Foundation Mathematics Units 1 & 2

Chapter 4 Algebra and formulas

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Theory

Algebra is a shorthand for arithmetic. When a quantity is unknown, or can change from one situation to the next, we represent it with a letter called a **pronumeral**. A pronumeral simply stands for a number: once we know the number, we can replace the letter with it. Following Australian senior convention, a pronumeral is a single italic letter, such as n, h or c.

Writing expressions from words.

An **expression** is a combination of numbers, pronumerals and operations that represents a quantity — for example $3n+5$. To turn a sentence into an expression, decide what the pronumeral represents, then translate each phrase into an operation.

In words	In algebra
8 more than a number p	$p+8$
6 less than a number p	$p-6$
twice the cost c	$2c$
the number n divided by 4	$\frac{n}{4}$
the total price of k tickets at $\$12$ each	$12k$
a $\$90$ fee plus $\$70$ for each hour h	$90+70h$

Two conventions save writing: we omit the multiplication sign ($6\times n$ is written $6n$) and we write division as a fraction ($n\div 4$ is written $\frac{n}{4}$).

Terms, coefficients and constants.

An expression is built from **terms** — the parts separated by + or − signs. In the expression

$$20n+45,$$

there are two terms: $20n$ and 45.

- The number multiplying a pronumeral is called its **coefficient**. In $20n$ the coefficient of n is 20.
- A term that is just a number, with no pronumeral, is called a **constant** (or constant term). Here the constant term is 45.

So an expression like $20n+45$ might describe a worker's pay: $\$20$ for each of n items made, plus a fixed $\$45$ payment.

Substitution.

To **substitute** is to replace a pronumeral with a given number and then evaluate the expression. Work through the arithmetic using the normal order of operations (brackets, then × and ÷, then + and −). For example, substituting $n=12$ into $20n+45$:

$$20\times 12+45=240+45=285.$$

Substitution is how a general formula produces a specific answer.

Worked examples

Example 1 — scaffolded

A mobile phone-repair service charges a fixed booking fee of $\$40$ plus $\$25$ for each hour of labour. Let h be the number of hours a repair takes.

Working	Comment
Labour for h hours costs $25\times h=25h$ dollars.	"$\$25$ for each hour" means 25 multiplied by h.
Add the fixed fee: total charge $=40+25h$.	The booking fee does not depend on h, so it is a constant term.
The coefficient of h is 25; the constant term is 40.	Coefficient = number multiplying the pronumeral.
If a $\$15$ parts charge is also added, the total is $40+25h+15=55+25h$.	Combine the two constants: $40+15=55$.

Example 2 — scaffolded

A prepaid phone plan costs $\$15$ per month plus $\$0.10$ for each text message sent. The monthly cost, in dollars, is $C=15+0.1t$, where t is the number of texts sent.

Working	Comment
For $t=60$: $C=15+0.1\times 60$.	Substitute $t=60$ into the expression.
$=15+6=21$, so the cost is $\$21$.	Do the multiplication before the addition.
For $t=150$: $C=15+0.1\times 150=15+15=30$, so $\$30$.	Same method with a larger value of t.
The constant 15 is the fixed monthly charge, paid even if no texts are sent.	Interpreting the constant term in context.

Example 3 — unscaffolded

A market stallholder's takings for a day, in dollars, are given by $T=18n+60$, where n is the number of baskets sold. State the number of terms, the coefficient of n and the constant term, then find the takings when 50 baskets are sold.

The expression $18n+60$ has **two terms**: $18n$ and 60.

The coefficient of n is 18 (the price per basket), and the constant term is 60 (a fixed amount, such as an appearance payment).

Substituting $n=50$:

$$T=18\times 50+60=900+60=960.$$

$\therefore$ the takings for the day are $\$960$.

Example 4 — unscaffolded

An electrician quotes jobs using the rule $Q=85+60h$ dollars, where h is the number of hours worked. Find the quote for a 3-hour job and for a 5-hour job, and state how much more the longer job costs.

For $h=3$:

$$Q=85+60\times 3=85+180=265.$$

For $h=5$:

$$Q=85+60\times 5=85+300=385.$$

The difference is $385-265=120$.

$\therefore$ the quotes are $\$265$ and $\$385$; the 5-hour job costs $\$120$ more.

Practice questions

Scaffolded

Q1. Write an algebraic expression for each description. (a) $\$17$ more than m dollars. (b) 6 times a number n. (c) the total cost of t movie tickets at $\$16$ each.

Q2. A café pays a casual worker $\$24$ per hour. (a) Write an expression for the pay, in dollars, for h hours. (b) Write an expression if the worker also receives a fixed $\$30$ travel allowance. (c) State the coefficient of h and the constant term in your answer to (b).

Q3. Consider the expression $7x+9$. (a) How many terms does it have? (b) What is the coefficient of x? (c) What is the constant term?

Q4. Evaluate $5n+12$ when (a) $n=4$, (b) $n=10$, (c) $n=0$.

Q5. The cost of hiring a marquee, in dollars, is $C=150+45d$, where d is the number of days. (a) Find the cost for 3 days. (b) Find the cost for 7 days. (c) State what the number 150 represents in this context.

Q6. A plumber charges a $\$75$ callout fee plus $\$50$ for each hour worked. (a) Write an expression for the charge, in dollars, for h hours. (b) Find the charge for a job lasting 2 hours. (c) Find the charge for a job lasting 4.5 hours.

Unscaffolded

Q7. Write an expression for the perimeter of a rectangle whose length is l and whose width is w.

Q8. A number n is multiplied by 3 and then 7 is subtracted. Write the expression, and evaluate it when $n=10$.

Q9. A worker is paid $\$22$ for each ordinary hour h and $\$33$ for each overtime hour v. Write an expression for the total pay, then find the pay for 38 ordinary hours and 4 overtime hours.

Q10. Evaluate $4a+3b$ when $a=5$ and $b=2$.

Q11. A courier charges a $\$6.00$ pick-up fee plus $\$1.80$ per kilometre. Write an expression for the cost C, in dollars, of a d-km delivery, then find the cost of a 15 km delivery.

Q12. A gym charges a $\$60$ joining fee plus $\$25$ per month. (a) Write an expression for the total cost after m months. (b) Find the total cost after 6 months. (c) State the coefficient of m.

Q13. Evaluate $\frac{n}{4}+5$ when (a) $n=20$, (b) $n=2$.

Q14. A hardware store prices its stock by adding a 10% markup to the wholesale price p, giving a selling price of $1.1p$ dollars. Find the selling price when $p=250$.

Q15. A quick rule to convert a temperature from degrees Celsius C to degrees Fahrenheit is $F=1.8C+32$. (a) Find F when $C=25$. (b) Find F when $C=0$.

Q16. A removalist charges $\$120$ plus $\$95$ for each hour. Find the cost of a 3-hour job and a 6-hour job, and state the difference.

Q17. Evaluate $2x+7$ when (a) $x=-3$, (b) $x=-5$.

Q18. A phone plan has a fixed monthly charge of $\$19$ plus $\$0.15$ for each minute m used beyond the included cap. (a) Write an expression for the monthly cost, in dollars. (b) Find the cost in a month with 40 minutes over the cap. (c) Find the cost in a month with 120 minutes over the cap.

Answers

Q1. (a) $m+17$. (b) $6n$. (c) $16t$. **Q2.** (a) $24h$. (b) $24h+30$. (c) coefficient 24, constant 30. **Q3.** (a) 2. (b) 7. (c) 9. **Q4.** (a) 32. (b) 62. (c) 12. **Q5.** (a) $\$285$. (b) $\$465$. (c) the fixed hire fee, paid regardless of the number of days. **Q6.** (a) $75+50h$. (b) $\$175$. (c) $\$300$. **Q7.** $2l+2w$ (equivalently $2(l+w)$). **Q8.** $3n-7$; value 23. **Q9.** $22h+33v$; pay $=\$968$. **Q10.** 26. **Q11.** $C=6.00+1.80d$; $\$33.00$. **Q12.** (a) $60+25m$. (b) $\$210$. (c) 25. **Q13.** (a) 10. (b) 5.5. **Q14.** $\$275$. **Q15.** (a) 77. (b) 32. **Q16.** $\$405$ and $\$690$; difference $\$285$. **Q17.** (a) 1. (b) -3. **Q18.** (a) $19+0.15m$. (b) $\$25$. (c) $\$37$.

Fully-worked solutions

Q1. (a) “More than” means addition: $m+17$. (b) “6 times” means multiplication, written without the sign: $6n$. (c) Each ticket costs $\$16$, so t tickets cost $16\times t=16t$.

Q2. (a) At $\$24$ per hour for h hours the pay is $24\times h=24h$. (b) Add the fixed allowance: $24h+30$. (c) The coefficient of h (the number multiplying h) is 24; the constant term is 30.

Q3. The expression $7x+9$ is separated by the + sign into (a) two terms, $7x$ and 9. (b) The coefficient of x is 7. (c) The constant term is 9.

Q4. Substitute each value into $5n+12$. (a) $5\times 4+12=20+12=32$. (b) $5\times 10+12=50+12=62$. (c) $5\times 0+12=0+12=12$.

Q5. Substitute into $C=150+45d$. (a) $150+45\times 3=150+135=285$, so $\$285$. (b) $150+45\times 7=150+315=465$, so $\$465$. (c) 150 is the fixed part of the cost — a set fee charged no matter how many days the marquee is hired.

Q6. (a) The hourly charge for h hours is $50h$; adding the callout fee gives $75+50h$. (b) $75+50\times 2=75+100=175$, so $\$175$. (c) $75+50\times 4.5=75+225=300$, so $\$300$.

Q7. A rectangle has two lengths and two widths, so the perimeter is $l+l+w+w=2l+2w$, which can also be written $2(l+w)$.

Q8. “Multiplied by 3” gives $3n$; “then 7 subtracted” gives $3n-7$. Substituting $n=10$: $3\times 10-7=30-7=23$.

Q9. Ordinary pay is $22h$ and overtime pay is $33v$, so the total is $22h+33v$. Substituting $h=38$ and $v=4$:

$$22\times 38+33\times 4=836+132=968,$$

so the pay is $\$968$.

Q10. $4a+3b=4\times 5+3\times 2=20+6=26$.

Q11. The cost is the pick-up fee plus $\$1.80$ per km, so $C=6.00+1.80d$. Substitute $d=15$:

$$6.00+1.80\times 15=6.00+27.00=33.00,$$

so the delivery costs $\$33.00$.

Q12. (a) The monthly charge for m months is $25m$; adding the joining fee gives $60+25m$. (b) $60+25\times 6=60+150=210$, so $\$210$. (c) The coefficient of m is 25.

Q13. (a) $\frac{20}{4}+5=5+5=10$. (b) $\frac{2}{4}+5=0.5+5=5.5$.

Q14. Substitute $p=250$ into $1.1p$: $1.1\times 250=275$, so the selling price is $\$275$.

Q15. Substitute into $F=1.8C+32$. (a) $1.8\times 25+32=45+32=77$. (b) $1.8\times 0+32=0+32=32$.

Q16. The cost for h hours is $120+95h$. For $h=3$: $120+95\times3=120+285=405$, so $\$405$. For $h=6$: $120+95\times6=120+570=690$, so $\$690$. The difference is $690-405=285$, so $\$285$.

Q17. Substitute the negative values into $2x+7$, taking care with signs. (a) $2\times(-3)+7=-6+7=1$. (b) $2\times(-5)+7=-10+7=-3$.

Q18. (a) The charge for m minutes over the cap is $0.15m$; adding the fixed charge gives $19+0.15m$. (b) $19+0.15\times40=19+6=25$, so $\$25$. (c) $19+0.15\times120=19+18=37$, so $\$37$.

