Foundation Mathematics Units 1 & 2

Chapter 8 Taxation, interest and credit

Aboriginal and Torres Strait Islander content has been excluded as accuracy cannot be guaranteed, and it is better to be respectful than cover the entire curriculum inaccurately.

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Theory

The **Goods and Services Tax (GST)** is a tax of **10%** added to the price of most goods and services sold in Australia. The business collecting it passes the GST on to the Australian Taxation Office. Because GST is built into the prices we pay every day, being able to work with a $10\,\%$ tax quickly is a genuinely useful skill.

Adding GST to a price.

When GST is added to a price, the tax is $10\,\%$ of that price:

$$\text{GST} = 10\,\% \text{ of the price} = \text{price} \times 0.1\,.$$

The **GST-inclusive price** (what the customer actually pays) is the original price plus the tax, which is the same as multiplying by 1.1:

$$\text{price including GST} = \text{price} + \text{GST} = \text{price} \times 1.1\,.$$

For example, a service priced at $\$200$ before GST has $\$20$ of GST added, giving a total of $\$220$.

GST-exclusive and GST-inclusive quoting.

Prices can be quoted two ways. A price **before GST** (also called *GST-exclusive* or *plus GST*) does not yet include the tax — businesses often quote each other this way. A price **including GST** (*GST-inclusive*) already contains the tax — this is how prices are shown to ordinary shoppers. Always check which kind of price you are given before you calculate.

Working backwards from a GST-inclusive price.

If a price **already includes** GST, the tax is not $10\,\%$ of the amount shown, because the $10\,\%$ was added to the *smaller* pre-tax price. Since the inclusive price is 1.1 times the pre-tax price, we divide to undo it:

$$\text{price before GST} = \frac{\text{price including GST}}{1.1}\,.$$

The GST hidden inside an inclusive price is found even more simply. The inclusive price is 1.1 times the pre-tax price and the GST is 0.1 times it, so the GST is $\frac{0.1}{1.1} = \frac{1}{11}$ of the inclusive price:

$$\text{GST component} = \frac{\text{price including GST}}{11}\,.$$

So on a $\$220$ inclusive price the GST is $220 \div 11 = \$20$, and the price before GST is $220 \div 1.1 = \$200$ — matching the example above.

GST-free items.

Not everything attracts GST. **Basic food** (fresh fruit and vegetables, plain milk, bread, meat), and most **health**, **education** and some childcare services are **GST-free**. When a bill mixes GST-free and taxable items, add GST **only** to the taxable part.

Worked examples

Example 1 — scaffolded

An electrician quotes $\$340$ for a repair job. This is the price **before GST**. Find the GST to be added and the total price the customer pays.

Working	Comment
GST $= 10\%$ of $\$340 = 0.1 \times 340 = \$34$.	GST added is 10% of the pre-tax price.
Total $= 340 + 34 = \$374$.	Price before GST plus the GST.
Check: $340 \times 1.1 = \$374$.	Multiplying by 1.1 adds the 10% in one step.

Example 2 — scaffolded

A hardware store receipt shows a **GST-inclusive** total of $\$88$ for a power tool. Find the GST included in that price, and the price before GST.

Working	Comment
GST component $= 88 \div 11 = \$8$.	For a price that already includes GST, the tax is $1/11$ of it.
Price before GST $= 88 \div 1.1 = \$80$.	Divide by 1.1 to remove the 10%.
Check: $80 + 8 = \$88$.	Pre-tax price plus GST returns the inclusive price.

Example 3 — unscaffolded

A phone is advertised at $\$913$, a price that already includes GST. Find the price before GST and the amount of GST the buyer pays.

The advertised price is GST-inclusive, so it is 1.1 times the pre-tax price. Dividing,

$$\text{price before GST} = \frac{913}{1.1} = \$830.$$

The GST is one-eleventh of the inclusive price:

$$\text{GST} = \frac{913}{11} = \$83.$$

Checking, $830 + 83 = 913$. ✓

$\therefore$ the price before GST is $\$830$ and the GST paid is $\$83$.

Example 4 — unscaffolded

A small café buys $\$66$ of crockery, on which GST applies, and $\$45$ of fresh milk and bread, which are GST-free. Both amounts are before any GST. Find the total GST payable and the total cost.

GST is added only to the taxable crockery:

$$\text{GST} = 0.1 \times 66 = \$6.60.$$

The milk and bread are GST-free, so no tax is added to the $\$45$. The total cost is the two pre-tax amounts plus the GST:

$$66 + 6.60 + 45 = \$117.60.$$

$\therefore$ the total GST is $\$6.60$ and the total cost is $\$117.60$.

Practice questions

Scaffolded

Q1. A plumber’s labour charge is $\$260$ before GST. (a) Find the GST to be added. (b) Find the total charge including GST. (c) The plumber also supplies parts costing $\$88$ before GST. Find the total charge for labour and parts including GST.

Q2. A restaurant bill comes to $\$99$, including GST. (a) Find the GST included in the bill. (b) Find the price before GST. (c) What fraction of the GST-inclusive bill is GST?

Q3. A supplier quotes prices **before GST**. Find the GST-inclusive price of each item. (a) A chair priced at $\$150$. (b) A desk priced at $\$1240$. (c) A cable priced at $\$18.50$.

Q4. Each of these prices **includes** GST. Find the GST component of each. (a) $\$44$. (b) $\$253$. (c) $\$16.50$.

Q5. Each of these prices **includes** GST. Find the price before GST. (a) $\$385$. (b) $\$71.50$. (c) $\$3850$.

Q6. A shopping trolley holds $\$60$ of fresh fruit and vegetables, which are GST-free, and $\$30$ of packaged snacks, on which GST applies. Both amounts are before GST. (a) On which items is GST charged? (b) Find the GST added. (c) Find the total to pay.

Unscaffolded

Q7. A furniture store advertises a table for $\$660$ including GST. Find the GST component and the price before GST.

Q8. An accountant charges $\$1200$ before GST for preparing a tax return. Find the total the client pays including GST.

Q9. A tradesperson's invoice shows a subtotal of $\$840$ before GST. Find the GST and the invoice total.

Q10. A phone-screen repair costs $\$148.50$ including GST. Find the price before GST and the GST paid.

Q11. A registered business buys equipment for $\$3520$ including GST. A registered business can claim back the GST it pays. How much GST can the business claim?

Q12. A household order is made up of $\$80$ of GST-free food and $\$50$ of GST-applicable cleaning products, both before GST. Find the total GST and the total cost.

Q13. Two shops sell the same appliance. Shop A advertises it as "$\$500$ plus GST". Shop B advertises it as "$\$544$ including GST". Which shop is cheaper, and by how much?

Q14. A café's takings for a month are $\$6600$ including GST. Find the amount of GST the café must send to the tax office.

Q15. A builder quotes $\$12\,400$ before GST for a renovation. The client has budgeted $\$13\,000$ including GST. Determine whether the client can afford the job, and by how much they are over or under budget.

Q16. The GST component of an item is $\$7$. Find the price before GST and the GST-inclusive price.

Q17. A supermarket receipt shows a GST-inclusive total of $\$93.50$. Of this, $\$22$ was spent on GST-free food. Find the GST included in the receipt.

Q18. A tradesperson completes a job with $\$520$ of labour and $\$180$ of materials, both before GST. (a) Find the total before GST. (b) Find the GST. (c) Find the total including GST, and state what fraction of the final bill is GST.

Answers

Q1. (a) $\$26$. (b) $\$286$. (c) $\$382.80$. **Q2.** (a) $\$9$. (b) $\$90$. (c) $1/11$ (about 9.1%). **Q3.** (a) $\$165$. (b) $\$1364$. (c) $\$20.35$. **Q4.** (a) $\$4$. (b) $\$23$. (c) $\$1.50$. **Q5.** (a) $\$350$. (b) $\$65$. (c) $\$3500$. **Q6.** (a) the packaged snacks only. (b) $\$3$. (c) $\$93$. **Q7.** GST $\$60$; before GST $\$600$. **Q8.** $\$1320$. **Q9.** GST $\$84$; total $\$924$. **Q10.** before GST $\$135$; GST $\$13.50$. **Q11.** $\$320$. **Q12.** GST $\$5$; total $\$135$. **Q13.** Shop B, by $\$6$ (A costs $\$550$, B costs $\$544$). **Q14.** $\$600$. **Q15.** No; the job costs $\$13\,640$ including GST, so the client is $\$640$ over budget. **Q16.** before GST $\$70$; including GST $\$77$. **Q17.** $\$6.50$. **Q18.** (a) $\$700$. (b) $\$70$. (c) $\$770$; the GST is $1/11$ of the bill (about 9.1%).

Fully-worked solutions

Q1. (a) GST $=0.1\times260=\$26$. (b) Total $=260+26=\$286$ (or $260\times1.1=\$286$). (c) Labour and parts before GST $=260+88=\$348$; including GST, $348\times1.1=\$382.80$.

Q2. The bill of $\$99$ already includes GST. (a) GST $=99\div11=\$9$. (b) Price before GST $=99\div1.1=\$90$. (c) The GST is $\dfrac{9}{99}=\dfrac{1}{11}$ of the inclusive bill, about 9.1%.

Q3. Multiply each pre-GST price by 1.1. (a) $150\times1.1=\$165$. (b) $1240\times1.1=\$1364$. (c) $18.50\times1.1=\$20.35$.

Q4. Each price includes GST, so the GST is $1/11$ of it. (a) $44\div11=\$4$. (b) $253\div11=\$23$. (c) $16.50\div11=\$1.50$.

Q5. Each price includes GST, so divide by 1.1 to remove it. (a) $385\div1.1=\$350$. (b) $71.50\div1.1=\$65$. (c) $3850\div1.1=\$3500$.

Q6. (a) Fresh fruit and vegetables are GST-free, so GST is charged on **the packaged snacks only**. (b) GST $=0.1\times30=\$3$. (c) Total $=60+30+3=\$93$.

Q7. The advertised $\$660$ includes GST. GST component $=660\div11=\$60$; price before GST $=660\div1.1=\$600$ (check $600+60=660$).

Q8. The charge is before GST, so the total is $1200\times1.1=\$1320$.

Q9. GST $=0.1\times840=\$84$; invoice total $=840+84=\$924$.

Q10. The $\$148.50$ includes GST. Price before GST $=148.50\div1.1=\$135$; GST $=148.50\div11=\$13.50$ (check $135+13.50=148.50$).

Q11. The claimable GST is the GST inside the inclusive price: $3520\div11=\$320$.

Q12. GST applies only to the $\$50$ of cleaning products: GST $=0.1\times50=\$5$. Total $=80+50+5=\$135$.

Q13. Shop A's inclusive price is $500\times1.1=\$550$. Shop B's inclusive price is $\$544$. Since $544<550$, **Shop B** is cheaper, by $550-544=\$6$.

Q14. The takings include GST, so the GST portion is $6600\div11=\$600$.

Q15. The quote is before GST, so the job costs $12\,400\times1.1=\$13\,640$ including GST. Since $13\,640>13\,000$, the client **cannot** afford it and is $13\,640-13\,000=\$640$ over budget.

Q16. The GST is 10% of the pre-GST price, so the pre-GST price $=7\div0.1=\$70$. The GST-inclusive price $=70+7=\$77$.

Q17. The GST-free food carries no tax, so remove it first: the taxable, GST-inclusive part is $93.50-22=\$71.50$. The GST inside that is $71.50\div11=\$6.50$.

Q18. (a) Total before GST $=520+180=\$700$. (b) GST $=0.1\times700=\$70$. (c) Total including GST $=700+70=\$770$; the GST is $\dfrac{70}{770}=\dfrac{1}{11}$ of the final bill, about 9.1%.

Theory

Most of the money that pays for hospitals, schools and roads is raised through **taxation**. If you earn an income, some of it is paid to the government as **income tax**. This section shows how income tax is worked out from a **tax table**, how the **Medicare levy** and **superannuation** are added, and how the **PAYG** system means the tax you pay through the year is squared up at the end of it.

Taxable income and the tax table.

Your **taxable income** is what is left of your income after any allowable deductions. Australia uses a **progressive** tax system: you pay a higher rate only on the part of your income that falls in a higher **bracket**, not on your whole income. The table below (used throughout this section) sets out the brackets for the **2026–27 financial year**.

