : 40. (a) Find the plan dimensions of the shed in centimetres. (b) A door in the real shed is 0.8 m wide. Find its width on the plan.

**Q15.** A backyard 15 m by 10 m contains a rectangular pool 6 m by 3 m. The yard is drawn at 1 : 200. (a) Find the plan dimensions of the yard. (b) Find the plan dimensions of the pool. (c) Find the real area of grass (the yard area not taken up by the pool).

**Q16.** An L-shaped room is drawn at 1 : 100. On the plan it is made of two rectangles, one 5 cm by 4 cm and the other 3 cm by 2 cm. Find the real floor area of the whole room.

**Q17.** A map is drawn at a scale of 1 : 250. (a) Complete: “1 cm represents \_\_\_\_ m”. (b) Find the real length of a road drawn 6 cm long. (c) Find the plan length of a 100 m straight road.

**Q18.** A house floor plan is drawn at 1 : 100. On the plan the living area is a rectangle 7 cm by 5 cm and a bedroom is 4 cm by 3.5 cm. (a) Find the real dimensions of each room in metres. (b) Find the real area of each room. (c) Find the total real floor area of the two rooms. (d) Find the cost of tiling both rooms at \$ 38 per square metre.

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## Answers

**Q1.** (a) 3 cm. (b) 5.5 cm. (c) 24 cm. **Q2.** (a) 2 m. (b) 3.25 m. (c) 6 m. **Q3.** (a) 4 cm. (b) 3 cm. (c)  $48 \text{ m}^2$ . **Q4.** (a) 4000 cm. (b) 1:200. (c) 20 cm. **Q5.** (a) 7 m. (b) 5 m. (c)  $35 \text{ m}^2$ . **Q6.** (a) 1:200. (b) 8 cm. (c) 18 m. **Q7.** 7.2 cm. **Q8.** 11.5 m. **Q9.** 6.0 cm by 4.7 cm. **Q10.** 42 m. **Q11.** 12 cm by 5 cm; area  $15 \text{ m}^2$ . **Q12.** 1:200, plan length 15 cm. **Q13.** (a) 4 m by 3 m. (b)  $12 \text{ m}^2$ . (c) \$540. **Q14.** (a) 12.5 cm by 7.5 cm. (b) 2 cm. **Q15.** (a) 7.5 cm by 5 cm. (b) 3 cm by 1.5 cm. (c)  $132 \text{ m}^2$ . **Q16.**  $26 \text{ m}^2$ . **Q17.** (a) 2.5 m. (b) 15 m. (c) 40 cm. **Q18.** (a) living 7 m by 5 m, bedroom 4 m by 3.5 m. (b)  $35 \text{ m}^2$  and  $14 \text{ m}^2$ . (c)  $49 \text{ m}^2$ . (d) \$1862.

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## Fully-worked solutions

**Q1.** At 1:100, divide each real length (in cm) by 100; since 1 m = 100 cm, each metre becomes 1 cm. (a)  $300 \div 100 = 3 \text{ cm}$ . (b)  $550 \div 100 = 5.5 \text{ cm}$ . (c)  $2400 \div 100 = 24 \text{ cm}$ .

**Q2.** At 1:50, multiply each plan length by 50, then convert to metres. (a)  $4 \times 50 = 200 \text{ cm} = 2 \text{ m}$ . (b)  $6.5 \times 50 = 325 \text{ cm} = 3.25 \text{ m}$ . (c)  $12 \times 50 = 600 \text{ cm} = 6 \text{ m}$ .

**Q3.** Scale factor 200; work in cm. 8 m = 800 cm, 6 m = 600 cm. (a) Plan length =  $800 \div 200 = 4 \text{ cm}$ . (b) Plan width =  $600 \div 200 = 3 \text{ cm}$ . (c) Real area =  $8 \times 6 = 48 \text{ m}^2$ .

**Q4.** (a) 40 m =  $40 \times 100 = 4000 \text{ cm}$ . (b) At 1:100 the plan width would be  $4000 \div 100 = 40 \text{ cm}$ , too wide for the 20 cm space; at 1:200 it is  $4000 \div 200 = 20 \text{ cm}$ , which fits. Choose 1:200. (c) Plan width = 20 cm.

**Q5.** At 1:100, multiply each plan length by 100. (a)  $7 \times 100 = 700 \text{ cm} = 7 \text{ m}$ . (b)  $5 \times 100 = 500 \text{ cm} = 5 \text{ m}$ . (c) Real area =  $7 \times 5 = 35 \text{ m}^2$ .