Theory

Mathematics is full of **relationships** — pairs of quantities that change together. The cost of a taxi depends on how far you travel; the number of matchsticks in a pattern depends on how many shapes you build. This section is about spotting a relationship, organising it in a **table of values**, describing it **in words**, and writing it as a short **rule** or **formula**.

Number and geometric patterns.

A **pattern** (or sequence) is a list of numbers in a set order, such as

$$5, 8, 11, 14, \ldots$$

Each number is a **term**. A **term-to-term rule** says how to get from one term to the next — here, "**add 3**". Some patterns are **geometric** in the everyday sense that they are built from shapes: rows of tiles, matchstick figures, or growing borders. Continuing a pattern means applying its rule again and again.

Tables of values.

A **table of values** lines up the two quantities in a relationship so you can see how they change together. For the pattern above, if n counts the position of the term, then:

Position n	1	2	3	4
Term	5	8	11	14

Reading **down** the table shows the pairs; reading **along** the bottom row shows the term-to-term rule (each term is 3 more than the last).

Describing a relationship in words and as a rule.

The same relationship can be written three ways:

- **in words** — "start at 5 and add 3 for each step";
- **as a formula** — using a pronumeral, $\text{term} = 3n + 2$ (check: $n = 1$ gives 5);
- **as a graph** — a set of points that, for these relationships, lie on a straight line.

A **formula** is the most useful form because it lets you jump straight to any term without listing all the ones before it. Choose a letter for each quantity (for example C for cost, n for the number of items) and state clearly what each letter stands for.

Linear patterns — a constant change.

A relationship is **linear** when the quantity goes up (or down) by the **same amount** each step. That constant step is the **first difference**. Every linear rule has the shape

$$\text{quantity} = (\text{starting value}) + (\text{step}) \times (\text{number of steps}).$$

For example, a gym that charges a $\$30$ joining fee plus $\$15$ each week has the rule $C = 30 + 15w$: the **starting value** 30 is the cost at $w = 0$, and the **step** 15 is the weekly increase. If the first difference is **not** constant, the relationship is not linear.

Reading a relationship from a graph.

Plotting the pairs from a table gives a picture of the relationship. For a linear rule the points lie on a **straight line**, so you can read off values in between or predict ahead: find the input on the horizontal axis, go up to the line, then across to the vertical axis. Always check the axis labels and the scale before reading a value.

Worked examples

Example 1 — scaffolded

Matchsticks are used to build a row of squares that share sides: 1 square uses 4 matchsticks, 2 squares use 7, and 3 squares use 10. Find a rule for the number of matchsticks m needed for n squares, and use it to find the number for 8 squares.

Working	Comment
Table: $n=1,2,3 \Rightarrow m=4,7,10$.	List the pairs you are given.
Each extra square adds 3 matchsticks (first difference $=3$).	Look at the step between terms.
Starting value: at $n=1$ the count is 4, and $4=3(1)+1$.	Test the shape (step) $\times n+$ (starting number).
Rule: $m=3n+1$.	Check $n=2$: $3(2)+1=7$. ✓
For 8 squares: $m=3(8)+1=25$ matchsticks.	Substitute $n=8$.

Example 2 — scaffolded

A plumber charges a $\$60$ call-out fee plus $\$45$ for each hour worked. Write a rule for the total cost C for a job lasting h hours, complete a table for $h=0,1,2,3$, and find the cost of a 5-hour job.

Working	Comment
Fixed part $\$60$; variable part $\$45$ per hour $=45h$.	Separate the once-off charge from the per-hour charge.
Rule: $C=60+45h$.	Starting value 60, step 45.
$h=0,1,2,3 \Rightarrow C=60,105,150,195$.	Add 45 each time, starting at 60.
5-hour job: $C=60+45(5)=60+225=\$285$.	Substitute $h=5$.

Example 3 — unscaffolded

A saver opens an account with $\$150$ and adds the same amount every week, so the balance forms the pattern $150,220,290,360,\ldots$ Describe the pattern in words, write a rule for the balance B (in dollars) after n weeks, find the balance after 10 weeks, and find the first week in which the balance is above $\$500$.

The balance starts at $\$150$ and increases by $\$70$ each week (the first difference is $220-150=70$), so the rule is

$$B=150+70n.$$

After 10 weeks, $B=150+70(10)=150+700=\$850$. To pass $\$500$, solve $150+70n>500$, so $70n>350$ and $n>5$; the first whole week is $n=6$, where $B=150+70(6)=\$570$.

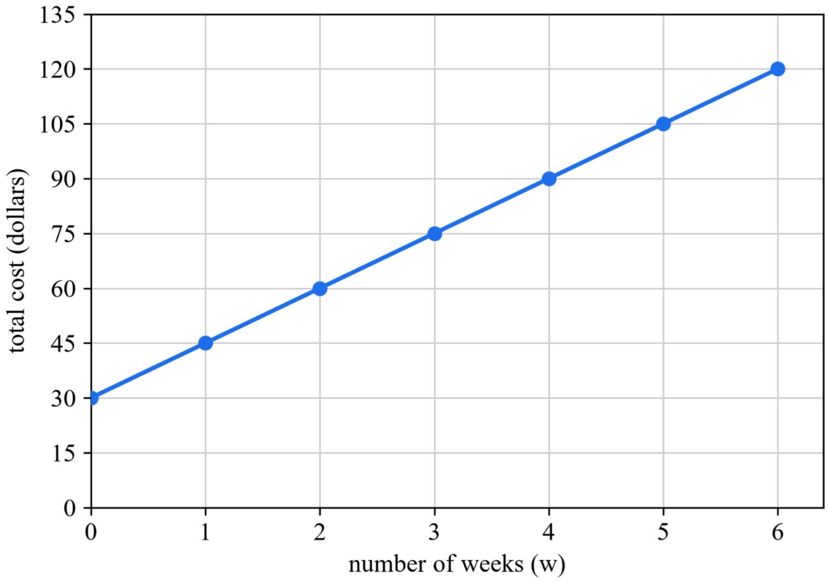
$\therefore$ the rule is $B=150+70n$, the balance after 10 weeks is $\$850$, and it first exceeds $\$500$ in week 6.

Example 4 — unscaffolded

A gym charges a $\$30$ joining fee plus $\$15$ per week. The total cost C after w weeks is $C=30+15w$. Complete a table for $w=0$ to 6, plot the relationship, and use the graph to find the total cost after 4 weeks and the week in which the cost reaches $\$105$.

Building the table by adding $\$15$ each week:

w	0	1	2	3	4	5	6
C (\$)	30	45	60	75	90	105	120



The points lie on a straight line because the cost rises by the same $\$15$ each week. Reading the graph: at $w=4$ the line is level with $C=90$, and the cost reaches $\$105$ at $w=5$.

$\therefore$ the cost after 4 weeks is $\$90$, and it reaches $\$105$ in week 5.

Practice questions

Scaffolded

Q1. A pattern of triangles made from matchsticks uses $3, 5, 7, 9, \ldots$ matchsticks for $1, 2, 3, 4, \ldots$ triangles. (a) Write down the next two terms. (b) Describe the term-to-term rule in words. (c) Write a rule for the number of matchsticks t for n triangles, and use it to find the number for 10 triangles.

Q2. A car-hire firm charges a $\$40$ hire fee plus $\$0.25$ per kilometre driven. (a) Write a rule for the total cost C (dollars) for k kilometres. (b) Complete a table for $k=0, 100, 200$. (c) Find the cost of driving 500 km.

Q3. A number pattern begins $8, 15, 22, 29, \ldots$ (a) State the term-to-term rule. (b) Write a rule for the nth term a. (c) Find the 10th term.

Q4. A relationship between x and y is shown in the table.

x	1	2	3	4
y	5	9	13	17

(a) State the first difference.
(b) Write a rule for y in terms of x.
(c) Use the rule to find y when $x=10$.

Q5. A taxi charges a $\$4.20$ flagfall plus $\$2.10$ per kilometre. The fare F (dollars) for a trip of k kilometres is $F=4.20+2.10k$. (a) Find the fare for a 6 km trip. (b) A trip cost $\$27.30$. Set up and solve an equation to find its length.

Q6. A market stall's takings are modelled by $C=12+8t$, where t is the number of hours open. (a) Complete a table for $t=0, 1, 2, 3, 4$. (b) State whether the points would lie on a straight line, and how you can tell from the table.

Unscaffolded

Q7. A row of pentagons that share sides uses 5 matchsticks for 1 pentagon, 9 for 2 and 13 for 3. (a) Write a rule for the number of matchsticks m for n pentagons. (b) Find the number of matchsticks for 20 pentagons. (c) How many pentagons can be built with exactly 45 matchsticks?

Q8. A catering company charges a $\$250$ hire fee plus $\$18$ per guest. (a) Write a rule for the total cost C (dollars) for g guests. (b) Find the cost for 30 guests. (c) A host has a budget of $\$1000$. Find the greatest number of guests they can cater for.

Q9. A relationship is shown in the table.

x	0	1	2	3
y	7	11	15	19

Write a rule for y in terms of x, and use it to predict y when $x = 8$.

Q10. A pattern begins $5, 10, 20, 40, \ldots$ (a) Describe the term-to-term rule in words. (b) Write down the next two terms. (c) Explain why this pattern is **not** linear.

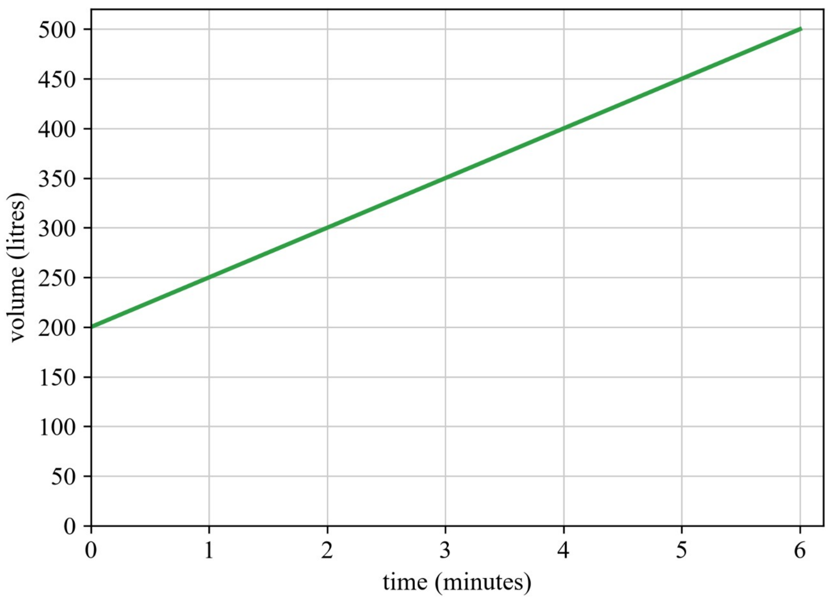
Q11. Two phone plans have monthly costs (dollars) after n months given by

$$\text{Plan A: } A = 20 + 12n, \text{Plan B: } B = 50 + 6n.$$

(a) Complete a table of both costs for $n = 0$ to 6. (b) Which plan is cheaper after 3 months? (c) Find the number of months at which the two plans cost the same, and that cost.

Q12. A number pattern begins $12, 19, 26, 33, \ldots$ (a) Write a rule for the nth term. (b) Find the first term that is greater than 100.

Q13. The graph shows the volume of water V (litres) in a tank t minutes after a pump is switched on.



(a) State the volume of water in the tank when the pump was switched on.
(b) By how many litres does the volume rise each minute?
(c) Use the graph to find the volume after 4 minutes.
(d) Use the graph to find the time at which the volume reaches 450 litres.