Taxable income	Tax on this income
\$0 – \$18 200	Nil
\$18 201 – \$45 000	15c for each \$1 over \$18 200
\$45 001 – \$135 000	\$4020 plus 30c for each \$1 over \$45 000
\$135 001 – \$190 000	\$31 020 plus 37c for each \$1 over \$135 000
\$190 001 and over	\$51 370 plus 45c for each \$1 over \$190 000

The first \$18 200 is the **tax-free threshold**. To find the tax, locate the bracket your taxable income falls in, then add the fixed amount for that bracket to the marginal rate applied to the part of your income above the bracket's lower edge. For example, on a taxable income of \$45 000 the tax is $0.15\times\left(45\,000-18\,200\right)=0.15\times 26\,800=\$4020$, which is exactly the fixed amount that starts the next bracket.

The Medicare levy.

Most taxpayers also pay the **Medicare levy**, which helps fund public health care. It is

$$\text{Medicare levy}=2\,\%\,\text{of taxable income}.$$

Your **total tax** for the year is the income tax from the table **plus** the Medicare levy.

PAYG and your tax return.

Under **Pay As You Go (PAYG)**, your employer withholds an estimated amount of tax from each pay and sends it to the government during the year. At the end of the financial year you lodge a **tax return** that works out your actual total tax. Then:

- if the tax **withheld** is **more** than the total tax payable, you receive the difference back as a **refund**;
- if the tax withheld is **less** than the total tax payable, you must pay the difference as a **tax bill**.

Superannuation.

Superannuation ("super") is money set aside for your retirement. By law an employer must pay a super contribution **on top of** your wage, into your super fund. The rate is set by the Australian Government's superannuation guarantee, and has been 12% of ordinary earnings since 1 July 2025:

$$\text{super contribution}=12\,\%\,\text{of ordinary earnings}.$$

Super is **extra**, not taken out of your pay, so the true cost of employing someone is their wage plus their super.

Average and marginal rates.

The **marginal rate** is the rate charged on your next dollar of income (the rate for your top bracket). The **average rate** is the total tax divided by the total income, written as a percentage. Because the first parts of your income are taxed lightly (or not at all), anyone earning above the tax-free threshold has an average rate lower than their marginal rate.

(At or below the threshold the two rates are equal — both 0% — and at very high incomes the 2% Medicare levy can push the average rate above the top bracket rate.)

Worked examples

Example 1 — scaffolded

Liam has a taxable income of $52 000. Using the tax table, find his income tax, his Medicare levy, and his total tax for the year.

Working	Comment
$52 000 is in the $45 001–$135 000 bracket.	Locate the bracket first.
Income tax $= 4020 + 0.30 \times (52\,000 - 45\,000)$.	Fixed amount + marginal rate on the excess.
$= 4020 + 0.30 \times 7000 = 4020 + 2100 = \$6120$.	Only the $7000 above $45 000 is taxed at 30c.
Medicare levy $= 0.02 \times 52\,000 = \$1040$.	2% of taxable income.
Total tax $= 6120 + 1040 = \$7160$.	Income tax plus Medicare levy.

Example 2 — scaffolded

Maya's ordinary earnings are $64 000 a year. Her employer pays superannuation at 12%. Find her yearly super contribution, the monthly amount paid into her fund, and the total yearly cost of employing her.

Working	Comment
Yearly super $= 0.12 \times 64\,000 = \$7680$.	Super is 12% of ordinary earnings.
Monthly super $= 7680 \div 12 = \$640$.	Spread evenly over 12 months.
Total cost $= 64\,000 + 7680 = \$71\,680$.	Super is paid **on top of** the wage.

Example 3 — unscaffolded

Priya has a taxable income of $88 000. Her employer withheld $22 000 in PAYG tax during the year. Work out her total tax for the year, and state whether she receives a refund or has a bill, and how much.

Her income is in the $45 001–$135 000 bracket, so the income tax is

$$4020 + 0.30 \times (88\,000 - 45\,000) = 4020 + 0.30 \times 43\,000 = 4020 + 12\,900 = \$16\,920.$$

The Medicare levy is $0.02 \times 88\,000 = \$1760$, so the total tax payable is

$$16\,920 + 1760 = \$18\,680.$$

She had $22 000 withheld, which is more than $18 680, so she is refunded the difference: $22\,000 - 18\,680 = \$3320$.

∴ Priya's total tax is $18 680, and because more was withheld she receives a refund of $3320.

Example 4 — unscaffolded

Sam earns a salary of $60 000 (all taxable). Find his total tax, his net (take-home) income for the year, and his take-home pay per month. His employer also pays 12% super — state how much goes into his super fund.

The income tax is $4020 + 0.30 \times (60\,000 - 45\,000) = 4020 + 0.30 \times 15\,000 = 4020 + 4500 = \$8520$. The Medicare levy is $0.02 \times 60\,000 = \$1200$, so the total tax is $8520 + 1200 = \$9720$. His net income is

$$60\,000 - 9720 = \$50\,280,$$

which per month is $50\,280 \div 12 = \$4190$. Separately, his super is $0.12 \times 60\,000 = \$7200$, paid on top into his fund.

∴ Sam's take-home pay is $50 280 a year ($4190 a month), and $7200 of super is added to his fund.

Practice questions

Scaffolded

Q1. Aisha has a taxable income of $30 000. (a) State which bracket of the tax table her income falls in. (b) Find her income tax using the table. (c) Find her Medicare levy (2% of taxable income).

Q2. Ben's ordinary earnings are $48 000 and his employer pays super at 12%. (a) Find his yearly super contribution. (b) Find the monthly amount paid into his fund. (c) Find the total yearly cost of employing Ben (wage plus super).

Q3. Chloe has a taxable income of $95 000. (a) Find her income tax using the table. (b) Find her Medicare levy. (c) Find her total tax for the year.

Q4. Dan has a taxable income of $40 000, and $4000 was withheld in PAYG during the year. (a) Find his income tax. (b) Find his total tax (income tax plus Medicare levy). (c) State whether he receives a refund or has a bill, and how much.

Q5. Ella has a taxable income of exactly $45 000. (a) Find her income tax. (b) Find her Medicare levy. (c) Find her net income for the year (after income tax and Medicare levy).

Q6. Farid earns $55 000 and his employer pays 12% super. (a) Find his super contribution for one year. (b) Assuming his salary stays the same, find the total super paid over 3 years (ignore any investment growth). (c) Write one year's super as a percentage of his salary.

Unscaffolded

Q7. Grace has a taxable income of $22 000. Find her income tax using the table and her Medicare levy.

Q8. Hugo has a taxable income of $150 000. Find his income tax using the table, and his total tax including the Medicare levy.

Q9. Ivy's ordinary earnings are $72 000, with super paid at 12%. Find her yearly super contribution and the monthly amount paid into her fund.

Q10. Jack has a taxable income of $68 000, and $15 000 was withheld in PAYG. Find his total tax (income tax plus Medicare levy), and the refund or bill.

Q11. Two workers pay tax under the table: Kim's taxable income is $40 000 and Leo's is $80 000. (a) Find each person's income tax. (b) Find each person's income tax as a percentage of their income (the average rate, to 1 d.p.). (c) Comment on what this shows about the progressive tax system.

Q12. Mia has a taxable income of $84 000. Find her income tax, her Medicare levy, her net income for the year, and her take-home pay per month (to the nearest cent).

Q13. Noah's ordinary earnings rise from $58 000 to $64 000. Super is paid at 12%. Find the increase in his yearly super contribution.

Q14. Olivia has a taxable income of $50 000. Find her income tax, her Medicare levy, her total tax, and her total tax as a percentage of her income (to 1 d.p.).

Q15. Pat has a taxable income of $60 000, and his employer withholds tax in 12 equal monthly amounts. (a) Find his total tax for the year (income tax plus Medicare levy). (b) Find the monthly amount withheld (to the nearest cent). (c) If the Medicare levy were **not** charged, how much less would be withheld each month?

Q16. Quinn has a taxable income of $210 000. Find her income tax using the table, and her total tax including the Medicare levy.

Q17. Riya earns a salary of $70 000 (all taxable); her employer also pays 12% super. (a) Find her total tax (income tax plus Medicare levy). (b) Find her net income for the year. (c) Find her yearly super contribution. (d) Find the total value to Riya of one year's work (take-home pay plus super).

Q18. Sara earns $44 000 and Tom earns $46 000, so Tom's income crosses the $45 000 bracket edge. (a) Find each person's income tax using the table. (b) Find how much more income tax Tom pays than Sara. (c) Tom earns $2000 more than Sara. How much of that $2000 does he keep after the extra tax, and what does this show about earning income above a bracket edge?

Answers

Q1. (a) \$18 201–\$45 000. (b) \$1770. (c) \$600. **Q2.** (a) \$5760. (b) \$480. (c) \$53 760. **Q3.** (a) \$19 020. (b) \$1900. (c) \$20 920. **Q4.** (a) \$3270. (b) \$4070. (c) a bill of \$70. **Q5.** (a) \$4020. (b) \$900. (c) \$40 080. **Q6.** (a) \$6600. (b) \$19 800. (c) 12%. **Q7.** income tax \$570, Medicare levy \$440. **Q8.** income tax \$36 570, total tax \$39 570. **Q9.** yearly super \$8640, monthly \$720. **Q10.** total tax \$12 280; refund of \$2720. **Q11.** (a) Kim \$3270, Leo \$14 520. (b) Kim 8.2%, Leo 18.2%. (c) The higher earner pays a larger *proportion* of income, because more of it falls in higher brackets — that is what "progressive" means. **Q12.** income tax \$15 720, Medicare \$1680, net income \$66 600, monthly take-home \$5550. **Q13.** \$720. **Q14.** income tax \$5520, Medicare \$1000, total tax \$6520, which is 13.0% of income. **Q15.** (a) \$9720. (b) \$810. (c) \$100 less per month. **Q16.** income tax \$60 370, total tax \$64 570. **Q17.** (a) \$12 920. (b) \$57 080. (c) \$8400. (d) \$65 480. **Q18.** (a) Sara \$3870, Tom \$4320. (b) \$450. (c) He keeps $2000-450=\$1550$; only the \$1000 above \$45 000 is taxed at the higher rate, so earning more still leaves him better off.

Fully-worked solutions

Q1. (a) \$30 000 lies between \$18 201 and \$45 000, so it is in the second bracket. (b) Income tax $=0.15\times(30\,000-18\,200)=0.15\times 11\,800=\$1770$. (c) Medicare levy $=0.02\times 30\,000=\$600$.

Q2. (a) Super $=0.12\times 48\,000=\$5760$. (b) Monthly $=5760\div 12=\$480$. (c) Total cost $=48\,000+5760=\$53\,760$ (wage plus super).

Q3. (a) \$95 000 is in the \$45 001–\$135 000 bracket: income tax $=4020+0.30\times(95\,000-45\,000)=4020+0.30\times 50\,000=4020+15\,000=\$19\,020$. (b) Medicare levy $=0.02\times 95\,000=\$1900$. (c) Total tax $=19\,020+1900=\$20\,920$.

Q4. (a) \$40 000 is in the second bracket: income tax $=0.15\times(40\,000-18\,200)=0.15\times 21\,800=\$3270$. (b) Total tax $=3270+0.02\times 40\,000=3270+800=\$4070$. (c) Only \$4000 was withheld, which is less than \$4070, so Dan has a bill of $4070-4000=\$70$.

Q5. (a) At \$45 000 the income tax is $0.15\times(45\,000-18\,200)=0.15\times 26\,800=\$4020$. (b) Medicare levy $=0.02\times 45\,000=\$900$. (c) Net income $=45\,000-(4020+900)=45\,000-4920=\$40\,080$.

Q6. (a) Super $=0.12\times 55\,000=\$6600$. (b) Over 3 years $=3\times 6600=\$19\,800$. (c) $\frac{6600}{55\,000}\times 100\%=12\%$ — as expected, since super is defined as 12% of the salary.

Q7. \$22 000 is in the second bracket: income tax $=0.15\times(22\,000-18\,200)=0.15\times 3800=\$570$. Medicare levy $=0.02\times 22\,000=\$440$.

Q8. \$150 000 is in the \$135 001–\$190 000 bracket: income tax $=31\,020+0.37\times(150\,000-135\,000)=31\,020+0.37\times 15\,000=31\,020+5550=\$36\,570$. Total tax $=36\,570+0.02\times 150\,000=36\,570+3000=\$39\,570$.