**Q6.** (a) 1 cm represents 2 m = 200 cm, so the ratio is 1:200. (b) Plan length =  $16 \div 2 = 8 \text{ cm}$  (each 2 m is 1 cm). (c) Real length =  $9 \times 2 = 18 \text{ m}$ .

**Q7.** 3.6 m = 360 cm; plan length =  $360 \div 50 = 7.2 \text{ cm}$ .

**Q8.** Real length =  $11.5 \times 100 = 1150 \text{ cm} = 11.5 \text{ m}$ .

**Q9.** Scale factor 150. 9 m = 900 cm  $\rightarrow 900 \div 150 = 6.0 \text{ cm}$ ; 7 m = 700 cm  $\rightarrow 700 \div 150 = 4.67 \approx 4.7 \text{ cm}$ . So 6.0 cm by 4.7 cm.

**Q10.** Real length =  $8.4 \times 500 = 4200 \text{ cm} = 42 \text{ m}$ .

**Q11.** 6 m = 600 cm  $\rightarrow 600 \div 50 = 12 \text{ cm}$ ; 2.5 m = 250 cm  $\rightarrow 250 \div 50 = 5 \text{ cm}$ . Real area =  $6 \times 2.5 = 15 \text{ m}^2$ .

**Q12.** 30 m = 3000 cm. At 1:100:  $3000 \div 100 = 30 \text{ cm}$  (too wide). At 1:200:  $3000 \div 200 = 15 \text{ cm}$  (fits exactly). At 1:500:  $3000 \div 500 = 6 \text{ cm}$  (fits but smaller). The largest drawing that still fits is at 1:200, giving a plan length of 15 cm.

**Q13.** At 1:50: (a)  $8 \times 50 = 400 \text{ cm} = 4 \text{ m}$  and  $6 \times 50 = 300 \text{ cm} = 3 \text{ m}$ . (b) Real area =  $4 \times 3 = 12 \text{ m}^2$ . (c) Carpet cost =  $12 \times \$45 = \$540$ .

**Q14.** Scale factor 40. (a) 5 m = 500 cm  $\rightarrow 500 \div 40 = 12.5 \text{ cm}$ ; 3 m = 300 cm  $\rightarrow 300 \div 40 = 7.5 \text{ cm}$ . (b) Door 0.8 m = 80 cm  $\rightarrow 80 \div 40 = 2 \text{ cm}$ .

**Q15.** Scale factor 200. (a) Yard: 15 m = 1500 cm  $\rightarrow 7.5 \text{ cm}$ ; 10 m = 1000 cm  $\rightarrow 5 \text{ cm}$ . (b) Pool: 6 m = 600 cm  $\rightarrow 3 \text{ cm}$ ; 3 m = 300 cm  $\rightarrow 1.5 \text{ cm}$ . (c) Grass =  $15 \times 10 - 6 \times 3 = 150 - 18 = 132 \text{ m}^2$ .

**Q16.** At 1:100 each plan centimetre is 1 m. First rectangle:  $5 \text{ cm} \times 4 \text{ cm} \rightarrow 5 \text{ m} \times 4 \text{ m} = 20 \text{ m}^2$ . Second rectangle:  $3 \text{ cm} \times 2 \text{ cm} \rightarrow 3 \text{ m} \times 2 \text{ m} = 6 \text{ m}^2$ . Total floor area =  $20 + 6 = 26 \text{ m}^2$ .

**Q17.** (a)  $1:250$  means 1 cm represents  $250\text{ cm} = 2.5\text{ m}$ . (b) Real length  $= 6 \times 2.5 = 15\text{ m}$ . (c)  $100\text{ m} = 10\,000\text{ cm} \rightarrow 10\,000 \div 250 = 40\text{ cm}$ .

**Q18.** At  $1:100$  each plan centimetre is 1 m. (a) Living area  $7\text{ cm} \times 5\text{ cm} \rightarrow 7\text{ m} \times 5\text{ m}$ ; bedroom  $4\text{ cm} \times 3.5\text{ cm} \rightarrow 4\text{ m} \times 3.5\text{ m}$ . (b) Living area  $= 7 \times 5 = 35\text{ m}^2$ ; bedroom  $= 4 \times 3.5 = 14\text{ m}^2$ . (c) Total floor area  $= 35 + 14 = 49\text{ m}^2$ . (d) Tiling cost  $= 49 \times \$38 = \$1862$ .