Q14. The takings C (dollars) of a food van are given by $C = 15 + 9t$, where t is the number of hours open. Copy and complete the table for $t = 0$ to 5.

t	0	1	2	3	4	5
C						

Q15. A phone's battery charge (percent) is modelled by $B = 100 - 12n$, where n is the number of hours of use. (a) Describe in words how the charge changes. (b) Find the charge after 5 hours of use. (c) Find the number of hours of use for which the charge falls to 4%.

Q16. A mobile data plan costs a $\$20$ base fee plus $\$8$ per gigabyte of data used. (a) Write a rule for the monthly cost C (dollars) for g gigabytes. (b) Find the cost of using 5 GB. (c) One month's bill was $\$52$. Find how many gigabytes were used.

Q17. A row of n unit squares placed side by side forms a long rectangle whose perimeter P (in unit lengths) is $4, 6, 8, \ldots$ for $n = 1, 2, 3, \ldots$ (a) Write a rule for P in terms of n. (b) Find the perimeter when $n = 10$. (c) Find the value of n that gives a perimeter of 50.

Q18. Two savers record their balances (dollars) each week in the tables below.

Week w	0	1	2
Saver 1	40	65	90
Saver 2	100	115	130

(a) Write a rule for each saver's balance after w weeks.
(b) Find each saver's balance after 4 weeks.
(c) Determine which saver first reaches a balance of at least $\$250$, and in which week.

Answers

Q1. (a) $11, 13$. (b) add 2 each time. (c) $t = 2n + 1$; 21 matchsticks. **Q2.** (a) $C = 40 + 0.25k$. (b) $\$40, \$65, \$90$. (c) $\$165$. **Q3.** (a) add 7. (b) $a = 7n + 1$. (c) 71. **Q4.** (a) 4. (b) $y = 4x + 1$. (c) 41. **Q5.** (a) $\$16.80$. (b) 11 km. **Q6.** (a) $12, 20, 28, 36, 44$. (b) yes — the first difference is a constant 8. **Q7.** (a) $m = 4n + 1$. (b) 81. (c) 11 pentagons. **Q8.** (a) $C = 250 + 18g$. (b) $\$790$. (c) 41 guests. **Q9.** $y = 4x + 7$; $y = 39$ when $x = 8$. **Q10.** (a) multiply by 2 (double) each time. (b) $80, 160$. (c) the first difference is not constant ($5, 10, 20, \ldots$), so it is not linear. **Q11.** (a) A: $20, 32, 44, 56, 68, 80, 92$; B: $50, 56, 62, 68, 74, 80, 86$. (b) Plan A ($\$56$ vs $\$68$). (c) 5 months, both $\$80$. **Q12.** (a) $7n + 5$. (b) 103 (the 14th term). **Q13.** (a) 200 L. (b) 50 L per minute. (c) 400 L. (d) 5 minutes. **Q14.** $15, 24, 33, 42, 51, 60$. **Q15.** (a) it starts at 100% and drops by 12 percentage points each hour. (b) 40%. (c) 8 hours. **Q16.** (a) $C = 20 + 8g$. (b) $\$60$. (c) 4 GB. **Q17.** (a) $P = 2n + 2$. (b) 22. (c) $n = 24$. **Q18.** (a) Saver 1: $B = 40 + 25w$; Saver 2: $B = 100 + 15w$. (b) Saver 1 $\$140$, Saver 2 $\$160$. (c) Saver 1, in week 9 ($\$265$, versus Saver 2 first reaching $\$250$ in week 10).

Fully-worked solutions

Q1. The pattern $3, 5, 7, 9$ increases by 2 each time. (a) Next two terms: $9 + 2 = 11$, then $11 + 2 = 13$. (b) The term-to-term rule is "**add** 2". (c) The step is 2 and at $n = 1$ the count is $3 = 2(1) + 1$, so $t = 2n + 1$. For 10 triangles, $t = 2(10) + 1 = 21$.

Q2. (a) A fixed $\$40$ plus $\$0.25$ for each of k kilometres: $C = 40 + 0.25k$. (b) $k = 0$: $\$40$; $k = 100$: $40 + 0.25(100) = \$65$; $k = 200$: $40 + 0.25(200) = \$90$. (c) $k = 500$: $C = 40 + 0.25(500) = 40 + 125 = \$165$.

Q3. (a) Each term is 7 more than the last, so the rule is "**add** 7". (b) The step is 7 and $8 = 7(1) + 1$, so $a = 7n + 1$. (c) The 10th term is $a = 7(10) + 1 = 71$.

Q4. (a) The values rise $5 \to 9 \to 13 \to 17$, a constant first difference of 4. (b) The step is 4 and at $x = 1$, $y = 5 = 4(1) + 1$, so $y = 4x + 1$. (c) $y = 4(10) + 1 = 41$.

Q5. (a) $F = 4.20 + 2.10(6) = 4.20 + 12.60 = \$16.80$. (b) Solve $4.20 + 2.10k = 27.30$. Then $2.10k = 27.30 - 4.20 = 23.10$, so $k = \dfrac{23.10}{2.10} = 11$ km.

Q6. (a) Starting at 12 and adding 8: $t = 0, 1, 2, 3, 4$ give $C = 12, 20, 28, 36, 44$. (b) The first difference is 8 each time — a constant step — so the points lie on a straight line.

Q7. (a) One pentagon uses 5; each extra pentagon adds 4, and $5 = 4(1) + 1$, so $m = 4n + 1$. (b) $m = 4(20) + 1 = 81$ matchsticks. (c) Solve $4n + 1 = 45$: $4n = 44$, so $n = 11$ pentagons.

Q8. (a) Fixed $\$250$ plus $\$18$ per guest: $C = 250 + 18g$. (b) $C = 250 + 18(30) = 250 + 540 = \$790$. (c) Solve $250 + 18g \leq 1000$: $18g \leq 750$, so $g \leq 41.67$. The greatest whole number of guests is 41 (costing $250 + 18(41) = \$988$).

Q9. The values rise by 4 each step and at $x = 0$, $y = 7$, so $y = 4x + 7$. When $x = 8$, $y = 4(8) + 7 = 39$.

Q10. (a) Each term is **double** the one before ($5 \to 10 \to 20 \to 40$). (b) $40 \times 2 = 80$, then $80 \times 2 = 160$. (c) The first differences are $5, 10, 20, \ldots$ — they are not constant — so the pattern is not linear (a linear pattern must increase by the same amount each step).

Q11. (a)

n	0	1	2	3	4	5	6
A	20	32	44	56	68	80	92
B	50	56	62	68	74	80	86

(b) After 3 months, Plan A costs $\$56$ and Plan B costs $\$68$, so **Plan A** is cheaper.

(c) Setting $20+12n=50+6n$ gives $6n=30$, so $n=5$; both cost $20+12(5)=\$80$.

Q12. (a) The terms rise by 7 and $12=7(1)+5$, so the nth term is $7n+5$. (b) Solve $7n+5>100$: $7n>95$, so $n>13.57$; the first whole value is $n=14$, giving $7(14)+5=103$.

Q13. (a) When $t=0$ the line meets the vertical axis at 200, so the tank held 200 L. (b) The line rises from 200 L to 500 L over 6 minutes, that is $\frac{500-200}{6}=50$ L per minute. (c) At $t=4$, reading up to the line and across gives $V=400$ L. (d) Reading across from $V=450$ to the line and down gives $t=5$ minutes.

Q14. Starting at 15 and adding 9 each hour: $C=15,24,33,42,51,60$ for $t=0,1,2,3,4,5$.

Q15. (a) The charge begins at 100% and falls by 12 percentage points for each hour of use. (b) $B=100-12(5)=100-60=40\%$. (c) Solve $100-12n=4$: $12n=96$, so $n=8$ hours.

Q16. (a) A fixed $\$20$ plus $\$8$ per gigabyte: $C=20+8g$. (b) $C=20+8(5)=\$60$. (c) Solve $20+8g=52$: $8g=32$, so $g=4$ GB.

Q17. (a) Each square adds 2 to the perimeter and a single square has perimeter $4=2(1)+2$, so $P=2n+2$. (b) $P=2(10)+2=22$. (c) Solve $2n+2=50$: $2n=48$, so $n=24$.

Q18. (a) Saver 1 starts at 40 and rises by 25 each week: $B=40+25w$. Saver 2 starts at 100 and rises by 15: $B=100+15w$. (b) At $w=4$: Saver 1 has $40+25(4)=\$140$; Saver 2 has $100+15(4)=\$160$. (c) Saver 1 reaches $\$250$ when $40+25w\geq250$, i.e. $w\geq8.4$, so week 9 (balance $\$265$). Saver 2 reaches $\$250$ when $100+15w\geq250$, i.e. $w\geq10$, so week 10. Saver 1 reaches $\$250$ first.

(The two savers happen to have equal balances of $\$190$ in week 6.)

Theory

Algebra is a shorthand for describing relationships using letters in place of numbers. A **pronumeral** (or **variable**) is a letter, such as x or n, that stands for a number we do not yet know or that is allowed to change. Learning the **conventions** — the agreed rules for writing algebra — means everyone reads an expression the same way.

Writing products.

A few simple rules cover almost everything you will write.

- The multiplication sign is left out: $3 \times n$ is written $3n$, and $a \times b$ is written ab.
- The number written in front of a pronumeral is the **coefficient**: in $5y$ the coefficient is 5.
- $1 \times x$ is written simply as x (a coefficient of 1 is not shown), and $-1 \times x$ is written $-x$.
- A pronumeral multiplied by itself is written with an index: $x \times x = x^2$ and $x \times x \times x = x^3$.
- Division is written as a fraction: $m \div 4 = \frac{m}{4}$.
- Write the number first, then the pronumerals in alphabetical order: $b \times 2 \times a = 2ab$.

Terms, expressions, equations and formulas.

These four words are used constantly, so it is worth being precise.

- A **term** is a single number, a pronumeral, or a product of them — for example 7, x, $3ab$ or $\frac{n}{2}$. Terms in an expression are separated by $+$ and $-$ signs.
- A **constant** is a term that is just a number, such as the 5 in $3x+5$.
- An **expression** is one or more terms joined by $+$ or $-$, such as $3x+5$. An expression has **no** equals sign.
- An **equation** states that two expressions are equal, such as $3x+5=20$.
- A **formula** is an equation, usually connecting two or more pronumerals, that gives a rule for calculating one quantity from others — for example $C=40+25h$ for the cost of a job lasting h hours.

Like terms.

Two terms are **like terms** when they have exactly the same pronumeral part — the same letters raised to the same powers. Only their coefficients may differ.

- $4x$ and $9x$ are like terms; $4x$ and $9y$ are not.
- $3ab$ and $7ab$ are like terms; $3ab$ and $3a$ are not.
- $2x^2$ and $5x^2$ are like terms; $2x^2$ and $5x$ are not.

Collecting like terms.

To **simplify** an expression we **collect like terms** — add or subtract their coefficients while keeping the pronumeral part unchanged. Unlike terms cannot be combined.

$$4x+9x=13x, 8y-3y=5y, 6a+2b+a=7a+2b.$$

It helps to read $4x+9x$ aloud as "4 lots of x plus 9 lots of x makes 13 lots of x": the pronumeral stands for a number, so only terms with the same pronumeral part can be added.

Multiplying a term by a number.

To multiply a term by a number, multiply the **coefficient** and leave the pronumeral part unchanged:

$$3 \times 4x=12x, 5 \times 2ab=10ab.$$

This is often the first step before collecting like terms in a longer calculation.

Worked examples

Example 1 — scaffolded

Rewrite each of the following using the conventions of algebra: (a) $7 \times k$, (b) $y \times 4 \times x$, (c) $m \times m$, (d) $p \div 5$.