Q9. Super $=0.12\times 72\,000=\$8640$; monthly $=8640\div 12=\$720$.

Q10. \$68 000 is in the \$45 001–\$135 000 bracket: income tax $=4020+0.30\times(68\,000-45\,000)=4020+0.30\times 23\,000=4020+6900=\$10\,920$. Total tax $=10\,920+0.02\times 68\,000=10\,920+1360=\$12\,280$. Since \$15 000 was withheld and $15\,000>12\,280$, Jack receives a refund of $15\,000-12\,280=\$2720$.

Q11. (a) Kim: $0.15\times(40\,000-18\,200)=0.15\times 21\,800=\$3270$. Leo: $4020+0.30\times(80\,000-45\,000)=4020+0.30\times 35\,000=4020+10\,500=\$14\,520$. (b) Kim: $\frac{3270}{40\,000}\times 100\%=8.2\%$ (1 d.p.). Leo: $\frac{14\,520}{80\,000}\times 100\%=18.2\%$ (1 d.p.). (c) Leo earns twice as much as Kim but pays a much larger *share* of his income in tax (about 18.2% versus 8.2%), because more of his income falls in higher brackets. That is what a progressive tax system does.

Q12. \$84 000 is in the \$45 001–\$135 000 bracket: income tax $=4020+0.30\times(84\,000-45\,000)=4020+0.30\times 39\,000=4020+11\,700=\$15\,720$. Medicare levy $=0.02\times 84\,000=\$1680$. Total tax $=15\,720+1680=\$17\,400$. Net income $=84\,000-17\,400=\$66\,600$; per month $=66\,600\div 12=\$5550$.

Q13. Old super $=0.12\times 58\,000=\$6960$; new super $=0.12\times 64\,000=\$7680$. Increase $=7680-6960=\$720$.

Q14. \$50 000 is in the \$45 001–\$135 000 bracket: income tax $=4020+0.30\times(50\,000-45\,000)=4020+0.30\times 5000=4020+1500=\$5520$. Medicare levy $=0.02\times 50\,000=\$1000$. Total tax $=5520+1000=\$6520$. As a percentage: $\dfrac{6520}{50\,000}\times 100\,\%=13.0\,\%$ (1 d.p.).

Q15. (a) Income tax $=4020+0.30\times(60\,000-45\,000)=4020+4500=\$8520$; Medicare levy $=0.02\times 60\,000=\$1200$; total tax $=8520+1200=\$9720$. (b) Monthly $=9720\div 12=\$810$. (c) The Medicare levy is \$1200 a year, or $1200\div 12=\$100$ a month, so without it \$100 less would be withheld each month.

Q16. \$210 000 is in the top bracket: income tax $=51\,370+0.45\times(210\,000-190\,000)=51\,370+0.45\times 20\,000=51\,370+9000=\$60\,370$. Total tax $=60\,370+0.02\times 210\,000=60\,370+4200=\$64\,570$.

Q17. (a) Income tax $=4020+0.30\times(70\,000-45\,000)=4020+0.30\times 25\,000=4020+7500=\$11\,520$; Medicare levy $=0.02\times 70\,000=\$1400$; total tax $=11\,520+1400=\$12\,920$. (b) Net income $=70\,000-12\,920=\$57\,080$. (c) Super $=0.12\times 70\,000=\$8400$. (d) Total value $=57\,080+8400=\$65\,480$ (take-home pay plus super).

Q18. (a) Sara (\$44 000, second bracket): $0.15\times(44\,000-18\,200)=0.15\times 25\,800=\$3870$. Tom (\$46 000, third bracket): $4020+0.30\times(46\,000-45\,000)=4020+300=\$4320$. (b) Extra tax $=4320-3870=\$450$. (c) Tom keeps $2000-450=\$1550$ of his extra \$2000. Crossing the \$45 000 edge does **not** tax his whole income at the higher rate — only the \$1000 above \$45 000 is taxed at 30c — so earning more always leaves him better off overall.

Theory

Whenever you keep money in a savings account the bank pays you **interest**; whenever you borrow money — a personal loan, a car loan — you pay the lender interest. Interest is the *cost of using someone else's money for a period of time*. The simplest kind, used throughout Units 1 & 2, is **simple interest**.

The simple interest formula.

Simple interest is worked out on the original amount only — called the **principal** — and is the same each year. If a principal P dollars is invested or borrowed at an interest rate of $r\,\%$ per year for t years, the interest is

$$I=\frac{P\,r\,t}{100}.$$

Here P is in dollars, r is the **percentage rate per year** (per annum, "p.a."), and t is the **time in years**. The rate and the time must match the same unit of time — always convert a time given in months to years by dividing by 12.

Savings and loans.

For a **savings** account, the interest is added to your money, so the **total amount** after t years is

$$A=P+I.$$

For a **loan**, the interest is added to what you owe, so the **total to repay** is also $P+I$ (the amount borrowed plus the interest charged).

Rearranging the formula.

If you know the interest and two of the other quantities, rearrange $I=\frac{P\,r\,t}{100}$ to find the third:

$$P=\frac{100\,I}{r\,t},\ r=\frac{100\,I}{P\,t},\ t=\frac{100\,I}{P\,r}.$$

How interest depends on P, r and t.

Because P, r and t are all *multiplied* together, the interest is **proportional** to each of them: double the principal (or the rate, or the time) and the interest doubles. A larger loan, a higher rate, or a longer term all increase the interest paid.

Time and risk.

A lender sets the interest rate according to **risk** and **time**. Lending to a borrower who is less likely to repay, or lending for a longer term, is riskier, so the lender charges a **higher rate**. But a higher rate does not always mean more interest overall: a loan at a higher rate over a shorter time can cost *less* interest than a loan at a lower rate over a longer time, because the interest depends on the rate **and** the time together.

Worked examples

Example 1 — scaffolded

Priya invests $\$2500$ in a savings account paying $4\,\%$ per annum simple interest. Find the interest earned after 3 years and the total amount in the account.

Working	Comment
List the values: $P=2500$, $r=4$, $t=3$.	Rate is per year; time is already in years.

Working	Comment
$I=\frac{Prt}{100}=\frac{2500\times4\times3}{100}$.	Substitute into the simple interest formula.
$I=\frac{30\,000}{100}=\$300$.	The interest earned over the 3 years.
$A=P+I=2500+300=\$2800$.	Total = principal + interest.

Example 2 — scaffolded

Sam borrows $\$6000$ at 9% per annum simple interest and repays it after 8 months. Find the interest charged and the total amount he must repay.

Working	Comment
Convert the time to years: $t=\frac{8}{12}$ year.	Time must be in years to match the "per annum" rate.
$I=\frac{6000\times9\times8/12}{100}$.	Substitute $P=6000$, $r=9$, $t=8/12$.
$I=\frac{6000\times9\times8}{12\times100}=\frac{432\,000}{1200}=\$360$.	Work the top out, then divide.
Total to repay $=6000+360=\$6360$.	Amount borrowed plus the interest.

Example 3 — unscaffolded

A savings account of $\$4000$ earns $\$480$ in simple interest over 3 years. Find the annual interest rate.

Rearranging $I=\frac{Prt}{100}$ for the rate gives $r=\frac{100I}{Pt}$. Substituting $I=480$, $P=4000$ and $t=3$,

$$r=\frac{100\times480}{4000\times3}=\frac{48\,000}{12\,000}=4.$$

∴ the account pays 4% per annum simple interest.

Example 4 — unscaffolded

Two lenders each offer Dev a $\$8000$ personal loan. Loan A charges 6% per annum for 3 years; Loan B charges 8% per annum for 2 years. Find the interest on each loan and state which costs less.

For Loan A, $I_A=\frac{8000\times6\times3}{100}=\frac{144\,000}{100}=\$1440$. For Loan B, $I_B=\frac{8000\times8\times2}{100}=\frac{128\,000}{100}=\$1280$. The difference is $1440-1280=\$160$.

∴ Loan B costs $\$160$ less interest, even though its rate is higher, because it runs for a shorter time.

Practice questions

Scaffolded

Q1. Maya invests $\$1500$ at 5% per annum simple interest. (a) Find the interest earned after 4 years. (b) Find the total amount in the account after 4 years. (c) Find the interest earned if the money is instead left for 6 years.

Q2. Leo borrows $\$3600$ at 10% per annum simple interest and repays it after 6 months. (a) Write the time as a fraction of a year. (b) Find the interest charged. (c) Find the total amount Leo repays.

Q3. A principal of $\$2000$ is invested for 3 years. (a) Find the interest at 4% per annum. (b) Find the interest at 6% per annum. (c) State the difference between the two amounts of interest.

Q4. A deposit of $\$5000$ earns $\$750$ simple interest over 5 years. (a) Write the rearranged formula for the rate r. (b) Find the annual interest rate. (c) Using that rate, find the interest the deposit would earn over 8 years.

Q5. A loan charged at 8% per annum simple interest for 2 years produces $\$640$ in interest. (a) Write the rearranged formula for the principal P. (b) Find the amount borrowed. (c) Find the total amount repaid.

Q6. An investment of $\$2500$ at 6% per annum simple interest earns $\$450$. (a) Write the rearranged formula for the time t. (b) Find the number of years the money was invested. (c) If the interest rate had been higher, would it take more or less time to earn the same $\$450$? Explain.

Unscaffolded

Q7. Nina invests $\$8000$ at 3.5% per annum simple interest for 5 years. Find the interest earned and the total amount in the account.

Q8. A business borrows $\$12\,000$ at 11% per annum simple interest for 9 months. Find the interest charged and the total amount to repay.

Q9. A savings account of $\$6000$ earns $\$900$ in simple interest over 4 years. Find the annual interest rate.

Q10. A personal loan is charged at 5% per annum simple interest for 6 years and produces $\$1650$ in interest. Find the amount borrowed.

Q11. An investment of $\$4000$ at 7.5% per annum simple interest earns $\$900$. Find how many years the money was invested.

Q12. A $\$10\,000$ loan can be taken as Option A at 6.5% per annum for 4 years, or Option B at 7% per annum for 3 years. Find the interest on each option and state which costs less, and by how much.

Q13. Aleksandar borrows $\$9000$ at 8% per annum simple interest over 3 years, repaid in equal monthly instalments. (a) Find the total interest and the total amount to repay. (b) Find the size of each monthly instalment (there are 36 months).

Q14. A deposit of $\$5000$ is invested at 4.5% per annum simple interest. Find how many years it takes to earn $\$1350$ in interest.

Q15. Two customers each borrow $\$7000$ for 2 years. A lower-risk customer is charged 6% per annum and a higher-risk customer 9% per annum simple interest. (a) Find the interest each customer pays. (b) State the difference, and explain in one sentence why the higher-risk customer pays more.

Q16. A loan is charged at 8% per annum simple interest for 5 years. The total amount repaid (principal plus interest) is $\$7000$. (a) Explain why the total is 1.4 times the principal. (b) Find the principal and the interest.

Q17. How long does it take for $\$2000$ invested at 5% per annum simple interest to grow to $\$3000$?

Q18. A car loan of $\$18\,000$ is charged at 7.5% per annum simple interest over 5 years, repaid in equal monthly instalments. (a) Find the total interest. (b) Find the total amount to repay. (c) Find the size of each monthly instalment (there are 60 months).

Answers

Q1. (a) $\$300$. (b) $\$1800$. (c) $\$450$. **Q2.** (a) $6/12=1/2$ year. (b) $\$180$. (c) $\$3780$. **Q3.** (a) $\$240$. (b) $\$360$. (c) $\$120$. **Q4.** (a) $r=\frac{100\,I}{P\,t}$. (b) 3% per annum. (c) $\$1200$. **Q5.** (a) $P=\frac{100\,I}{r\,t}$. (b) $\$4000$. (c) $\$4640$. **Q6.** (a) $t=\frac{100\,I}{P\,r}$. (b) 3 years. (c) Less time — a higher rate earns interest faster, so the same $\$450$ is reached sooner. **Q7.** $I=\$1400$; total $\$9400$. **Q8.** $I=\$990$; total $\$12\,990$. **Q9.** 3.75% per annum. **Q10.** $\$5500$. **Q11.** 3 years. **Q12.** Option A $\$2600$, Option B $\$2100$; Option B costs $\$500$ less. **Q13.** (a) interest $\$2160$, total $\$11\,160$. (b) $\$310$ per month. **Q14.** 6 years. **Q15.** (a) lower-risk $\$840$, higher-risk $\$1260$. (b) difference $\$420$; the higher-risk customer is charged a higher rate because the lender is more likely to lose money, so more interest is paid on the same loan. **Q16.** (a) see solution. (b) principal $\$5000$, interest $\$2000$. **Q17.** 10 years. **Q18.** (a) $\$6750$. (b) $\$24\,750$. (c) $\$412.50$ per month.

Fully-worked solutions

Q1. $P=1500$, $r=5$. (a) $I=\frac{1500\times5\times4}{100}=\frac{30\,000}{100}=\$300$. (b) $A=1500+300=\$1800$. (c) $I=\frac{1500\times5\times6}{100}=\frac{45\,000}{100}=\$450$.

Q2. $P=3600$, $r=10$. (a) 6 months $=\frac{6}{12}=\frac{1}{2}$ year. (b) $I=\frac{3600\times10\times1/2}{100}=\frac{18\,000}{100}=\$180$. (c) Total $=3600+180=\$3780$.

Q3. $P=2000$, $t=3$. (a) At 4%: $I=\frac{2000\times4\times3}{100}=\$240$. (b) At 6%: $I=\frac{2000\times6\times3}{100}=\$360$. (c) Difference $=360-240=\$120$ — raising the rate from 4% to 6% raises the interest in the same proportion.

Q4. $I=750$, $P=5000$, $t=5$. (a) Rearranging $I=\frac{P\,r\,t}{100}$ gives $r=\frac{100\,I}{P\,t}$. (b) $r=\frac{100\times750}{5000\times5}=\frac{75\,000}{25\,000}=3$, i.e. 3% per annum. (c) $I=\frac{5000\times3\times8}{100}=\frac{120\,000}{100}=\$1200$.

Q5. $r=8$, $t=2$, $I=640$. (a) Rearranging gives $P=\frac{100\,I}{r\,t}$. (b) $P=\frac{100\times640}{8\times2}=\frac{64\,000}{16}=\$4000$. (c) Total repaid $=4000+640=\$4640$.

Q6. $P=2500$, $r=6$, $I=450$. (a) Rearranging gives $t=\frac{100\,I}{P\,r}$. (b) $t=\frac{100\times450}{2500\times6}=\frac{45\,000}{15\,000}=3$ years. (c) Less time. The interest is earned faster at a higher rate, so the same $\$450$ would be reached in fewer years.

Q7. $I=\frac{8000\times3.5\times5}{100}=\frac{140\,000}{100}=\$1400$; total $=8000+1400=\$9400$.

Q8. Time $=\frac{9}{12}$ year. $I=\frac{12\,000\times11\times9/12}{100}=\frac{12\,000\times11\times9}{12\times100}=\frac{1\,188\,000}{1200}=\$990$; total $=12\,000+990=\$12\,990$.

Q9. $r=\frac{100\times900}{6000\times4}=\frac{90\,000}{24\,000}=3.75$, i.e. 3.75% per annum.

Q10. $P=\frac{100\times1650}{5\times6}=\frac{165\,000}{30}=\$5500$.

Q11. $t=\frac{100\times900}{4000\times7.5}=\frac{90\,000}{30\,000}=3$ years.

Q12. Option A: $I=\dfrac{10\,000\times 6.5\times 4}{100}=\dfrac{260\,000}{100}=\$\,2600$. Option B: $I=\dfrac{10\,000\times 7\times 3}{100}=\dfrac{210\,000}{100}=\$\,2100$. Option B costs $2600-2100=\$\,500$ less, because although its rate is higher it runs for one year fewer.

Q13. (a) $I=\dfrac{9000\times 8\times 3}{100}=\dfrac{216\,000}{100}=\$\,2160$; total $=9000+2160=\$\,11\,160$. (b) Each monthly instalment $=\dfrac{11\,160}{36}=\$\,310$.

Q14. $t=\dfrac{100\times 1350}{5000\times 4.5}=\dfrac{135\,000}{22\,500}=6$ years.

Q15. (a) Lower-risk: $I=\dfrac{7000\times 6\times 2}{100}=\$\,840$. Higher-risk: $I=\dfrac{7000\times 9\times 2}{100}=\$\,1260$. (b) Difference $=1260-840=\$\,420$. The higher-risk customer is charged a higher rate because the lender is more likely to lose money on the loan, so pays more interest on the same amount over the same time.

Q16. (a) The total is the principal plus the interest, $P+\dfrac{P\times 8\times 5}{100}=P+0.4\,P=1.4\,P$, so the total is 1.4 times the principal. (b) $1.4\,P=7000$, so $P=\dfrac{7000}{1.4}=\$\,5000$, and the interest is $7000-5000=\$\,2000$.

Q17. Growing from $\$\,2000$ to $\$\,3000$ means earning $I=1000$. Then $t=\dfrac{100\times 1000}{2000\times 5}=\dfrac{100\,000}{10\,000}=10$ years.

Q18. (a) $I=\dfrac{18\,000\times 7.5\times 5}{100}=\dfrac{675\,000}{100}=\$\,6750$. (b) Total $=18\,000+6750=\$\,24\,750$. (c) Each monthly instalment $=\dfrac{24\,750}{60}=\$\,412.50$.

Theory

Large items — a fridge, a laptop, a lounge suite — do not always have to be paid for all at once. An **instalment plan** lets a buyer take the goods (or put them aside) and pay in a series of smaller, regular amounts. The convenience almost always costs more than paying cash, so it pays to work out exactly what a plan adds up to.

Paying by instalments.

Most plans begin with a **deposit** (an up-front part-payment) and then a fixed number of equal **instalments** paid weekly, fortnightly or monthly. The total amount paid is

$$\text{total cost} = \text{deposit} + \left(\text{number of instalments} \times \text{instalment amount}\right).$$

For example, a \$150 deposit followed by 12 monthly instalments of \$60 costs $150 + 12 \times 60 = \$870$ in total.

The extra cost.

Buying on a plan usually costs more than the **cash price** (the price if you pay in full straight away). The difference is the **extra cost**:

$$\text{extra cost} = \text{total cost of plan} - \text{cash price}.$$

It is often written as a percentage of the cash price, so different plans can be compared:

$$\text{extra as a percentage} = \frac{\text{extra cost}}{\text{cash price}} \times 100\,\%.$$

Finding an instalment.

If the plan's total (or the balance to be spread over the instalments) is known, rearrange to find one instalment:

$$\text{instalment} = \frac{\text{amount to be paid by instalments}}{\text{number of instalments}},$$

where the amount paid by instalments is the total cost minus the deposit.

Layby.

With **layby**, the store keeps the goods until they are fully paid off. There is usually a deposit and a small **service fee**, and the balance is paid in regular instalments — but normally **no interest** is charged, so the only extra cost is the fee.

Plans with a simple interest charge.

Some plans add **interest** to the amount owing. In Units 1 & 2 this is always **simple interest**, found with

$$I = \frac{P r t}{100},$$

where P is the amount owing (the principal), r is the yearly rate and t is the time in years. The interest is added once to the balance, and the new total is then divided into equal instalments. (Interest is never added on top of interest at this level.)

Worked examples

Example 1 — scaffolded

A laptop has a cash price of \$1500. On an instalment plan the buyer pays a \$300 deposit and 12 monthly instalments of \$110. Find the total cost of the plan, the extra cost compared with the cash price, and the extra as a percentage of the cash price.

Working	Comment
Instalments total $=12\times 110=\$1320$.	Number of instalments × instalment amount.
Total cost $=300+1320=\$1620$.	Add the deposit.
Extra cost $=1620-1500=\$120$.	Total cost − cash price.
Extra as a percentage $=\dfrac{120}{1500}\times 100\%=8\%$.	Compare the extra with the cash price.

Example 2 — scaffolded

A dining setting is advertised on a plan whose total cost is \$2100, made up of a \$500 deposit and 16 equal monthly instalments. The cash price is \$1900. Find each monthly instalment and the extra paid compared with the cash price.

Working	Comment
Amount paid by instalments $=2100-500=\$1600$.	Total cost − deposit.
Each instalment $=\dfrac{1600}{16}=\$100$.	Divide by the number of instalments.
Extra cost $=2100-1900=\$200$.	Plan total − cash price.
As a percentage: $\dfrac{200}{1900}\times 100\%=10.5\%$ (1 d.p.).	The plan costs about 10.5% more.

Example 3 — unscaffolded

A bicycle with a cash price of \$840 is bought on layby. The store requires a 10% deposit, a \$15 service fee, and the balance in 8 equal fortnightly payments. Find the deposit, each fortnightly payment, and the total cost.

The deposit is 10% of the cash price: $0.10\times 840=\$84$. The balance owing after the deposit is $840-84=\$756$, so each of the 8 fortnightly payments is

$$\frac{756}{8}=\$94.50.$$

Because layby charges no interest, the only extra cost is the \$15 fee, so the total cost is $840+15=\$855$.

∴ the deposit is \$84, each payment is \$94.50, and the total cost is \$855.

Example 4 — unscaffolded

A refrigerator has a cash price of \$1800. A store plan requires no deposit but adds simple interest at 9% per year for 2 years to the price, and the total is then paid in 24 equal monthly instalments. Find the monthly instalment and the extra paid as a percentage of the cash price.

The simple interest added is

$$I=\frac{P\,r\,t}{100}=\frac{1800\times 9\times 2}{100}=\$324.$$

The total to repay is $1800+324=\$2124$, so each monthly instalment is

$$\frac{2124}{24}=\$88.50.$$

The extra paid is the \$324 of interest, which as a percentage of the cash price is $\frac{324}{1800} \times 100\% = 18\%$.

$\therefore$ each instalment is \$88.50, and the plan costs 18% more than paying cash.

Practice questions

Scaffolded

Q1. A phone has a cash price of \$720. On a plan you pay a \$120 deposit and 12 monthly instalments of \$55. (a) Find the total of the 12 instalments. (b) Find the total cost of the plan. (c) How much more than the cash price does the plan cost?

Q2. A fridge has a cash price of \$1600. A plan charges a \$250 deposit and 18 monthly instalments of \$86. (a) Find the total cost of the plan. (b) Find the extra cost compared with the cash price. (c) Express the extra cost as a percentage of the cash price (1 d.p.).

Q3. A sofa is sold on a plan whose total cost is \$1800: a \$300 deposit and the balance in 15 equal monthly instalments. (a) How much is paid through the instalments? (b) Find the amount of each monthly instalment. (c) The cash price is \$1650. Find the extra paid on the plan.

Q4. A laptop has a cash price of \$1250. A layby plan needs a 20% deposit, then the balance in 10 equal weekly payments. (a) Find the deposit. (b) Find the balance owing after the deposit. (c) Find each weekly payment.

Q5. A \$2000 sound system is bought on a plan with no deposit. The store adds simple interest at 8% per year for 1 year, then splits the total into 12 monthly instalments. (a) Find the simple interest added, using $I = \frac{P\,r\,t}{100}$. (b) Find the total amount to repay. (c) Find each monthly instalment.

Q6. A washing machine has a cash price of \$900. It can also be bought for a \$100 deposit and 8 monthly instalments of \$110. (a) Find the total cost of the instalment plan. (b) How much more does the plan cost than paying cash? (c) Express the extra as a percentage of the cash price (1 d.p.).

Unscaffolded

Q7. A television has a cash price of \$1400. On an instalment plan you pay a \$200 deposit and 24 monthly instalments of \$58. Find the total cost of the plan and the extra paid compared with the cash price.

Q8. A \$3200 lounge is bought with an \$800 deposit and the balance in 20 equal fortnightly instalments, with no extra charge. Find each fortnightly instalment.

Q9. A camera has a cash price of \$960. A plan charges a \$160 deposit and 12 monthly payments of \$75. Find the extra cost of the plan and express it as a percentage of the cash price (1 d.p.).

Q10. A jeweller offers a \$1500 ring on layby: a 15% deposit, a \$20 service fee, and the balance in 6 equal monthly payments. Find the deposit, each monthly payment, and the total cost.

Q11. A \$2400 motorbike is financed with no deposit at simple interest of 10% per year for 2 years, repaid in 24 equal monthly instalments. Find the total to repay and each monthly instalment.