Working	Comment
(a) $7 \times k = 7k$.	Drop the × sign; the coefficient 7 goes in front.
(b) $y \times 4 \times x = 4xy$.	Number first, then pronumerals in alphabetical order.
(c) $m \times m = m^2$.	A pronumeral times itself is written with an index.
(d) $p \div 5 = \frac{p}{5}$.	Division is written as a fraction.

Example 2 — scaffolded

For the expression $9a + 4b - 7$, state (a) the number of terms, (b) the coefficient of a, (c) the constant term, and (d) whether $9a$ and $4b$ are like terms.

Working	Comment
(a) The terms are $9a$, $4b$ and -7, so there are **3 terms**.	Terms are separated by + and − signs.
(b) In $9a$ the number in front of a is 9, so the coefficient of a is **9**.	The coefficient is the multiplying number.
(c) The term that is just a number is -7, so the constant term is -7.	A constant has no pronumeral.
(d) $9a$ has pronumeral a and $4b$ has pronumeral b; the pronumerals differ, so they are **not** like terms.	Like terms need the same pronumeral part.

Example 3 — unscaffolded

A fruit box holds a apples and b bananas. A shop receives 3 boxes, then a separate delivery of 5 extra apples and 4 extra bananas, and finally 2 more boxes. Write an expression for the total number of pieces of fruit and simplify it.

The 3 boxes give $3a$ apples and $3b$ bananas; the extra delivery gives 5 apples and 4 bananas; the 2 further boxes give $2a$ apples and $2b$ bananas. Adding everything,

$$3a + 3b + 5 + 4 + 2a + 2b.$$

Collecting like terms — the a terms together, the b terms together, and the numbers together:

$$(3a + 2a) + (3b + 2b) + (5 + 4) = 5a + 5b + 9.$$

∴ the total number of pieces of fruit is $5a + 5b + 9$.

Example 4 — unscaffolded

A triangular garden bed has sides of length $2n$, $3n + 1$ and $n + 5$ metres. Write a simplified expression for its perimeter.

The perimeter is the sum of the three side lengths:

$$P = 2n + (3n + 1) + (n + 5).$$

Collecting the n terms and the constant terms separately,

$$P = (2n + 3n + n) + (1 + 5) = 6n + 6.$$

$\therefore$ the perimeter is $(6n+6)$ metres.

Practice questions

Scaffolded

Q1. Rewrite each expression using the conventions of algebra. (a) $8\times t$ (b) $a\times b\times 5$ (c) $w\times w\times w$

Q2. For the expression $5x+7y-3$: (a) state the number of terms; (b) state the coefficient of x; (c) state the constant term.

Q3. State whether each pair are like terms. (a) $6m$ and $2m$ (b) $4p$ and $4q$ (c) $3x^2$ and $8x^2$

Q4. Simplify by collecting like terms. (a) $7a+2a$ (b) $10c-4c$ (c) $3x+5x+x$

Q5. Simplify each expression. (a) $4m+3n+2m$ (b) $8p-3p+2q$ (c) $5x+6+2x-1$

Q6. Classify each of the following as an expression, an equation, or a formula. (a) $3x+7$ (b) $2x-1=9$ (c) $A=lw$

Unscaffolded

Q7. Rewrite each using the conventions of algebra. (a) $n\times 6$ (b) $3\times a\times a$ (c) $y\div 8$ (d) $q\times p\times 2$

Q8. Write an algebraic expression for each description. (a) 5 more than x (b) the product of 7 and m (c) a number n divided by 4 (d) 3 less than twice y

Q9. For the expression $12a-5b+9$, state (a) the number of terms, (b) the coefficient of b, and (c) the constant term.

Q10. Simplify each expression. (a) $9k+4k-2k$ (b) $6x+3y-2x+y$ (c) $10+4n-3+2n$

Q11. A café buys x litres of milk on Monday, $2x$ litres on Tuesday and $3x$ litres on Wednesday. Write and simplify an expression for the total number of litres bought over the three days.

Q12. A rectangle has length $5m$ and width $2m$. By adding all four side lengths, write a simplified expression for its perimeter.

Q13. Multiply each term by the number in front. (a) $6\times 3x$ (b) $2\times 6ab$ (c) $5\times 2n$

Q14. A gardener charges a $\$50$ call-out fee plus $\$38$ for each hour h worked. (a) Write a rule for the total cost C in dollars. (b) State whether your answer to (a) is an expression, an equation or a formula.

Q15. Simplify each expression. (a) $8x+3+2x+9$ (b) $7a+4b-a-4b$ (c) $6p+2q+3p-q$

Q16. Explain why $5x$ and $5x^2$ are not like terms, then simplify $5x+5x^2+2x$ as far as possible.

Q17. A crate holds a apples. A shop has 4 full crates and 6 loose apples, then sells 9 apples. Write and simplify an expression for the number of apples remaining.

Q18. At a concert stall a ticket costs $\$t$ and a drink costs $\$d$. Write and simplify an expression for the total cost of 3 tickets and 2 drinks, followed by a second purchase of 2 more tickets and 1 more drink.

Answers

Q1. (a) $8t$. (b) $5ab$. (c) w^3. **Q2.** (a) 3. (b) 5. (c) -3. **Q3.** (a) like. (b) not like. (c) like. **Q4.** (a) $9a$. (b) $6c$. (c) $9x$. **Q5.** (a) $6m+3n$. (b) $5p+2q$. (c) $7x+5$. **Q6.** (a) expression. (b) equation. (c) formula. **Q7.** (a) $6n$. (b) $3a^2$. (c) $\frac{y}{8}$. (d) $2pq$. **Q8.** (a) $x+5$. (b) $7m$. (c) $\frac{n}{4}$. (d) $2y-3$. **Q9.** (a) 3. (b) -5. (c) 9. **Q10.** (a) $11k$. (b) $4x+4y$. (c) $6n+7$. **Q11.** $x+2x+3x=6x$ litres. **Q12.** $14m$. **Q13.** (a) $18x$. (b) $12ab$. (c) $10n$. **Q14.** (a) $C=50+38h$. (b) a formula. **Q15.** (a) $10x+12$. (b) $6a$. (c) $9p+q$. **Q16.** They have different pronumeral parts (x versus x^2); simplified: $7x+5x^2$. **Q17.** $4a-3$. **Q18.** $5t+3d$.

Fully-worked solutions

Q1. (a) Drop the multiplication sign: $8\times t=8t$. (b) Write the number first, then the pronumerals in alphabetical order: $a\times b\times 5=5ab$. (c) A pronumeral multiplied by itself three times uses index 3: $w\times w\times w=w^3$.

Q2. The expression $5x+7y-3$ has terms $5x$, $7y$ and -3. (a) There are 3 terms. (b) The coefficient of x (the number in front of x) is 5. (c) The constant term (the term with no pronumeral) is -3.

Q3. (a) $6m$ and $2m$ both have pronumeral part m, so they are like terms. (b) $4p$ and $4q$ have different pronumerals (p and q), so they are not like terms. (c) $3x^2$ and $8x^2$ both have pronumeral part x^2, so they are like terms.

Q4. (a) $7a+2a=(7+2)a=9a$. (b) $10c-4c=(10-4)c=6c$. (c) $3x+5x+x=(3+5+1)x=9x$ (remember the coefficient of x alone is 1).

Q5. (a) Collect the m terms: $4m+2m=6m$; the n term is unchanged, giving $6m+3n$. (b) Collect the p terms: $8p-3p=5p$; the q term is unchanged, giving $5p+2q$. (c) Collect the x terms $5x+2x=7x$ and the numbers $6-1=5$, giving $7x+5$.

Q6. (a) $3x+7$ has no equals sign, so it is an **expression**. (b) $2x-1=9$ sets two expressions equal, so it is an **equation**. (c) $A=lw$ is a rule connecting several pronumerals, so it is a **formula**.

Q7. (a) $n\times 6=6n$ (number in front). (b) $3\times a\times a=3a^2$. (c) $y\div 8=\frac{y}{8}$. (d) $q\times p\times 2=2pq$ (number first, pronumerals alphabetical).

Q8. (a) "5 more than x" means add 5: $x+5$. (b) "the product of 7 and m" means multiply: $7m$. (c) "n divided by 4" is the fraction $\frac{n}{4}$. (d) "twice y" is $2y$, and "3 less than" it is $2y-3$.

Q9. The expression $12a-5b+9$ has terms $12a$, $-5b$ and 9. (a) There are 3 terms. (b) The coefficient of b is -5 (the sign belongs to the coefficient). (c) The constant term is 9.

Q10. (a) All three terms are like: $9k+4k-2k=(9+4-2)k=11k$. (b) Collect x terms $6x-2x=4x$ and y terms $3y+y=4y$, giving $4x+4y$. (c) Collect n terms $4n+2n=6n$ and numbers $10-3=7$, giving $6n+7$.

Q11. Total litres $=x+2x+3x$. These are like terms, so add the coefficients: $(1+2+3)x=6x$ litres.

Q12. A rectangle has two lengths and two widths, so the perimeter is $5m+5m+2m+2m$. Collecting like terms, $(5+5+2+2)m=14m$.

Q13. Multiply the coefficient of each term by the number in front, leaving the pronumeral part unchanged. (a) $6\times 3x=18x$. (b) $2\times 6ab=12ab$. (c) $5\times 2n=10n$.

Q14. (a) The cost is a fixed $\$50$ plus $\$38$ for each of the h hours: $C=50+38h$. (b) It is a rule connecting the quantities C and h, written with an equals sign, so it is a **formula**.

Q15. (a) Collect x terms $8x+2x=10x$ and numbers $3+9=12$: $10x+12$. (b) Collect a terms $7a-a=6a$ and b terms $4b-4b=0$: $6a$. (c) Collect p terms $6p+3p=9p$ and q terms $2q-q=q$: $9p+q$.

Q16. In $5x$ the pronumeral part is x, but in $5x^2$ it is x^2; because these pronumeral parts differ, the two terms are not like terms and cannot be combined. Collecting only the genuine like terms $5x+2x=7x$ and keeping $5x^2$ separate gives $7x+5x^2$.

Q17. Each crate holds a apples, so 4 crates hold $4a$ apples. With 6 loose apples the shop starts with $4a+6$. Selling 9 apples subtracts 9: $4a+6-9=4a-3$ apples remaining.

Q18. The first purchase costs $3t+2d$ and the second costs $2t+d$. Adding, $3t+2d+2t+d$; collect t terms $3t+2t=5t$ and d terms $2d+d=3d$, giving $5t+3d$ dollars.

Theory

A **formula** is a rule, written with pronumerals, that connects two or more quantities. Formulas appear everywhere in everyday life — the cost of hiring a van, the interest earned on savings, the area of a garden bed, the conversion of one temperature scale to another. Being able to **apply** a formula means two things: substituting known values to find an unknown output, and, when you already know the output, solving a simple equation to find an input.

Substituting into a formula.

To **substitute** is to replace each pronumeral with its known value and then evaluate, following the usual order of operations (brackets, then powers, then × and ÷, then + and −).

For example, a company hires out a small van for a $45 booking fee plus $30 for each hour. The cost $$C$ for h hours is

$$C = 45 + 30h.$$

For a 4-hour hire, substitute $h = 4$:

$$C = 45 + 30 \times 4 = 45 + 120 = 165,$$

so the hire costs $165. Always write the **units** in your final answer, and only round when the question asks you to.

Some formulas you will meet often:

- **Simple interest:** $I = \frac{Prt}{100}$, where P is the principal (dollars), r the rate (% per year) and t the time (years).
- **Temperature:** $F = 1.8C + 32$ converts degrees Celsius to degrees Fahrenheit.
- **Rectangle:** area $A = lw$ and perimeter $P = 2(l + w)$.
- **Distance:** $d = st$, where s is speed and t is time.