Q12. A treadmill can be bought for \$1800 cash, or for a \$300 deposit plus 15 monthly instalments of \$115. Find how much extra the instalment plan costs and express it as a percentage of the cash price (1 d.p.).

Q13. On an instalment plan a customer pays a \$400 deposit and 12 equal monthly instalments, for a total plan cost of \$1960. Find the amount of each monthly instalment.

Q14. A fridge with a cash price of \$1200 is bought on a plan whose total cost is \$1350, made up of a deposit and 10 equal monthly instalments of \$120. Find the deposit, and the extra paid over the cash price.

Q15. A furniture store adds a flat $12\,\%$ charge to the balance owing on a plan. A customer buys a \$2000 bedroom suite, pays a \$200 deposit, and the $12\,\%$ charge is added to the remaining balance, which is then paid in 16 equal fortnightly instalments. Find the total cost and each fortnightly instalment.

Q16. Two plans are offered for a \$1000 appliance. Plan A: a \$100 deposit and 12 monthly instalments of \$85. Plan B: no deposit, with simple interest of $15\,\%$ for 1 year added, then 12 equal monthly instalments. Find the total cost of each plan and state which costs less, and by how much.

Q17. A store advertises a television as "\$50 deposit and 52 weekly payments of \$12". Its cash price is \$560. (a) Find the total cost of the plan. (b) Find the extra cost over the cash price and express it as a percentage of the cash price (1 d.p.).

Q18. A home-theatre system has a cash price of \$4000. On a plan the buyer pays a $10\,\%$ deposit, then simple interest of $8\,\%$ per year for 2 years is added to the balance, and the result is paid in 24 equal monthly instalments. Find the deposit, the monthly instalment, and the total cost of the plan.

Answers

Q1. (a) \$660. (b) \$780. (c) \$60. **Q2.** (a) \$1798. (b) \$198. (c) $12.4\,\%$. **Q3.** (a) \$1500. (b) \$100. (c) \$150. **Q4.** (a) \$250. (b) \$1000. (c) \$100. **Q5.** (a) \$160. (b) \$2160. (c) \$180. **Q6.** (a) \$980. (b) \$80. (c) 8.9%. **Q7.** total \$1592; extra \$192. **Q8.** \$120. **Q9.** extra \$100; $10.4\,\%$. **Q10.** deposit \$225; each payment \$212.50; total cost \$1520. **Q11.** total \$2880; each instalment \$120. **Q12.** extra \$225; $12.5\,\%$. **Q13.** \$130. **Q14.** deposit \$150; extra \$150. **Q15.** total cost \$2216; each instalment \$126. **Q16.** Plan A \$1120, Plan B \$1150; Plan A costs \$30 less. **Q17.** (a) \$674. (b) extra \$114; $20.4\,\%$. **Q18.** deposit \$400; each instalment \$174; total cost \$4576.

Fully-worked solutions

Q1. (a) Instalments total $=12\times 55=\$660$. (b) Total cost $=\text{deposit}+\text{instalments}=120+660=\$780$. (c) Extra $=780-720=\$60$.

Q2. (a) Total cost $=250+18\times 86=250+1548=\$1798$. (b) Extra $=1798-1600=\$198$. (c) $\frac{198}{1600}\times 100\,\%=12.375\,\%=12.4\,\%$ (1 d.p.).

Q3. (a) Amount paid by instalments $=1800-300=\$1500$. (b) Each instalment $=\frac{1500}{15}=\$100$. (c) Extra $=1800-1650=\$150$.

Q4. (a) Deposit $=20\,\%$ of $1250=0.20\times 1250=\$250$. (b) Balance $=1250-250=\$1000$. (c) Each weekly payment $=\frac{1000}{10}=\$100$.

Q5. (a) $I=\frac{P\,r\,t}{100}=\frac{2000\times 8\times 1}{100}=\$160$. (b) Total to repay $=2000+160=\$2160$. (c) Each instalment $=\frac{2160}{12}=\$180$.

Q6. (a) Total cost $=100+8\times 110=100+880=\$980$. (b) Extra $=980-900=\$80$. (c) $\frac{80}{900}\times 100\,\%=8.888\ldots\,\%=8.9\,\%$ (1 d.p.).

Q7. Total cost $=200+24\times 58=200+1392=\$1592$. Extra $=1592-1400=\$192$.

Q8. Balance after deposit $=3200-800=\$2400$. Each fortnightly instalment $=\frac{2400}{20}=\$120$.

Q9. Total cost $=160+12\times 75=160+900=\$1060$. Extra $=1060-960=\$100$, which is $\frac{100}{960}\times 100\,\%=10.4166\ldots\,\%=10.4\,\%$ (1 d.p.).

Q10. Deposit $=15\,\%$ of $1500=0.15\times 1500=\$225$. Balance $=1500-225=\$1275$, so each monthly payment $=\frac{1275}{6}=\$212.50$. Layby charges no interest, so the total cost is the cash price plus the fee: $1500+20=\$1520$.

Q11. Simple interest $=\frac{2400\times 10\times 2}{100}=\$480$. Total to repay $=2400+480=\$2880$. Each monthly instalment $=\frac{2880}{24}=\$120$.

Q12. Plan total $=300+15\times 115=300+1725=\$2025$. Extra $=2025-1800=\$225$, which is $\frac{225}{1800}\times 100\,\%=12.5\,\%$.

Q13. Amount paid by instalments $= 1960 - 400 = \$1560$. Each instalment $= \frac{1560}{12} = \$130$.

Q14. Instalments total $= 10 \times 120 = \$1200$. Deposit $= 1350 - 1200 = \$150$. Extra over the cash price $= 1350 - 1200 = \$150$.

Q15. Balance after deposit $= 2000 - 200 = \$1800$. The 12% charge is $0.12 \times 1800 = \$216$, so the amount paid by instalments is $1800 + 216 = \$2016$, and each fortnightly instalment is $\frac{2016}{16} = \$126$. The total cost is $200 + 2016 = \$2216$ (equivalently $2000 + 216$).

Q16. Plan A total $= 100 + 12 \times 85 = 100 + 1020 = \$1120$. Plan B: simple interest $= \frac{1000 \times 15 \times 1}{100} = \$150$, so the total is $1000 + 150 = \$1150$ (with instalments of $1150 \div 12 \approx \$95.83$). Plan A costs $1150 - 1120 = \$30$ less than Plan B.

Q17. (a) Total cost $= 50 + 52 \times 12 = 50 + 624 = \$674$. (b) Extra $= 674 - 560 = \$114$, which is $\frac{114}{560} \times 100\% = 20.357\ldots\% = 20.4\%$ (1 d.p.).

Q18. Deposit $= 10\%$ of $4000 = \$400$. Balance $= 4000 - 400 = \$3600$. Simple interest $= \frac{3600 \times 8 \times 2}{100} = \$576$, so the amount paid by instalments is $3600 + 576 = \$4176$, giving each monthly instalment $\frac{4176}{24} = \$174$. The total cost is $400 + 4176 = \$4576$.

Theory

Many Australians pay for university or TAFE study with a government **study loan** (a **HECS-HELP** or **HELP** loan). The government pays the tuition fees to the institution, and the amount becomes a **debt** that the student repays later, through the tax system, once they are earning enough. A study loan is unusual in two ways: it charges **no interest**, and repayments are **income-contingent** — you only repay when your income is high enough.

How the debt builds up.

Each subject (or unit) you study has a fee. Deferring these fees to a study loan simply **adds** them to your debt:

$$\text{study debt} = \text{sum of the deferred subject fees}.$$

Indexation.

A study loan charges no interest, but once a year (on 1 June) the balance is **indexed** — increased by a small percentage so that the debt keeps pace with the rising cost of living. Indexation works exactly like a **percentage increase**:

$$\text{indexed balance} = \text{balance} + \text{indexation rate} \times \text{balance} = \text{balance} \times \left(1 + \frac{r}{100}\right),$$

where r is the indexation rate for that year. Indexation is applied to whatever balance is **still owing** on 1 June, so paying some of the debt off *before* that date reduces the amount that gets indexed.

Compulsory repayments.

While you are studying you make no repayments. Once your **repayment income** rises above a threshold, your employer withholds a **compulsory repayment** and sends it to the tax office. The repayment is a **percentage of your whole repayment income**, and the percentage rises as income rises. The rates used throughout this book are:

Repayment income	Repayment rate
Below $55 000	Nil
$55 000 – $64 999	1.5%
$65 000 – $74 999	2.5%
$75 000 – $84 999	3.5%
$85 000 – $99 999	4.5%
$100 000 and over	6.0%

So a person earning $70 000 repays $2.5\% \times \$70\,000 = \$1750$ for the year. Notice the rate is applied to the **entire** income, not just the part above the threshold — so crossing into a higher band can raise the repayment sharply.

Voluntary repayments.

You may also make a **voluntary repayment** at any time. This comes straight off the balance, reducing both the debt and the amount that will later be indexed.

A worked model. A debt of $10 000 is reduced by a $2000 voluntary repayment to $8000, then indexed by 4%: the indexed debt is $\$8000 \times 1.04 = \$8320$.

Worked examples

Example 1 — scaffolded

Maya enrols in first-year study and defers the fees for four subjects to a HECS-HELP loan: three subjects at $1240 each and one at $1685. Find her study debt, then find the debt after it is indexed by $4.0\,\%$ on 1 June.

Working	Comment
Add the four subject fees: $3\times\$1240+\$1685$.	Three cost the same; one is different.
Study debt $=\$3720+\$1685=\$5405$.	This is the amount borrowed.
Indexation of $4.0\,\%$: increase $=0.04\times\$5405=\$216.20$.	Indexation is a percentage increase.
Indexed debt $=\$5405+\$216.20=\$5621.20$.	Same as $\$5405\times1.04$.

Example 2 — scaffolded

Sam's repayment income for the year is $78 400. Use the repayment table to find Sam's compulsory repayment for the year.

Working	Comment
Locate the band: $78 400 lies in $75 000 – $84 999.	Compare the income with each row.
The repayment rate for that band is $3.5\,\%$.	Read across to the rate.
Repayment $=3.5\,\%\times\$78\,400=0.035\times78\,400$.	The rate applies to the whole income.
$=\$2744$.	Sam repays $2744 over the year.

Example 3 — unscaffolded

Ravi has a study debt of $14 300. Just before the 1 June indexation he makes a voluntary repayment of $3000, and the remaining balance is then indexed by $3.5\,\%$. Find his debt after indexation.

The voluntary repayment reduces the balance first:

$$\$14\,300-\$3000=\$11\,300.$$

Indexation of $3.5\,\%$ is applied to this remaining balance:

$$0.035\times\$11\,300=\$395.50,$$

$$\$11\,300+\$395.50=\$11\,695.50.$$

∴ after the voluntary repayment and indexation Ravi owes $11 695.50.

Example 4 — unscaffolded

At the start of a financial year Priya owes $21 600 on her study loan. On 1 June the balance is indexed by $4.2\,\%$. Her repayment income for the year is $69 500, and the compulsory repayment is credited to the loan after the indexation. Find the indexed balance, and the balance after the compulsory repayment.

The indexation increases the balance by $4.2\,\%$:

$$\$21\,600\times1.042=\$22\,507.20.$$

The repayment income $69 500 lies in the $65 000 – $74 999 band, so the rate is $2.5\,\%$:

$$\text{repayment}=0.025\times\$69\,500=\$1737.50.$$

Subtracting this from the indexed balance,

$$\$22\,507.20-\$1737.50=\$20\,769.70.$$

∴ the indexed balance is $22 507.20, and after the compulsory repayment Priya owes $20 769.70.

Practice questions

Use the repayment table in the Theory section for any question that needs a repayment rate.

Scaffolded

Q1. Tom defers the fees for five subjects to a HELP loan: two subjects at $1150 each, two at $1420 each, and one at $985. (a) Find the total fee for the two $1150 subjects. (b) Find the total fee for the two $1420 subjects. (c) Find Tom's total study debt.

Q2. Nadia owes $9800 on her study loan when it is indexed by 3.8%. (a) Find the indexation amount (the increase). (b) Find the indexed debt. (c) Write the indexed debt as a single multiplication of $9800.

Q3. For each repayment income, state the repayment rate and find the compulsory repayment for the year. (a) $58 000. (b) $81 200. (c) $104 000.

Q4. Leila owes $16 500. She makes a voluntary repayment of $4500, and the remaining balance is then indexed by 4.0%. (a) Find the balance after the voluntary repayment. (b) Find the indexation amount on that balance. (c) Find the balance after indexation.

Q5. Anya's repayment income is $63 500 and Ben's is $88 000. (a) State each person's repayment rate. (b) Find each person's compulsory repayment for the year. (c) How much more does Ben repay than Anya this year?

Q6. Marco owes $18 200. His repayment income is $76 000, and the compulsory repayment for the year is credited to his loan. (a) State Marco's repayment rate. (b) Find his compulsory repayment. (c) Find his loan balance after the repayment (ignore indexation).

Unscaffolded

Q7. Jess is charged $1380 for each of six subjects and defers them all to a HELP loan. Find her study debt.

Q8. A study debt of $12 400 is indexed by 4.5%. Find the indexed debt.

Q9. Dan's repayment income is $71 800. Find his compulsory repayment for the year.

Q10. Sara owes $10 000. She makes a voluntary repayment of $2500, then the remaining debt is indexed by 3.6%. Find the debt after indexation.

Q11. Two students each owe $15 000. Before an indexation of 4.0%, one makes a $5000 voluntary repayment and the other makes no repayment. Find each student's debt after indexation, and the difference between the two debts.

Q12. Kofi's repayment income rises from $64 000 one year to $66 000 the next. Find his compulsory repayment in each year, and the increase in his repayment.

Q13. Owen owes $19 400. He makes a voluntary repayment of $6400 just before the 1 June indexation of 4.2%. His friend claims the indexation is $814.80, which is 4.2% of the original $19 400. Explain the friend's error, then find the correct indexation amount and the balance after indexation.

Q14. A student's study debt is $11 250. After a year with no repayment it is indexed by 4.0%. The student says, "indexation is not interest, so I owe nothing extra." Find the indexed debt, and briefly explain why the statement is wrong.

Q15. Bianca's repayment income is $74 900 and Carlos's is $75 100 — just $200 apart, but on opposite sides of a band boundary. Find each person's compulsory repayment, and comment on the size of the jump.

Q16. At the start of the year Hana owes $24 800. On 1 June it is indexed by 3.7%. Her repayment income for the year is $92 500, and the compulsory repayment is credited to the loan after indexation. (a) Find the indexed balance. (b) Find the compulsory repayment. (c) Find the balance after the repayment.

Q17. A graduate with a study debt of \$30 000 has it indexed by 4.0% on 1 June. Over the same year the graduate's repayment income is \$96 000, and the compulsory repayment is credited after indexation. (a) Find the indexed debt. (b) Find the compulsory repayment. (c) Has the debt gone up or down over the year, and by how much?

Q18. A student is deciding whether to make a \$5000 voluntary repayment just before the 1 June indexation of 4.5%. (a) If she makes no repayment, her \$20 000 debt is indexed. Find the indexed debt. (b) If she makes the \$5000 repayment first, find the indexed debt on the remaining balance. (c) How much smaller is the indexation amount because she paid early?

Answers

Q1. (a) \$2300. (b) \$2840. (c) \$6125. **Q2.** (a) \$372.40. (b) \$10 172.40. (c) $\$9800 \times 1.038$. **Q3.** (a) 1.5%, \$870. (b) 3.5%, \$2842. (c) 6.0%, \$6240. **Q4.** (a) \$12 000. (b) \$480. (c) \$12 480. **Q5.** (a) Anya 1.5%, Ben 4.5%. (b) Anya \$952.50, Ben \$3960. (c) \$3007.50. **Q6.** (a) 3.5%. (b) \$2660. (c) \$15 540. **Q7.** \$8280. **Q8.** \$12 958. **Q9.** \$1795. **Q10.** \$7770. **Q11.** With repayment \$10 400; without \$15 600; difference \$5200. **Q12.** \$960 then \$1650; increase \$690. **Q13.** Indexation applies to the balance still owing (\$13 000), not the original \$19 400; correct indexation \$546; balance \$13 546. **Q14.** \$11 700; it still increases the amount owed by \$450, even though it is not called interest. **Q15.** Bianca \$1872.50, Carlos \$2628.50; a \$200 income difference gives \$756 more repayment. **Q16.** (a) \$25 717.60. (b) \$4162.50. (c) \$21 555.10. **Q17.** (a) \$31 200. (b) \$4320. (c) Down by \$3120. **Q18.** (a) \$20 900. (b) \$15 675. (c) \$225.

Fully-worked solutions

Q1. (a) Two subjects at \$1150: $2 \times \$1150 = \$2300$. (b) Two subjects at \$1420: $2 \times \$1420 = \$2840$. (c) Total study debt $= \$2300 + \$2840 + \$985 = \$6125$.

Q2. (a) Indexation $= 3.8\% \times \$9800 = 0.038 \times 9800 = \$372.40$. (b) Indexed debt $= \$9800 + \$372.40 = \$10172.40$. (c) Increasing by 3.8% is multiplying by 1.038, so the indexed debt is $\$9800 \times 1.038$.

Q3. (a) \$58 000 is in the \$55 000 – \$64 999 band, rate 1.5%: $0.015 \times 58000 = \$870$. (b) \$81 200 is in the \$75 000 – \$84 999 band, rate 3.5%: $0.035 \times 81200 = \$2842$. (c) \$104 000 is in the \$100 000-and-over band, rate 6.0%: $0.06 \times 104000 = \$6240$.

Q4. (a) After the voluntary repayment: $\$16500 - \$4500 = \$12000$. (b) Indexation $= 4.0\% \times \$12000 = 0.04 \times 12000 = \$480$. (c) Balance after indexation $= \$12000 + \$480 = \$12480$.

Q5. (a) Anya's \$63 500 is in the \$55 000 – \$64 999 band (1.5%); Ben's \$88 000 is in the \$85 000 – \$99 999 band (4.5%). (b) Anya: $0.015 \times 63500 = \$952.50$. Ben: $0.045 \times 88000 = \$3960$. (c) Ben repays $\$3960 - \$952.50 = \$3007.50$ more than Anya.

Q6. (a) \$76 000 is in the \$75 000 – \$84 999 band, so the rate is 3.5%. (b) Repayment $= 0.035 \times 76000 = \$2660$. (c) Balance after the repayment $= \$18200 - \$2660 = \$15540$.

Q7. Six subjects at \$1380 each: $6 \times \$1380 = \$8280$.

Q8. Increasing \$12 400 by 4.5%: $\$12400 \times 1.045 = \$12958$ (the increase is $0.045 \times 12400 = \$558$).

Q9. \$71 800 is in the \$65 000 – \$74 999 band, rate 2.5%: $0.025 \times 71800 = \$1795$.

Q10. After the voluntary repayment: $\$10000 - \$2500 = \$7500$. Indexing by 3.6%: $\$7500 \times 1.036 = \$7770$ (the increase is $0.036 \times 7500 = \$270$).

Q11. The student who repays first has $\$15000 - \$5000 = \$10000$ indexed: $\$10000 \times 1.04 = \$10400$. The student who does not repay has $\$15000 \times 1.04 = \$15600$. The difference is $\$15600 - \$10400 = \$5200$.

Q12. At \$64 000 (band \$55 000 – \$64 999, rate 1.5%): $0.015 \times 64000 = \$960$. At \$66 000 (band \$65 000 – \$74 999, rate 2.5%): $0.025 \times 66000 = \$1650$. The repayment increases by $\$1650 - \$960 = \$690$ — a \$2000 rise in income raises the yearly repayment by \$690 because it crosses into a higher band.

Q13. Indexation is applied only to the balance **still owing** on 1 June. After the \$6400 voluntary repayment the balance is $\$19400 - \$6400 = \$13000$, so the friend has wrongly indexed the original \$19 400 instead of the reduced \$13 000. The correct indexation is $0.042 \times \$13000 = \$546$, and the balance after indexation is $\$13000 + \$546 = \$13546$.

Q14. Indexed debt $= \$11250 \times 1.04 = \$11700$. The extra owed is $0.04 \times 11250 = \$450$. Even though indexation is not called interest, it still increases the amount the student owes (here by \$450), so the statement is wrong.

Q15. Bianca's \$74 900 is in the \$65 000 – \$74 999 band (2.5%): $0.025 \times 74\,900 = \$1872.50$. Carlos's \$75 100 is in the \$75 000 – \$84 999 band (3.5%): $0.035 \times 75\,100 = \$2628.50$. Although their incomes differ by only \$200, Carlos repays $\$2628.50 - \$1872.50 = \$756$ more, because the higher rate applies to his whole income.

Q16. (a) Indexed balance $= \$24\,800 \times 1.037 = \$25\,717.60$ (increase $0.037 \times 24\,800 = \$917.60$). (b) \$92 500 is in the \$85 000 – \$99 999 band (4.5%): $0.045 \times 92\,500 = \$4162.50$. (c) Balance after the repayment $= \$25\,717.60 - \$4162.50 = \$21\,555.10$.

Q17. (a) Indexed debt $= \$30\,000 \times 1.04 = \$31\,200$. (b) \$96 000 is in the \$85 000 – \$99 999 band (4.5%): $0.045 \times 96\,000 = \$4320$. (c) After the repayment the debt is $\$31\,200 - \$4320 = \$26\,880$, which is $\$30\,000 - \$26\,880 = \$3120$ **less** than at the start of the year — the compulsory repayment more than covered the indexation.

Q18. (a) No repayment: $\$20\,000 \times 1.045 = \$20\,900$. (b) With the \$5000 repayment first: $(\$20\,000 - \$5000) \times 1.045 = \$15\,000 \times 1.045 = \$15\,675$. (c) The indexation is $0.045 \times 20\,000 = \$900$ without the early repayment and $0.045 \times 15\,000 = \$675$ with it, so paying early makes the indexation $\$900 - \$675 = \$225$ smaller.

Theory

Credit means buying something now and paying for it later, using money the provider lends you. Credit is convenient, but it can add to the cost of a purchase through **interest** and **fees** if you do not repay on time. The main forms met in everyday life are the **credit card** and **buy-now-pay-later**.

Credit cards.

A credit card lets you borrow up to a set **credit limit**. The amount you already owe is the **balance**, and

$$\text{available credit} = \text{credit limit} - \text{balance owing}.$$

Each month the card issuer sends a **statement** listing the opening balance, purchases, payments and the **closing balance**. Two features decide whether the card costs you anything:

- **Interest-free period.** If you pay the **whole** closing balance by the due date, you are charged **no interest** on your purchases.
- **Interest on an unpaid balance.** If you do not pay in full, interest is charged on what is left. The card quotes an **annual** rate (for example 18% per year); the **monthly rate** is the annual rate divided by 12. The interest for one month is

$$\text{monthly interest} = \text{unpaid balance} \times \text{monthly rate}.$$

For example, 18% per year is $18 \div 12 = 1.5\%$ per month, so one month's interest on an unpaid $1000 is $1000 \times 0.015 = \$15$.

Minimum payment.

The **minimum payment** is the least you must pay each month. It is usually the **greater** of a fixed dollar amount and a percentage of the closing balance — for example, the greater of $25 or 2% of the balance. Paying only the minimum leaves most of the balance owing, so interest keeps being charged.

Annual fee.

Many cards charge a yearly **annual fee** just for having the card. Spread over the year this is the annual fee divided by 12 each month.

Buy-now-pay-later.

A **buy-now-pay-later** (BNPL) plan splits a purchase into equal instalments, often four equal fortnightly payments:

$$\text{instalment} = \frac{\text{purchase price}}{\text{number of instalments}}.$$

There is usually **no interest** if every instalment is paid on time, but a **late fee** is charged for each missed payment.

Credit versus saving up.

Paying cash (after saving up) avoids interest and fees, so it is the cheapest way to buy. Using credit and carrying a balance adds the interest and any fees to the price — always compare the **total** cost.

Worked examples

Example 1 — scaffolded

Maria's credit card has a limit of $4000, and she currently owes $1560. (a) Find her available credit. (b) She does not pay the $1560 closing balance in full. The card charges 18% per year. Find one month's interest on the $1560.

Working	Comment
(a) Available credit $= 4000 - 1560 = \$2440$.	Limit minus what is already owed.
(b) Monthly rate $= 18 \div 12 = 1.5\%$.	Annual rate shared over 12 months.
Interest $= 1560 \times 0.015 = \$23.40$.	Unpaid balance × monthly rate.