Solving a simple equation.

Sometimes you know the *result* of a formula and need to find one of the inputs. This means solving an **equation**. You undo the operations acting on the unknown, one at a time, using **inverse operations** — the reverse of what was done — while keeping both sides balanced.

- The inverse of + is −, and the inverse of − is +.
- The inverse of × is ÷, and the inverse of ÷ is ×.

A **one-step** equation needs a single inverse operation. To solve $5x = 60$, divide both sides by 5:

$$x = \frac{60}{5} = 12.$$

A **two-step** equation needs two. To solve $30h + 45 = 165$, first subtract 45 from both sides, then divide by 30:

$$30h = 120,\ h = \frac{120}{30} = 4.$$

Undo the operations in the **reverse order** to the order of operations: deal with the + or − term first, then the × or ÷.

Worked examples

Example 1 — scaffolded

A plumber charges a \$90 call-out fee plus \$75 for each hour worked. The total charge \$$C$ for a job lasting h hours is $C=90+75h$. Find the charge for a job lasting 3 hours.

Working	Comment
Write the formula: $C=90+75h$.	h is the number of hours.
Substitute $h=3$: $C=90+75\times 3$.	Replace the pronumeral with its value.
$C=90+225=315$.	Multiply before adding (order of operations).
The charge is \$315.	State the answer with the dollar sign.

Example 2 — scaffolded

The formula $F=1.8C+32$ converts a temperature of C degrees Celsius to F degrees Fahrenheit. Convert 20°C to degrees Fahrenheit.

Working	Comment
Write the formula: $F=1.8C+32$.	C is the Celsius temperature.
Substitute $C=20$: $F=1.8\times 20+32$.	Replace C with 20.
$F=36+32=68$.	Multiply first, then add.
So 20°C $=68^{\circ}$F.	A warm spring day.

Example 3 — unscaffolded

Using the same plumber's formula $C=90+75h$, a customer receives a bill for \$352.50. How many hours did the job take?

Here the output is known ($C=352.50$) and the unknown is h, so solve the equation

$$90+75h=352.50.$$

Subtract the \$90 call-out fee from both sides:

$$75h=352.50-90=262.50.$$

Divide both sides by 75:

$$h=\frac{262.50}{75}=3.5.$$

$\therefore$ the job took 3.5 hours.

Example 4 — unscaffolded

A rectangular vegetable bed has perimeter 26 m and width 4 m. Using $P=2(l+w)$, find its length.

Substitute the known values $P=26$ and $w=4$ into the formula:

$$26=2(l+4).$$

Divide both sides by 2:

$$13=l+4.$$

Subtract 4 from both sides:

$$l=13-4=9.$$

∴ the vegetable bed is 9 m long.

Practice questions

Scaffolded

Q1. A caterer charges using the formula $C=60+25h$, where \$$C$ is the total cost and h is the number of hours. Find the cost when (a) $h=2$, (b) $h=5$, (c) $h=8$.

Q2. Use $F=1.8C+32$ to convert each Celsius temperature to degrees Fahrenheit. (a) 10°C. (b) 30°C. (c) 0°C.

Q3. Use the simple interest formula $I=\dfrac{Prt}{100}$ to find the interest I when (a) $P=1500$, $r=5$, $t=2$, (b) $P=800$, $r=6$, $t=3$, (c) $P=5000$, $r=3$, $t=1$.

Q4. Find the unknown in each situation by solving a one-step equation. (a) After a \$7 deposit, a money box holds \$20. If it held \$$b$ before, then $b+7=20$. Find b. (b) Five friends share a prize of \$80 equally. If each receives \$$p$, then $5p=80$. Find p. (c) A length of ribbon is cut into 4 equal pieces, each 9 cm long. If the ribbon was n cm long, then $\dfrac{n}{4}=9$. Find n.

Q5. A plumber's charge is $C=90+75h$. For each bill, solve for the number of hours h. (a) \$540. (b) \$465. (c) \$240.

Q6. A rectangular courtyard is 8 m long and 5 m wide. (a) Use $A=lw$ to find its area. (b) Use $P=2(l+w)$ to find its perimeter. (c) A second courtyard has the same area of 40 m² but a width of 4 m. Use $A=lw$ to find its length.

Unscaffolded

Q7. A taxi fare is given by $F=3.60+1.75d$, where d is the distance in kilometres. Find the fare for a 14 km trip.

Q8. The formula $C=\dfrac{F-32}{1.8}$ converts degrees Fahrenheit to degrees Celsius. Convert 95°F to degrees Celsius.

Q9. Using $I=\dfrac{Prt}{100}$, a deposit of \$1200 earns \$180 of simple interest over 3 years. Find the annual interest rate r.

Q10. A phone plan costs $C=20+0.15m$ dollars, where m is the number of extra minutes used. Find the cost in a month with 40 extra minutes.

Q11. The area of a triangle is $A=\dfrac{1}{2}bh$. Find the area of a triangular sail with base $b=12$ m and height $h=5$ m.

Q12. A tool-hire shop charges $C=50+15h$ dollars for h hours. A customer is charged \$155. Find the number of hours.

Q13. The distance travelled is $d=st$. A car travels at a steady 90 km/h for 2.5 hours. Find the distance travelled.

Q14. A casual worker's pay is $W=25h+40$ dollars, where h is the hours worked and \$40 is a fixed travel allowance. In one shift the worker is paid \$340. Find the number of hours worked.

Q15. The area of a circle is $A=\pi r^2$. Find the area of a circular garden of radius 6 m, correct to two decimal places.

Q16. At a market stall, the profit formula is $3n-5=P$, where n is the number of baskets sold and \$5 covers the stall fee. On a day the profit was \$16. Find the number of baskets sold.

Q17. A printer charges $C=8n+12$ dollars to print n posters, where \$12 is a set-up fee. A school pays \$100. Find the number of posters printed.

Q18. Body temperature is about 37°C. Use $F=1.8C+32$ to express this in degrees Fahrenheit.

Answers

Q1. (a) \$110. (b) \$185. (c) \$260. **Q2.** (a) 50°F. (b) 86°F. (c) 32°F. **Q3.** (a) \$150. (b) \$144. (c) \$150. **Q4.** (a) $b = 13$. (b) $p = 16$. (c) $n = 36$. **Q5.** (a) 6 h. (b) 5 h. (c) 2 h. **Q6.** (a) 40 m². (b) 26 m. (c) 10 m. **Q7.** \$28.10. **Q8.** 35°C. **Q9.** 5% per year. **Q10.** \$26. **Q11.** 30 m². **Q12.** 7 h. **Q13.** 225 km. **Q14.** 12 h. **Q15.** 113.10 m². **Q16.** 7 baskets. **Q17.** 11 posters. **Q18.** 98.6°F.

Fully-worked solutions

Q1. $C = 60 + 25h$. (a) $C = 60 + 25 \times 2 = 60 + 50 = 110$, so \$110. (b) $C = 60 + 25 \times 5 = 60 + 125 = 185$, so \$185. (c) $C = 60 + 25 \times 8 = 60 + 200 = 260$, so \$260.

Q2. $F = 1.8C + 32$. (a) $F = 1.8 \times 10 + 32 = 18 + 32 = 50$, so 50°F. (b) $F = 1.8 \times 30 + 32 = 54 + 32 = 86$, so 86°F. (c) $F = 1.8 \times 0 + 32 = 0 + 32 = 32$, so 32°F.

Q3. $I = \dfrac{Prt}{100}$. (a) $I = \dfrac{1500 \times 5 \times 2}{100} = \dfrac{15\,000}{100} = 150$, so \$150. (b) $I = \dfrac{800 \times 6 \times 3}{100} = \dfrac{14\,400}{100} = 144$, so \$144. (c) $I = \dfrac{5000 \times 3 \times 1}{100} = \dfrac{15\,000}{100} = 150$, so \$150.

Q4. (a) $b + 7 = 20$; subtract 7: $b = 20 - 7 = 13$. (b) $5p = 80$; divide by 5: $p = \dfrac{80}{5} = 16$. (c) $\dfrac{n}{4} = 9$; multiply by 4: $n = 9 \times 4 = 36$.

Q5. $90 + 75h = C$. (a) $90 + 75h = 540 \Rightarrow 75h = 450 \Rightarrow h = \dfrac{450}{75} = 6$ h. (b) $90 + 75h = 465 \Rightarrow 75h = 375 \Rightarrow h = \dfrac{375}{75} = 5$ h. (c) $90 + 75h = 240 \Rightarrow 75h = 150 \Rightarrow h = \dfrac{150}{75} = 2$ h.

Q6. (a) $A = lw = 8 \times 5 = 40$ m². (b) $P = 2(l + w) = 2(8 + 5) = 2 \times 13 = 26$ m. (c) $A = lw \Rightarrow 40 = l \times 4 \Rightarrow l = \dfrac{40}{4} = 10$ m.

Q7. $F = 3.60 + 1.75 \times 14 = 3.60 + 24.50 = 28.10$, so the fare is \$28.10.

Q8. $C = \dfrac{F - 32}{1.8} = \dfrac{95 - 32}{1.8} = \dfrac{63}{1.8} = 35$, so 35°C.

Q9. $I = \dfrac{Prt}{100} \Rightarrow 180 = \dfrac{1200 \times r \times 3}{100} = 36r$. Divide by 36: $r = \dfrac{180}{36} = 5$, so the rate is 5% per year.

Q10. $C = 20 + 0.15 \times 40 = 20 + 6 = 26$, so \$26.

Q11. $A = \dfrac{1}{2}bh = \dfrac{1}{2} \times 12 \times 5 = \dfrac{1}{2} \times 60 = 30$ m².

Q12. $50 + 15h = 155 \Rightarrow 15h = 105 \Rightarrow h = \dfrac{105}{15} = 7$ h.

Q13. $d = st = 90 \times 2.5 = 225$ km.

Q14. $25h + 40 = 340 \Rightarrow 25h = 300 \Rightarrow h = \dfrac{300}{25} = 12$ h.

Q15. $A = \pi r^2 = \pi \times 6^2 = 36\pi \approx 113.10$ m² (to 2 d.p.).

Q16. $3n - 5 = 16 \Rightarrow 3n = 21 \Rightarrow n = \dfrac{21}{3} = 7$ baskets.

Q17. $8n + 12 = 100 \Rightarrow 8n = 88 \Rightarrow n = \dfrac{88}{8} = 11$ posters.

Q18. $F = 1.8 \times 37 + 32 = 66.6 + 32 = 98.6$, so 98.6°F.

Theory

A **spreadsheet** is a grid of boxes called **cells**. The columns are labelled with letters ($A, B, C, \ldots$) and the rows with numbers ($1, 2, 3, \ldots$), so every cell has an address called its **cell reference** — the cell in column B, row 2 is cell **B2**. A spreadsheet is really just a formula written in a new language: instead of $A = l \times w$ you type a formula that tells the computer which cells to combine.

Writing a formula.

Every formula begins with an **equals sign** =. After the = you build the calculation out of **cell references** and the operators

- + add, `-` subtract, `*` multiply, `/` divide, `^` raise to a power.

The key idea is that a formula refers to the **cells**, not to the numbers inside them. If cell B2 holds 6 and cell C2 holds 0.80, then typing `=B2*C2` into cell D2 makes D2 display $6 \times 0.80 = 4.80$. Change the number in B2 and D2 updates automatically.

Order of operations.

A spreadsheet follows the usual order of operations (brackets, then `^`, then `*` and `/`, then + and `-`). Use round brackets to force additions first. For a rectangle with the length in A2 and the width in B2, the perimeter $P = 2(l + w)$ is typed as `=2*(A2+B2)` — the brackets make the spreadsheet add before it multiplies.