Example 2 — scaffolded

A credit-card statement shows a closing balance of \$860. The minimum payment is the **greater** of \$30 or 3% of the closing balance. Find the minimum payment.

Working	Comment
3% of $860 = 0.03 \times 860 = \$25.80$.	Work out the percentage part first.
Compare \$30 and \$25.80.	The rule says take the **greater**.
Minimum payment $= \$30$.	\$30 is larger than \$25.80.

Example 3 — unscaffolded

Jordan buys a \$540 phone on a buy-now-pay-later plan of 4 equal fortnightly payments, with no interest if paid on time but a \$10 late fee for each missed payment. Find (a) the instalment amount, and (b) the total cost if Jordan misses one payment.

Each instalment is the price shared into four:

$$540 \div 4 = \$135.$$

If every payment is on time the total is just \$540. Missing one payment adds a single \$10 late fee:

$$540 + 10 = \$550.$$

∴ each instalment is \$135, and missing one payment makes the total cost \$550.

Example 4 — unscaffolded

Sam wants a \$900 laptop. If Sam buys it on a credit card charging 1.5% per month and pays the balance one month after the interest-free period ends, one month's interest is charged. If instead Sam saves up and pays cash, no interest is charged. How much more does using the card (and paying a month late) cost than saving up?

Saving up and paying cash costs the price only, \$900. Using the card and paying a month late adds one month's interest on the \$900:

$$900 \times 0.015 = \$13.50.$$

∴ using the card and paying a month late costs \$13.50 more than saving up.

Practice questions

Scaffolded

Q1. Lea's credit card has a limit of \$6000, and she owes \$2350. (a) Find her available credit. (b) She makes a \$500 purchase. Find her new balance and new available credit. (c) Could she then make a further \$3500 purchase on the card? Explain.

Q2. A card's closing balance of \$1240 is not paid in full. The card charges 24% per year. (a) Find the monthly interest rate. (b) Find one month's interest on \$1240. (c) Find the balance after this one month's interest is added.

Q3. A card's minimum payment is the greater of \$25 or 2% of the closing balance. Find the minimum payment when the closing balance is (a) \$900, (b) \$1500, (c) \$1100.

Q4. A $480 purchase is made on a buy-now-pay-later plan of 4 equal fortnightly payments, with an $8 late fee for each missed payment. (a) Find the instalment amount. (b) State the total cost if all payments are on time. (c) Find the total cost if two payments are late.

Q5. A $750 television is bought on a credit card charging $1.5\,\%$ per month, and the balance is paid one month late (one month's interest is charged). (a) Find one month's interest. (b) Find the total cost on the card. (c) State how much more this is than saving up and paying cash.

Q6. Priya's credit card has a $120 annual fee and charges $1.5\,\%$ per month on unpaid balances. This month she carries an unpaid balance of $800 and also pays the annual fee. (a) Find one month's interest on $800. (b) Find the total of the interest and the annual fee. (c) Express the annual fee as an amount per month.

Unscaffolded

Q7. A credit card has a limit of $3500, and the balance owing is $1890. Find the available credit.

Q8. A card charges $21\,\%$ per year. An unpaid balance of $2100 is carried for one month. Find the monthly interest rate and the interest charged for that month.

Q9. A card's minimum payment is the greater of $35 or $2.5\,\%$ of the closing balance. Find the minimum payment on a closing balance of $1800.

Q10. A $360 purchase is spread over 5 equal fortnightly buy-now-pay-later payments with no interest. Find the instalment amount and the total paid if every instalment is on time.

Q11. A $1500 sofa is bought on a credit card charging $2\,\%$ per month, and the balance is carried for one month. Find the interest charged for that month.

Q12. A credit-card statement has an opening balance of $450, purchases of $320 and a payment of $200. Find the closing balance.

Q13. A card's closing balance is $680 and it charges $1.6\,\%$ per month on unpaid balances. (a) State the interest charged if the whole balance is paid by the due date. (b) Find the interest charged if none of the balance is paid and it is carried for one month.

Q14. A card has a limit of $5000, and the balance owing is $4700. Explain, with a calculation, why a $500 purchase would be declined.

Q15. A $600 purchase is made on a buy-now-pay-later plan of 4 equal payments, with a $12 late fee for each missed payment. Find the instalment amount, and the total cost if three payments are late.

Q16. A card's closing balance is $1000. The minimum payment is the greater of $30 or $3\,\%$ of the balance. (a) Find the minimum payment. (b) Find the balance still owing after the minimum payment is made. (c) The card charges $1.5\,\%$ per month; find one month's interest on the balance still owing.

Q17. Two cards are compared for carrying a $1000 balance for one month. Card A charges $24\,\%$ per year with no annual fee. Card B charges $12\,\%$ per year plus a $60 annual fee. Using one month's share of any annual fee, find the one-month cost on each card and state which is cheaper.

Q18. An $800 fridge can be paid for in three ways: (1) save up and pay cash; (2) a buy-now-pay-later plan of 4 equal payments, all on time; (3) a credit card charging $1.5\,\%$ per month, with the balance carried for one month. (a) Find the total cost of each option. (b) State which option (or options) is cheapest. (c) State how much more the dearest option costs than the cheapest.

Answers

Q1. (a) \$3650. (b) balance \$2850, available \$3150. (c) No — available credit is only \$3150, less than \$3500. **Q2.** (a) 2% per month. (b) \$24.80. (c) \$1264.80. **Q3.** (a) \$25. (b) \$30. (c) \$25. **Q4.** (a) \$120. (b) \$480. (c) \$496. **Q5.** (a) \$11.25. (b) \$761.25. (c) \$11.25 more. **Q6.** (a) \$12. (b) \$132. (c) \$10 per month. **Q7.** \$1610. **Q8.** 1.75% per month; \$36.75. **Q9.** \$45. **Q10.** instalment \$72; total \$360. **Q11.** \$30. **Q12.** \$570. **Q13.** (a) \$0. (b) \$10.88. **Q14.** available credit $=5000-4700=\$300<\$500$, so the purchase exceeds the limit. **Q15.** instalment \$150; total \$636. **Q16.** (a) \$30. (b) \$970. (c) \$14.55. **Q17.** Card A \$20; Card B \$15; Card B is cheaper (by \$5). **Q18.** (a) cash \$800, BNPL \$800, credit \$812. (b) cash and BNPL (both \$800). (c) \$12 more.

Fully-worked solutions

Q1. (a) Available credit $=6000-2350=\$3650$. (b) New balance $=2350+500=\$2850$; new available credit $=6000-2850=\$3150$. (c) The available credit is now \$3150, which is less than \$3500, so a \$3500 purchase would take the balance over the \$6000 limit and be declined.

Q2. (a) Monthly rate $=24\div 12=2\%$. (b) Interest $=1240\times 0.02=\$24.80$. (c) Balance $=1240+24.80=\$1264.80$.

Q3. Take the greater of \$25 and 2% of the balance. (a) 2% of $900=\$18$; the greater of \$25 and \$18 is \$25. (b) 2% of $1500=\$30$; the greater of \$25 and \$30 is \$30. (c) 2% of $1100=\$22$; the greater of \$25 and \$22 is \$25.

Q4. (a) Instalment $=480\div 4=\$120$. (b) With every payment on time there is no fee, so the total is \$480. (c) Two late payments add $2\times 8=\$16$, so the total is $480+16=\$496$.

Q5. (a) Interest $=750\times 0.015=\$11.25$. (b) Total on the card $=750+11.25=\$761.25$. (c) Saving up costs \$750, so the card costs $761.25-750=\$11.25$ more.

Q6. (a) Interest $=800\times 0.015=\$12$. (b) Interest plus annual fee $=12+120=\$132$. (c) Annual fee per month $=120\div 12=\$10$.

Q7. Available credit $=3500-1890=\$1610$.

Q8. Monthly rate $=21\div 12=1.75\%$. Interest $=2100\times 0.0175=\$36.75$.

Q9. 2.5% of $1800=0.025\times 1800=\$45$. The greater of \$35 and \$45 is \$45.

Q10. Instalment $=360\div 5=\$72$. All five on time means no fees, so the total is \$360.

Q11. Interest $=1500\times 0.02=\$30$.

Q12. Closing balance $=450+320-200=\$570$.

Q13. (a) Paying the whole balance by the due date means the interest-free period applies, so the interest is \$0. (b) Carried for one month: interest $=680\times 0.016=\$10.88$.

Q14. Available credit $=5000-4700=\$300$. A \$500 purchase is more than the \$300 available, so it would push the balance past the \$5000 limit and be declined.

Q15. Instalment $=600\div 4=\$150$. Three late payments add $3\times 12=\$36$, so the total is $600+36=\$636$.

Q16. (a) 3% of $1000=\$30$; the greater of \$30 and \$30 is \$30. (b) Balance still owing $=1000-30=\$970$. (c) Interest $=970\times 0.015=\$14.55$.

Q17. Card A: monthly rate $24\div 12=2\%$, so interest $=1000\times 0.02=\$20$, with no fee, giving \$20. Card B: monthly rate $12\div 12=1\%$, so interest $=1000\times 0.01=\$10$; one month's share of the annual fee is $60\div 12=\$5$; total $=10+5=\$15$. Card B (\$15) is cheaper than Card A (\$20), by \$5.

Q18. (a) Option 1 (cash): \$800. Option 2 (BNPL, all on time): $4 \times 200 = \$800$, no extra. Option 3 (credit): interest $= 800 \times 0.015 = \$12$, so total $= 800 + 12 = \$812$. (b) Options 1 and 2 are cheapest, both \$800. (c) The dearest is the credit card at \$812, which is $812 - 800 = \$12$ more than the cheapest.

Theory

A **subscription** is a service you pay for regularly — a streaming service, a gym, a phone or internet plan, a software licence. Many come with a **lock-in contract**: you agree to stay for a fixed **term** (often 12, 24 or 36 months), and leaving early costs an **early-exit** (cancellation) fee. Working out the *true* cost of a subscription means adding up every payment over the whole term, not just looking at the monthly price.

Total cost over a term.

The **total cost** of a plan over its term is

$$\text{total cost} = (\text{regular fee} \times \text{number of payments}) + \text{any once-off fees}.$$

Once-off fees include a **set-up fee**, **connection fee**, **joining fee**, or an upfront **handset/modem** charge. For example, a plan of $50 per month for 24 months with a $120 set-up fee costs $120 + 24 \times 50 = \$1320$ in total.

Cost per month (or per year).

To compare plans fairly, it helps to find the **average cost per month** over the whole term:

$$\text{cost per month} = \frac{\text{total cost}}{\text{number of months}}.$$

Spreading a once-off fee across the term is what makes this average a bit higher than the advertised monthly price.

Monthly versus annual (prepaid) plans.

Many services offer a cheaper **annual** (paid-up-front) price instead of paying month by month. The **saving** is

$$\text{saving} = (\text{monthly fee} \times 12) - \text{annual price},$$

and the **percentage saving** is that saving written as a percentage of the monthly-plan cost:

$$\text{percentage saving} = \frac{\text{saving}}{\text{monthly fee} \times 12} \times 100\%.$$

Early-exit (cancellation) fees.

If you cancel a lock-in contract early, the provider usually charges an exit fee. A common rule is that the exit fee equals the **remaining monthly charges** (the fee for every month left on the contract). In that case leaving early often costs the **same total** as finishing the contract — so a lock-in contract is a real commitment.

Comparing two plans.

To choose between two plans, work out the **total cost of each over the same period** and compare. The plan with the lower total is cheaper, even if its monthly price looks higher — a low monthly fee with a large set-up fee can end up dearer.

Worked examples

Example 1 — scaffolded

A music streaming service costs $12.99 per month, or $129 if you pay for a whole year up front. Find the total cost of paying monthly for a year, the saving from paying annually, and the percentage saving (to 1 decimal place).

Working	Comment
Monthly for a year: $12 \times \$12.99 = \$155.88$.	Twelve monthly payments.

Working	Comment
Saving $= \$155.88 - \$129 = \$26.88$.	Monthly-plan cost minus the annual price.
Percentage saving $= \dfrac{26.88}{155.88} \times 100\% = 17.2\%$.	Saving as a percentage of the monthly-plan cost.