Functions: SUM and AVERAGE.

Adding a long list of cells one at a time is slow, so spreadsheets provide **functions**. A **range** of cells is written with a colon: `B2:B6` means every cell from B2 down to B6.

- `=SUM(B2:B6)` adds all the values from B2 to B6.
- `=AVERAGE(B2:B6)` finds their mean (the sum divided by how many there are).

Filling down.

When the same calculation is needed all the way down a column, you type the formula once and **fill down**. The cell references adjust automatically for each row: `=B2*C2` in D2 becomes `=B3*C3` in D3, `=B4*C4` in D4, and so on. This is why a wage sheet only needs one formula, copied down for every employee.

Checking by hand.

A spreadsheet only does what you tell it — a wrong formula gives a wrong answer with total confidence. Always predict, or check by hand, the value a cell *should* display. Every formula in this section can be checked on a scientific calculator.

Worked examples

Example 1 — scaffolded

A fruit stall uses the spreadsheet below to work out the value of each line. The **Value** column multiplies the number of boxes by the price per box.

	A	B	C	D
1	Item	Boxes	Price per box (dollars)	Value (dollars)
2	Oranges	7	12	

	A	B	C	D
3	Mangoes	4	15	

Write the formula that belongs in cell D2, and state the value it displays.

Working	Comment
The value is boxes × price per box, which are in cells B2 and C2.	Refer to the cells, not the numbers.
Formula for D2: `=B2*C2`.	Every formula starts with =; `*` means multiply.
D2 displays $7\times12=84$.	So the oranges are worth $84.

Example 2 — scaffolded

A student's five topic-test marks (out of 20) are entered in cells B2 to B6 as $15, 18, 12, 20, 17$. Write a formula for the **total** mark in cell B7 and a formula for the **average** mark in cell B8, and state each value.

Working	Comment
The marks fill the range B2 to B6, written `B2:B6`.	A colon gives a range of cells.
Total in B7: `=SUM(B2:B6)`.	`SUM` adds every cell in the range.
B7 displays $15+18+12+20+17=82$.	Check the addition by hand.
Average in B8: `=AVERAGE(B2:B6)`.	`AVERAGE` finds the mean.
B8 displays $\frac{82}{5}=16.4$.	Sum divided by the five marks.

Example 3 — unscaffolded

A tiler charges a **call-out fee** stored in cell A2 plus an **hourly rate** in cell B2 for each hour worked, with the number of hours in cell C2. Write a spreadsheet formula for the total charge in cell D2, and find the value it displays when $A2=60$, $B2=45$ and $C2=3$.

The total charge is the call-out fee plus the hourly rate times the hours: $60+45\times3$. In spreadsheet language this is

```
=A2+B2*C2
```

The spreadsheet multiplies before it adds, which is exactly what we want. With the given values it displays

$$60+45\times3=60+135=195.$$

∴ the formula is `=A2+B2*C2`, and cell D2 displays $195.

Example 4 — unscaffolded

A café pays each casual worker $\text{hours}\times\text{rate}$. The spreadsheet below has the formula `=B2*C2` typed in cell D2 and then **filled down** to D3 and D4. Write the formula that appears in D3 and in D4, find the three daily pays, and write a formula for the week's total pay in cell D5.

	A	B	C	D
1	Day	Hours	Rate (dollars)	Pay (dollars)
2	Friday	6	25	`=B2*C2`
3	Saturday	8	25	
4	Sunday	5	25	

Filling `=B2*C2` down makes the row numbers increase by one each time, so D3 holds `=B3*C3` and D4 holds `=B4*C4`. The three pays are

$$D2=6\times25=150, D3=8\times25=200, D4=5\times25=125.$$

The total pay adds the three cells: `=SUM(D2:D4)`, which displays $150+200+125=475$.

∴ D3 is `=B3*C3` and D4 is `=B4*C4`; the pays are \$150, \$200 and \$125, and the weekly total in D5 is `=SUM(D2:D4)` $=\$475$.

Practice questions

Scaffolded

Q1. A market gardener uses this spreadsheet, where **Value** is boxes × price per box.

	A	B	C	D
1	Item	Boxes	Price per box (dollars)	Value (dollars)
2	Pumpkins	9	8	
3	Melons	6	11	

(a) Which cell contains the number 11?
(b) Write the formula that belongs in cell D2.
(c) State the value that D2 displays.

Q2. A netball team's scores for five games are entered in cells B2 to B6 as $34, 41, 28, 45, 37$. (a) Write a formula in cell B7 for the total number of points. (b) State the value B7 displays. (c) Write a formula in cell B8 for the average score, and state its value.

Q3. A rectangle has its length in cell A2 and its width in cell B2. (a) Write a spreadsheet formula for the **area** $l\times w$. (b) Write a spreadsheet formula for the **perimeter** $2(l+w)$. (c) If $A2=12$ and $B2=5$, state the value each formula displays.

Q4. Cells hold $A2=10$, $B2=4$ and $C2=2$. Predict the value each formula displays. (a) `=A2+B2*C2` (b) `=(A2+B2)*C2` (c) `=A2-B2/C2`

Q5. A worker's pay each day is $\text{hours}\times\text{rate}$, using `=B2*C2` in D2.

	A	B	C	D
1	Day	Hours	Rate (dollars)	Pay (dollars)
2	Mon	7	22	=B2*C2
3	Tue	4	22	
4	Wed	9	22	

(a) When D2 is filled down to D3, what formula appears in D3?
(b) State the values displayed in D2, D3 and D4.
(c) Write a formula in cell D5 for the total pay, and state its value.

Q6. A mark book records three test marks per student. Row 2 is shown.

	A	B	C	D	E
1	Student	Test 1	Test 2	Test 3	Total
2	Aisha	16	14	18	=SUM(B2:D2)

(a) State the value cell E2 displays.
(b) Write a formula in cell F2 for Aisha's **average** test mark.
(c) State the value F2 displays.

Unscaffolded

Q7. A casual worker's pay is $\text{hours} \times \text{rate}$, with the hours in cell B2 ($=9$) and the rate in cell C2 ($=28$). Write the formula for the pay in cell D2 and state the value it displays.

Q8. A stall's daily takings for a week are entered in cells B2 to B6 as $45, 60, 38, 52, 70$ (dollars). (a) Write a formula in cell B7 for the total takings, and state its value. (b) Write a formula in cell B8 for the average daily takings, and state its value.

Q9. A taxi fare is a flagfall in cell A2 plus a charge per kilometre in cell B2 multiplied by the distance in cell C2. Write a spreadsheet formula for the total fare in cell D2, and find its value when $A2 = 3.50$, $B2 = 2.40$ and $C2 = 9$.

Q10. Cells hold $A2 = 20$ and $B2 = 5$. Predict the value each formula displays. (a) `=A2/B2` (b) `=A2-B2` (c) `=A2*B2-10`

Q11. A shopkeeper adds GST at 10% using `=B2*0.1` in cell C2, filled down.

	A	B	C
1	Item	Price (dollars)	GST (dollars)
2	Kettle	50	`=B2*0.1`
3	Toaster	120	
4	Charger	8	

(a) What formula appears in cell C3 after filling down?
(b) State the GST values displayed in C2, C3 and C4.

Q12. A weekly pay is worked out with `=B2*C2` in cell D2, where $B2 = 40$ (hours) and $C2 = 30$ (rate). (a) State the value D2 displays. (b) Cell E2 contains `=D2*0.115` to work out superannuation at 11.5%. State the value E2 displays.

Q13. An office order sheet uses `=B2*C2` (quantity × price) filled down the **Cost** column.

	A	B	C	D
1	Item	Quantity	Price (dollars)	Cost (dollars)
2	Pens	5	2	10
3	Notebooks	3	6	18
4	Folders	4	3	15

(a) One cost has been typed in by hand instead of by formula and is wrong. State which row.
(b) Give the correct value.
(c) Write a formula in cell D5 for the total cost, and state its correct value.

Q14. A worker is paid ordinary hours (cell A2) at a rate (cell B2), plus overtime hours (cell C2) at 1.5 times the rate. A spreadsheet formula for the total pay is `=A2*B2+C2*B2*1.5`. Find the value it displays when $A2 = 38$, $B2 = 20$ and $C2 = 4$.

Q15. A mark book finds each student's average of three tests with `=AVERAGE(B2:D2)`, filled down.

	A	B	C	D	E
1	Student	Test 1	Test 2	Test 3	Average
2	Ben	12	15	18	`=AVERAGE(B2:D2)`
3	Chloe	20	16	15	

(a) What formula appears in cell E3 after filling down?
(b) State the averages displayed in E2 and E3.

Q16. Cell A2 holds the side length of a square, $A2=7$. (a) Write a formula in cell B2 for the area of the square, using the ^ operator, and state its value. (b) Write a formula in cell C2 for the volume of a cube with that side length, and state its value.

Q17. A monthly budget has income in cell B2 ($=1500$) and four expenses in cells B3 to B6 as $600, 250, 180, 120$ (dollars). (a) Write a formula in cell B7 for the total expenses, and state its value. (b) Write a formula in cell B8 for the savings (income minus total expenses), and state its value.

Q18. A class's test scores are entered in cells B2 to B5 as $60, 70, 80, 90$. (a) Write a formula for the average in cell B6, and state its value. (b) A fifth student who scored 100 is added in cell B6, moving the average formula to cell B7 as `=AVERAGE(B2:B6)`. State the new average. (c) Explain, without re-adding all the numbers, why the average increased.

Answers

Q1. (a) C3. (b) `=B2*C2`. (c) 72. **Q2.** (a) `=SUM(B2:B6)`. (b) 185. (c) `=AVERAGE(B2:B6)`, 37. **Q3.** (a) `=A2*B2`. (b) `=2*(A2+B2)`. (c) area 60, perimeter 34. **Q4.** (a) 18. (b) 28. (c) 8. **Q5.** (a) `=B3*C3`. (b) 154, 88, 198. (c) `=SUM(D2:D4)`, 440. **Q6.** (a) 48. (b) `=AVERAGE(B2:D2)`. (c) 16. **Q7.** `=B2*C2`, value 252. **Q8.** (a) `=SUM(B2:B6)`, 265. (b) `=AVERAGE(B2:B6)`, 53. **Q9.** `=A2+B2*C2`, value 25.10 (dollars). **Q10.** (a) 4. (b) 15. (c) 90. **Q11.** (a) `=B3*0.1`. (b) 5, 12, 0.8. **Q12.** (a) 1200. (b) 138. **Q13.** (a) row 4 (Folders). (b) 12. (c) `=SUM(D2:D4)`, 40. **Q14.** 880. **Q15.** (a) `=AVERAGE(B3:D3)`. (b) 15 and 17. **Q16.** (a) `=A2^2`, 49. (b) `=A2^3`, 343. **Q17.** (a) `=SUM(B3:B6)`, 1150. (b) `=B2-B7`, 350. **Q18.** (a) `=AVERAGE(B2:B5)`, 75. (b) 80. (c) see solution.

Fully-worked solutions

Q1. Column C holds the price per box; row 3 is the Melons row, so the number 11 is in cell **C3**. The Value column is boxes × price, i.e. B2 × C2, so D2 is `=B2*C2`, which displays $9\times 8=72$.

Q2. The five scores fill B2 to B6. (a) Total: `=SUM(B2:B6)`. (b) $34+41+28+45+37=185$. (c) Average: `=AVERAGE(B2:B6)`, which displays $\frac{185}{5}=37$.

Q3. With length in A2 and width in B2: (a) area $l\times w$ is `=A2*B2`. (b) perimeter $2(l+w)$ is `=2*(A2+B2)` — the brackets force the addition first. (c) With $A2=12$, $B2=5$: area $=12\times 5=60$; perimeter $=2\times(12+5)=2\times 17=34$.

Q4. Using $A2=10$, $B2=4$, $C2=2$ and the order of operations: (a) `=A2+B2*C2`: multiply first, $4\times 2=8$, then $10+8=18$. (b) `=(A2+B2)*C2`: brackets first, $10+4=14$, then $14\times 2=28$. (c) `=A2-B2/C2`: divide first, $4\div 2=2$, then $10-2=8$.

Q5. The pay is hours × rate. (a) Filling `=B2*C2` down one row increases each row number by one, so D3 holds `=B3*C3`. (b) $D2=7\times 22=154$, $D3=4\times 22=88$, $D4=9\times 22=198$. (c) The total adds the three pays: `=SUM(D2:D4)` $=154+88+198=440$.

Q6. (a) `=SUM(B2:D2)` adds Aisha's three marks: $16+14+18=48$. (b) The average of the three tests is `=AVERAGE(B2:D2)`. (c) $\frac{48}{3}=16$.

Q7. Pay is hours × rate, i.e. B2 × C2, so the formula is `=B2*C2`. It displays $9\times 28=252$.

Q8. (a) `=SUM(B2:B6)` $=45+60+38+52+70=265$. (b) `=AVERAGE(B2:B6)` $=\frac{265}{5}=53$.

Q9. The fare is the flagfall plus the per-kilometre charge times the distance: `=A2+B2*C2`. The spreadsheet multiplies first: $2.40\times 9=21.60$, then adds the flagfall $3.50+21.60=25.10$. So D2 displays 25.10, i.e. $25.10.

Q10. With $A2=20$, $B2=5$: (a) `=A2/B2` $=20\div 5=4$. (b) `=A2-B2` $=20-5=15$. (c) `=A2*B2-10`: multiply first, $20\times 5=100$, then $100-10=90$.

Q11. GST is 10% of the price, i.e. price $\times 0.1$. (a) Filling `=B2*0.1` down one row gives `=B3*0.1` in C3. (b) $C2=50\times 0.1=5$; $C3=120\times 0.1=12$; $C4=8\times 0.1=0.8$.

Q12. (a) `=B2*C2` $=40\times 30=1200$. (b) `=D2*0.115` $=1200\times 0.115=138$, so the superannuation is $138.

Q13. The Cost column should be quantity × price. (a) Row 4 (Folders): $4\times 3=12$, but the sheet shows 15, so row 4 is wrong. (b) The correct cost is 12. (c) The total is `=SUM(D2:D4)`. Using the correct values, $10+18+12=40$.

Q14. `=A2*B2+C2*B2*1.5` with $A2=38$, $B2=20$, $C2=4$: the ordinary pay is $38\times 20=760$; the overtime is $4\times 20\times 1.5=120$; the total is $760+120=880$. So the cell displays 880, i.e. $880.

Q15. (a) Filling `=AVERAGE(B2:D2)` down one row gives `=AVERAGE(B3:D3)` in E3. (b) $E2=\frac{12+15+18}{3}=\frac{45}{3}=15$; $E3=\frac{20+16+15}{3}=\frac{51}{3}=17$.

Q16. (a) The area of a square is (side)², so `=A2^2` displays $7^2=49$. (b) The volume of a cube is (side)³, so `=A2^3` displays $7^3=343$.

Q17. (a) The four expenses fill B3 to B6, so the total is `=SUM(B3:B6)` $=600+250+180+120=1150$. (b) Savings are income minus total expenses: `=B2-B7` $=1500-1150=350$.

Q18. (a) The four scores fill B2 to B5, so `=AVERAGE(B2:B5)` $=\frac{60+70+80+90}{4}=\frac{300}{4}=75$. (b) Adding the fifth score 100 in B6 changes the formula to `=AVERAGE(B2:B6)` $=\frac{300+100}{5}=\frac{400}{5}=80$. (c) The new score of 100 is above the old average of 75, and any value added above the mean pulls the mean up — so the average increased from 75 to 80.

Theory

Algebra lets us write a rule once and use it for every set of numbers. To use a rule flexibly we often need to **manipulate** it — expand brackets, collect like terms, or **rearrange** it so that a different letter stands alone. This section covers those three skills.

Expanding a single bracket.

To **expand** a bracket, multiply the term outside by **every** term inside. This is the **distributive law**:

$$a(b+c)=ab+ac.$$

Watch the signs: a negative outside the bracket changes the sign of every term inside. For example,

$$-2(3x-5)=-6x+10.$$

Collecting like terms.

Like terms contain exactly the same pronumeral. Only like terms can be added or subtracted: $5x+3x=8x$, but $5x+3$ cannot be simplified further. After expanding, always collect the like terms:

$$4(x+2)+3x=4x+8+3x=7x+8.$$

Changing the subject of a formula.

The **subject** of a formula is the letter on its own, usually on the left. To make a different letter the subject we **transpose** (rearrange) the formula, using the rule:

- whatever operation you do to one side, you must do to the **other** side as well;
- **undo** the operations acting on the wanted letter in **reverse** order, using the **inverse** operation each time (the inverse of + is −, the inverse of × is ÷, and the inverse of squaring is taking the square root).

For example, to make r the subject of $C=2\pi r$, the letter r has been multiplied by 2π, so we divide both sides by 2π :

$$C=2\pi r \Rightarrow r=\frac{C}{2\pi}.$$

A two-step example — making w the subject of $P=2(l+w)$ — needs two inverse operations: divide by 2, then subtract l:

$$P=2(l+w) \Rightarrow \frac{P}{2}=l+w \Rightarrow w=\frac{P}{2}-l.$$

Substituting into a rearranged formula.

Once a formula is rearranged so the wanted letter is the subject, we can **substitute** the known values and evaluate on a scientific calculator. Rearranging first is usually quicker and less error-prone than substituting into the original formula and solving.

Worked examples

Example 1 — scaffolded

Expand and simplify $6(x+4)+3(2x-5)$.

Working	Comment
Expand the first bracket: $6(x+4)=6x+24$.	Multiply 6 by each term inside.
Expand the second bracket: $3(2x-5)=6x-15$.	$3\times 2x=6x$ and $3\times(-5)=-15$.
Add and collect like terms: $6x+24+6x-15=12x+9$.	Combine the x terms ($6x+6x$) and the numbers ($24-15$).

Example 2 — scaffolded

The circumference of a circle is $C=2\pi r$. Make r the subject, then find the radius of a circle whose circumference is 44 cm, correct to one decimal place.

Working	Comment
The letter r is multiplied by 2π, so divide both sides by 2π: $r=\frac{C}{2\pi}$.	Use the inverse of "$\times 2\pi$".
Substitute $C=44$: $r=\frac{44}{2\pi}=\frac{44}{6.2831\ldots}$.	Use the calculator value of 2π.
$r=7.00\ldots\approx 7.0$ cm.	Round to one decimal place.

Example 3 — unscaffolded

The perimeter of a rectangle is $P=2(l+w)$. Make w the subject, then find the width of a rectangle with perimeter 30 m and length 9 m.

Rearrange for w, one inverse operation per line:

$$P=2(l+w)$$

$$\frac{P}{2}=l+w$$

$$w=\frac{P}{2}-l.$$

Substitute $P=30$ and $l=9$:

$$w=\frac{30}{2}-9=15-9=6\,\text{m}.$$

$\therefore w=\frac{P}{2}-l$, and the width of the rectangle is 6 m.

Example 4 — unscaffolded

The simple interest earned on an investment is $I=\frac{Prt}{100}$, where P is the principal, r the annual interest rate (as a percentage) and t the time in years. Make t the subject, then find how long it takes a principal of $\$5000$ at 4% per annum to earn $\$800$ in simple interest.

Rearrange for t:

$$I=\frac{Prt}{100}$$

$$100I=Prt$$

$$t=\frac{100I}{Pr}.$$

Substitute $I=800$, $P=5000$ and $r=4$:

$$t=\frac{100\times800}{5000\times4}=\frac{80\,000}{20\,000}=4\text{ years}.$$

$\therefore t=\dfrac{100\,I}{P\,r}$, and the investment takes 4 years.

Practice questions

Scaffolded

Q1. Expand each bracket. (a) $3(x+4)$. (b) $6(2a-5)$. (c) $-2(3y+7)$.

Q2. Expand and simplify. (a) $4(x+2)+3x$. (b) $2(3a+1)+5(a-4)$. (c) $5(2m-3)-4m$.

Q3. Make x the subject of each formula. (a) $y=x+7$. (b) $y=5x$. (c) $y=\dfrac{x}{3}$.

Q4. Make the stated letter the subject. (a) $C=2\pi r$; make r the subject. (b) $A=lw$; make w the subject. (c) $V=Ah$; make h the subject.

Q5. Make x the subject of each formula. (a) $y=2x+3$. (b) $y=\dfrac{x-4}{5}$. (c) $P=2(l+x)$.

Q6. The simple interest formula is $I=\dfrac{Prt}{100}$. (a) Make t the subject. (b) A loan of $\$4000$ at 6% per annum earns $\$720$ in simple interest. Find the time t. (c) Check your answer by substituting back into the original formula.

Unscaffolded

Q7. Expand and simplify $3(2x+5)+2(4x-1)$.

Q8. Expand and simplify $7(a-3)-2(a-5)$.

Q9. The perimeter of a rectangle is $P=2(l+w)$. (a) Make w the subject. (b) A rectangular garden bed has perimeter 26 m and length 8 m. Find its width.

Q10. The circumference of a circle is $C=2\pi r$. (a) Make r the subject. (b) A circular pond has circumference 15 m. Find its radius, correct to one decimal place.

Q11. The simple interest formula is $I=\dfrac{Prt}{100}$. (a) Make r the subject. (b) An investment of $\$2500$ earns $\$300$ in simple interest over 4 years. Find the annual interest rate r.

Q12. For travel at a constant speed, distance is $d=st$ (d in km, s in km/h, t in hours). (a) Make t the subject. (b) A train travels 240 km at 80 km/h. Find the time taken.

Q13. To convert a temperature from degrees Celsius to degrees Fahrenheit, $F=1.8C+32$. (a) Make C the subject. (b) Find C when $F=98.6$, correct to one decimal place.

Q14. The area of a triangle is $A=\dfrac{1}{2}bh$. (a) Make h the subject. (b) A triangle has area 24 cm² and base 6 cm. Find its height.

Q15. Expand and simplify $4(2x+3)-(x-5)$.

Q16. A phone plan's monthly cost is $C=15+0.10n$ dollars, where n is the number of extra call-minutes used. (a) Make n the subject. (b) One month the bill was $\$23$. Find the number of extra call-minutes used.

Q17. The area of a circle is $A=\pi r^2$. (a) Make r the subject. (b) A circular tabletop has area 1.2 m². Find its radius, correct to two decimal places.

Q18. The volume of a cylinder is $V=\pi r^2 h$. (a) Make h the subject. (b) A cylindrical tank has volume 2.0 m³ and radius 0.5 m. Find its height, correct to two decimal places.