Example 2 — scaffolded

An internet plan is $75 per month on a 24-month contract, with a $120 modem paid up front. The early-exit fee is the total of the remaining monthly charges. Find the total cost over the full contract, and show that cancelling after 18 months costs the same total.

Working	Comment
Total over 24 months $= \$120 + 24 \times \$75 = \$120 + \$1800 = \$1920$.	Modem plus 24 monthly charges.
Cancel after 18 months: months left $= 24 - 18 = 6$, so exit fee $= 6 \times \$75 = \$450$.	Exit fee = remaining monthly charges.
Paid so far $= \$120 + 18 \times \$75 = \$1470$; with the exit fee, $\$1470 + \$450 = \$1920$.	Same total as finishing the contract.

Example 3 — unscaffolded

A shopper compares two 24-month phone plans. Plan A is $39 per month with no upfront cost. Plan B is $30 per month plus a $199 upfront handset fee. Find the total cost of each over the contract and state which is cheaper, and by how much.

Plan A total $= 24 \times \$39 = \$936$. Plan B total $= \$199 + 24 \times \$30 = \$199 + \$720 = \$919$. Comparing, Plan B costs $919 against Plan A's $936, so Plan B is cheaper by $\$936 - \$919 = \$17$ over the two years (an average of $\$919 \div 24 = \$38.29$ per month).

∴ Plan B is the cheaper plan over the 24-month contract, by $17.

Example 4 — unscaffolded

A small business needs a software subscription for 3 years. It can pay $24.99 per month, or $249 per year up front. Find the total cost each way and the saving from choosing the annual plan, giving the percentage saving to 1 decimal place.

Paying monthly for 3 years: $36 \times \$24.99 = \$899.64$. Paying annually for 3 years: $3 \times \$249 = \$747$. Saving $= \$899.64 - \$747 = \$152.64$, which as a percentage of the monthly cost is

$$\frac{152.64}{899.64} \times 100\% = 17.0\%.$$

∴ the annual plan saves the business $152.64 over three years, about 17.0%.

Practice questions

Scaffolded

Q1. A news website subscription is $8.50 per month, or $85 for a full year paid up front. (a) Find the total cost of paying monthly for 12 months. (b) Find the saving from paying for the year up front. (c) Express the saving as a percentage of the monthly-plan cost (to 1 decimal place).

Q2. A gym charges a $75 joining fee plus $25 per month on a 12-month membership. (a) Find the total cost of the first year. (b) Find the average cost per month over the first year. (c) In the second year there is no joining fee. Find the total second-year cost and the cost per month.

Q3. A phone plan is $40 per month on a 24-month contract. (a) Find the total cost over the 24 months. (b) How much has been paid after 15 months? (c) The early-exit fee equals the remaining monthly charges. Find the exit fee if the plan is cancelled after 15 months.

Q4. A family compares two streaming plans over one year. Plan P costs $17 per month. Plan Q costs $13 per month plus a one-off $50 set-up fee. (a) Find the total yearly cost of Plan P. (b) Find the total yearly cost of Plan Q. (c) State which plan is cheaper over the year, and by how much.

Q5. A magazine subscription is $9 per month, or $96 for a year up front. (a) Find the yearly cost of paying monthly. (b) Find the saving from paying annually. (c) Express the saving as a percentage of the monthly-plan cost (to the nearest whole per cent).

Q6. An internet contract is $65 per month for 18 months, with a $99 set-up fee. (a) Find the total cost over the contract. (b) Find the average cost per month over the contract. (c) Express the set-up fee as a percentage of the total cost (to 1 decimal place).

Unscaffolded

Q7. A video-streaming plan is $18.99 per month, or $199 per year prepaid. Find the saving from paying yearly, and express it as a percentage of the monthly-plan yearly cost (to 1 decimal place).

Q8. A phone contract is $49 per month for 24 months, plus a $149 upfront handset fee. Find (a) the total cost over the contract, and (b) the average cost per month over the 24 months (to the nearest cent).

Q9. A gym membership is $31 per month on a 12-month lock-in. A member cancels after 7 months and pays an early-exit fee of **half** the remaining monthly charges. Find the total amount the member has paid.

Q10. Compare two 24-month internet plans. Plan A is $70 per month with no set-up fee. Plan B is $60 per month plus a $180 modem fee. State which is cheaper over the 24 months and by how much, and find the average cost per month of each.

Q11. A software subscription is $34.99 per month or $349 per year up front. A user needs it for 2 years. Find the total cost each way and the saving from paying annually.

Q12. A music service costs $11.99 per month; a student plan is $5.99 per month. Over 12 months, how much does a student save, and what percentage is that of the full monthly plan's yearly cost (to 1 decimal place)?

Q13. A phone plan is $45 per month on a 36-month contract; the early-exit fee is the total of the remaining monthly charges. A customer cancels after 24 months. (a) Find the exit fee. (b) Find the total paid (24 months of charges plus the exit fee). (c) Compare this with the cost of completing the full contract.

Q14. A streaming bundle is $29.99 per month, or $299 per year prepaid. Over 3 years, find the total cost each way and the saving from prepaying each year.

Q15. A climbing gym offers pay-per-visit at $18 a visit, or a monthly membership of $54 for unlimited visits. (a) Find the cost of 4 visits in a month, paying per visit. (b) State which is cheaper for 4 visits in a month, and by how much. (c) Find the number of visits per month at which the two options cost the same.

Q16. A subscription box is $39 per month with a 6-month minimum term. A promotion gives $30 off the first month only. Find the minimum total cost with the promotion, and the average cost per month over the 6 months.

Q17. A design app can be bought as an annual licence for $480, or on a monthly plan at $44 per month. Find the total yearly cost of each, the saving from choosing the cheaper option, and the saving as a percentage of the dearer option (to 1 decimal place).

Q18. A phone plan is $60 per month for 24 months. A promotion gives 3 months free, so you pay for only 21 of the 24 months. Find (a) the total paid under the promotion, (b) the average cost per month over the full 24 months, and (c) the percentage saving compared with paying all 24 months (to 1 decimal place).

Answers

Q1. (a) \$102. (b) \$17. (c) $16.7\,\%$. **Q2.** (a) \$375. (b) \$31.25. (c) \$300; \$25 per month. **Q3.** (a) \$960. (b) \$600. (c) \$360. **Q4.** (a) \$204. (b) \$206. (c) Plan P, by \$2. **Q5.** (a) \$108. (b) \$12. (c) $11\,\%$. **Q6.** (a) \$1269. (b) \$70.50. (c) $7.8\,\%$. **Q7.** saving \$28.88; $12.7\,\%$. **Q8.** (a) \$1325. (b) \$55.21. **Q9.** \$294.50. **Q10.** Plan B is cheaper by \$60; Plan A \$70 per month, Plan B \$67.50 per month. **Q11.** monthly \$839.76, annual \$698; saving \$141.76. **Q12.** saving \$72.00; $50.0\,\%$. **Q13.** (a) \$540. (b) \$1620. (c) the same as the full contract, \$1620. **Q14.** monthly \$1079.64, annual \$897; saving \$182.64. **Q15.** (a) \$72. (b) membership, by \$18. (c) 3 visits. **Q16.** \$204; \$34 per month. **Q17.** annual \$480, monthly \$528; saving \$48; $9.1\,\%$ of the dearer option. **Q18.** (a) \$1260. (b) \$52.50. (c) $12.5\,\%$.

Fully-worked solutions

Q1. (a) Paying monthly: $12 \times \$8.50 = \$102$. (b) Saving $= \$102 - \$85 = \$17$. (c) $\frac{17}{102} \times 100\,\% = 16.7\,\%$ (1 d.p.).

Q2. (a) First year $= \$75 + 12 \times \$25 = \$75 + \$300 = \$375$. (b) Cost per month $= \$375 \div 12 = \$31.25$. (c) With no joining fee, the second year is $12 \times \$25 = \$300$, i.e. $\$300 \div 12 = \$25$ per month.

Q3. (a) Total $= 24 \times \$40 = \$960$. (b) After 15 months, $15 \times \$40 = \$600$ has been paid. (c) Months remaining $= 24 - 15 = 9$, so the exit fee $= 9 \times \$40 = \$360$.

Q4. (a) Plan P $= 12 \times \$17 = \$204$. (b) Plan Q $= \$50 + 12 \times \$13 = \$50 + \$156 = \$206$. (c) Plan P (\$204) is cheaper than Plan Q (\$206) by $\$206 - \$204 = \$2$.

Q5. (a) Monthly for a year $= 12 \times \$9 = \$108$. (b) Saving $= \$108 - \$96 = \$12$. (c) $\frac{12}{108} \times 100\,\% = 11.1\ldots\,\% \approx 11\,\%$ (nearest whole per cent).

Q6. (a) Total $= \$99 + 18 \times \$65 = \$99 + \$1170 = \$1269$. (b) Cost per month $= \$1269 \div 18 = \$70.50$. (c) $\frac{99}{1269} \times 100\,\% = 7.8\,\%$ (1 d.p.).

Q7. Paying monthly for a year $= 12 \times \$18.99 = \$227.88$. Saving $= \$227.88 - \$199 = \$28.88$, which is $\frac{28.88}{227.88} \times 100\,\% = 12.7\,\%$ (1 d.p.).

Q8. (a) Total $= \$149 + 24 \times \$49 = \$149 + \$1176 = \$1325$. (b) Cost per month $= \$1325 \div 24 = \$55.21$ (nearest cent).

Q9. Paid for 7 months $= 7 \times \$31 = \$217$. Months remaining $= 5$, so the remaining charges are $5 \times \$31 = \$155$; half of this is the exit fee, $\$155 \div 2 = \$77.50$. Total paid $= \$217 + \$77.50 = \$294.50$.

Q10. Plan A $= 24 \times \$70 = \$1680$. Plan B $= \$180 + 24 \times \$60 = \$180 + \$1440 = \$1620$. Plan B is cheaper by $\$1680 - \$1620 = \$60$. Average cost per month: Plan A $= \$1680 \div 24 = \$70$; Plan B $= \$1620 \div 24 = \$67.50$.

Q11. Monthly for 2 years $= 24 \times \$34.99 = \$839.76$. Annual for 2 years $= 2 \times \$349 = \$698$. Saving $= \$839.76 - \$698 = \$141.76$.

Q12. Full plan for a year $= 12 \times \$11.99 = \$143.88$. Student plan for a year $= 12 \times \$5.99 = \$71.88$. Saving $= \$143.88 - \$71.88 = \$72.00$, which is $\frac{72}{143.88} \times 100\,\% = 50.0\,\%$ (1 d.p.).

Q13. (a) Months remaining $= 36 - 24 = 12$, so the exit fee $= 12 \times \$45 = \$540$. (b) Paid $= 24 \times \$45 + \$540 = \$1080 + \$540 = \$1620$. (c) The full contract costs $36 \times \$45 = \$1620$, so cancelling early costs exactly the same as finishing it.

Q14. Monthly for 3 years $= 36 \times \$29.99 = \$1079.64$. Annual for 3 years $= 3 \times \$299 = \$897$. Saving $= \$1079.64 - \$897 = \$182.64$.

Q15. (a) Four visits $= 4 \times \$18 = \$72$. (b) The \$54 membership is cheaper than \$72 by $\$72 - \$54 = \$18$. (c) The costs are equal when $18 \times (\text{visits}) = 54$, i.e. $\text{visits} = 54 \div 18 = 3$. At 3 visits a month both options cost \$54.

Q16. Minimum term cost $= 6 \times \$39 = \$234$. With \$30 off the first month, the total is $\$234 - \$30 = \$204$, i.e. $\$204 \div 6 = \$34$ per month on average.

Q17. Annual licence $= \$480$. Monthly plan for a year $= 12 \times \$44 = \$528$. The annual licence is cheaper by $\$528 - \$480 = \$48$, which as a percentage of the dearer (monthly) option is $\frac{48}{528} \times 100\% = 9.1\%$ (1 d.p.).

Q18. (a) Paying for 21 months $= 21 \times \$60 = \$1260$. (b) Average over the full 24 months $= \$1260 \div 24 = \$52.50$ per month. (c) The full price is $24 \times \$60 = \$1440$, so the saving is $\$1440 - \$1260 = \$180$, which is $\frac{180}{1440} \times 100\% = 12.5\%$ (1 d.p.).