Answers

Q1. (a) $3x+12$. (b) $12a-30$. (c) $-6y-14$. **Q2.** (a) $7x+8$. (b) $11a-18$. (c) $6m-15$. **Q3.** (a) $x=y-7$. (b) $x=\frac{y}{5}$. (c) $x=3y$. **Q4.** (a) $r=\frac{C}{2\pi}$. (b) $w=\frac{A}{l}$. (c) $h=\frac{V}{A}$. **Q5.** (a) $x=\frac{y-3}{2}$. (b) $x=5y+4$. (c) $x=\frac{P}{2}-l$. **Q6.** (a) $t=\frac{100I}{Pr}$. (b) $t=3$ years. (c) $\frac{4000\times6\times3}{100}=\$720$ ✓. **Q7.** $14x+13$. **Q8.** $5a-11$. **Q9.** (a) $w=\frac{P}{2}-l$. (b) 5 m. **Q10.** (a) $r=\frac{C}{2\pi}$. (b) 2.4 m. **Q11.** (a) $r=\frac{100I}{Pt}$. (b) 3% per annum. **Q12.** (a) $t=\frac{d}{s}$. (b) 3 hours. **Q13.** (a) $C=\frac{F-32}{1.8}$. (b) 37.0 °C. **Q14.** (a) $h=\frac{2A}{b}$. (b) 8 cm. **Q15.** $7x+17$. **Q16.** (a) $n=\frac{C-15}{0.10}$. (b) 80 minutes. **Q17.** (a) $r=\sqrt{\frac{A}{\pi}}$. (b) 0.62 m. **Q18.** (a) $h=\frac{V}{\pi r^2}$. (b) 2.55 m.

Fully-worked solutions

Q1. Multiply the outside term by each term inside. (a) $3(x+4)=3\times x+3\times4=3x+12$. (b) $6(2a-5)=6\times2a-6\times5=12a-30$. (c) $-2(3y+7)=-2\times3y+(-2)\times7=-6y-14$.

Q2. Expand, then collect like terms. (a) $4(x+2)+3x=4x+8+3x=7x+8$. (b) $2(3a+1)+5(a-4)=6a+2+5a-20=11a-18$. (c) $5(2m-3)-4m=10m-15-4m=6m-15$.

Q3. Undo the operation on x. (a) $y=x+7$: subtract 7 from both sides, $x=y-7$. (b) $y=5x$: divide both sides by 5, $x=\frac{y}{5}$. (c) $y=\frac{x}{3}$: multiply both sides by 3, $x=3y$.

Q4. (a) $C=2\pi r$: divide both sides by 2π, $r=\frac{C}{2\pi}$. (b) $A=lw$: divide both sides by l, $w=\frac{A}{l}$. (c) $V=Ah$: divide both sides by A, $h=\frac{V}{A}$.

Q5. Undo the operations in reverse order. (a) $y=2x+3$: subtract 3, then divide by 2: $x=\frac{y-3}{2}$. (b) $y=\frac{x-4}{5}$: multiply by 5, then add 4: $x=5y+4$. (c) $P=2(l+x)$: divide by 2, then subtract l: $x=\frac{P}{2}-l$.

Q6. (a) $I=\frac{Prt}{100}$: multiply both sides by 100 to get $100I=Prt$, then divide both sides by Pr: $t=\frac{100I}{Pr}$. (b) Substitute $I=720$, $P=4000$, $r=6$: $t=\frac{100\times720}{4000\times6}=\frac{72\,000}{24\,000}=3$ years. (c) Substituting back: $\frac{Prt}{100}=\frac{4000\times6\times3}{100}=\frac{72\,000}{100}=\$720$, which matches the given interest. ✓

Q7. $3(2x+5)+2(4x-1)=6x+15+8x-2=14x+13$.

Q8. $7(a-3)-2(a-5)=7a-21-2a+10=5a-11$. (Note $-2\times(-5)=+10$.)

Q9. (a) $P=2(l+w)$: divide by 2, then subtract l: $w=\frac{P}{2}-l$. (b) Substitute $P=26$, $l=8$: $w=\frac{26}{2}-8=13-8=5$ m.

Q10. (a) $C=2\pi r$: divide both sides by 2π: $r=\frac{C}{2\pi}$. (b) Substitute $C=15$: $r=\frac{15}{2\pi}=2.387\ldots\approx2.4$ m.

Q11. (a) $I=\frac{P\,r\,t}{100}$: multiply by 100, then divide by $P\,t$: $r=\frac{100\,I}{P\,t}$. (b) Substitute $I=300$, $P=2500$, $t=4$: $r=\frac{100\times 300}{2500\times 4}=\frac{30\,000}{10\,000}=3$, i.e. $3\,\%$ per annum.

Q12. (a) $d=s\,t$: divide both sides by s: $t=\frac{d}{s}$. (b) Substitute $d=240$, $s=80$: $t=\frac{240}{80}=3$ hours.

Q13. (a) $F=1.8\,C+32$: subtract 32, then divide by 1.8: $C=\frac{F-32}{1.8}$. (b) Substitute $F=98.6$: $C=\frac{98.6-32}{1.8}=\frac{66.6}{1.8}=37.0$ °C.

Q14. (a) $A=\frac{1}{2}\,b\,h$: multiply both sides by 2 to get $2\,A=b\,h$, then divide by b: $h=\frac{2\,A}{b}$. (b) Substitute $A=24$, $b=6$: $h=\frac{2\times 24}{6}=\frac{48}{6}=8$ cm.

Q15. $4\left(2\,x+3\right)-\left(x-5\right)=8\,x+12-x+5=7\,x+17$. (The $-$ in front of $\left(x-5\right)$ changes both signs: $-x+5$.)

Q16. (a) $C=15+0.10\,n$: subtract 15, then divide by 0.10: $n=\frac{C-15}{0.10}$. (b) Substitute $C=23$: $n=\frac{23-15}{0.10}=\frac{8}{0.10}=80$ minutes.

Q17. (a) $A=\pi\,r^2$: divide both sides by π to get $r^2=\frac{A}{\pi}$, then take the square root: $r=\sqrt{\frac{A}{\pi}}$. (b) Substitute $A=1.2$: $r=\sqrt{\frac{1.2}{\pi}}=\sqrt{0.3819\ldots}=0.618\ldots\approx 0.62$ m.

Q18. (a) $V=\pi\,r^2\,h$: divide both sides by $\pi\,r^2$: $h=\frac{V}{\pi\,r^2}$. (b) Substitute $V=2.0$, $r=0.5$: $h=\frac{2.0}{\pi\times 0.5^2}=\frac{2.0}{\pi\times 0.25}=\frac{2.0}{0.7853\ldots}=2.546\ldots\approx 2.55$ m.

Topic 1 Test A — Multiple-choice

Topic 1 (Algebra, number and structure). Materials: one scientific calculator; one bound reference that may be annotated. Section A: 15 multiple-choice questions (15 marks). Approximate time: 25 minutes. Unless otherwise indicated, diagrams are not drawn to scale.

Section A — Multiple-choice questions

Choose the response that is correct for the question.

Question 1

At dawn the temperature at a ski resort is -4°C. By noon it has risen to 9°C. The rise in temperature is

A. 5°C

B. 6°C

C. 13°C

D. -13°C

Question 2

The value of $20-6\div 2$ is

A. 7

B. 14

C. 17

D. 4

Question 3

Rounded to two decimal places, 6.847 is

A. 6.90

B. 6.85

C. 6.80

D. 6.84

Question 4

The order of magnitude of 6800, written as a power of 10, is

A. 10^3

B. 10^4

C. 10^6

D. 10^2

Question 5

12% of $\$150$ is

A. $\$138$

B. $\$162$

C. $\$18$

D. $\$1.80$

Question 6

A phone plan costing $\$60$ per month is increased by 20%. The new monthly cost is

A. $\$12$

B. $\$72$

C. $\$80$

D. $\$48$

Question 7

A tyre marked at $\$180$ is discounted by 15%. The sale price is

A. $\$153$

B. $\$150$

C. $\$27$

D. $\$165$

Question 8

An amount of $\$800$ is shared between two people in the ratio $3:5$. The larger share is

A. $\$480$

B. $\$160$

C. $\$300$

D. $\$500$

Question 9

A 2-litre bottle of juice costs $\$3.60$. The cost per litre is

A. $\$1.60$

B. $\$1.80$

C. $\$7.20$

D. $\$0.56$

Question 10

The value of $\frac{3}{4}-\frac{1}{2}$ is

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{1}{8}$

Question 11

The value of $\sqrt{144}+3^2$ is

A. 20

B. 21

C. 156

D. 15

Question 12

If $P=4n-30$, then the value of P when $n=15$ is

A. 90

B. 45

C. -30

D. 30

Question 13

The solution of the equation $4x-3=17$ is

A. $x=20$

B. $x=4$

C. $x=3.5$

D. $x=5$

Question 14

When fully simplified, $7y+3-2y$ is

A. $5y+3$

B. $9y+3$

C. $8y$

D. $5y$

Question 15

In a spreadsheet, cell D2 contains the formula `=B2*C2`. If cell B2 holds 6 and cell C2 holds 8, the value displayed in D2 is

A. 48

B. 68

C. 2

D. 14

End of Section A

Topic 1 Test A — Solutions

Question	Answer	Working
1	**C**	$9-(-4)=9+4=13^{\circ}$C.
2	**C**	Division first: $6\div 2=3$, then $20-3=17$.
3	**B**	The third decimal is $7\geq 5$, so round up: 6.85.
4	**A**	$6800=6.8\times 10^3$, so the order of magnitude is 10^3.
5	**C**	$0.12\times 150=\$18$.
6	**B**	$60\times 1.20=\$72$.
7	**A**	$180\times 0.85=\$153$.
8	**D**	$800\div 8=100$; larger share $=5\times 100=\$500$.
9	**B**	$3.60\div 2=\$1.80$ per litre.
10	**C**	$\frac{3}{4}-\frac{2}{4}=\frac{1}{4}$.
11	**B**	$\sqrt{144}+3^2=12+9=21$.
12	**D**	$P=4\times 15-30=60-30=30$.
13	**D**	$4x=20$, so $x=5$.
14	**A**	$7y-2y=5y$, and the $+3$ is unlike, giving $5y+3$.
15	**A**	`=B2*C2` multiplies: $6\times 8=48$.

Topic 1 Test B — Short answer

Topic 1 (Algebra, number and structure). Materials: one scientific calculator; one bound reference. Section B: 6 questions (30 marks). Approximate time: 40 minutes. Answer all questions in the spaces provided. In all questions where a numerical answer is required, you should only round when instructed to do so. Where more than one mark is available, appropriate working must be shown.

Section B

Question 1 (5 marks)

a. Write $4\,500\,000$ in the form $a \times 10^n$, where $1 \le a < 10$. 1 mark

b. Evaluate $\sqrt{121} + 2^3$. 1 mark

c. Round $46\,820$ to (i) the nearest hundred and (ii) two significant figures. 2 marks

d. State the place value of the digit 7 in the number $3\,486.7$. 1 mark

Question 2 (5 marks)

a. Mortar is made from cement and sand in the ratio $1:4$. Find the mass of each ingredient in a 30 kg batch. 2 marks

b. Paint is mixed white to blue in the ratio $3:1$. If 12 L of white is used, find the volume of blue paint required. 1 mark

c. A car travels 240 km on 20 L of petrol. Show that its fuel consumption is 12 km per litre. 2 marks

Question 3 (5 marks)

A worker is paid at a constant hourly rate. She earns $\$95.40$ for a 6-hour shift.

a. Find her hourly rate of pay. 2 marks

b. Hence find her pay for a 9-hour shift. 1 mark

c. A tap fills a 240-litre tank at 8 litres per minute. Show that the tank fills in 30 minutes. 2 marks

Question 4 (5 marks)

a. Find 20% of $\$85$. 1 mark

b. A pair of boots costs $\$120$. In a sale it is reduced by 15%. Find the sale price. 2 marks

c. A gym's monthly membership fee rose from $\$40$ to $\$46$. Find the percentage increase. 2 marks

Question 5 (5 marks)

a. Solve the equation $5x - 3 = 2x + 18$. 2 marks

b. The perimeter of a rectangle is $P = 2(l + w)$. Make w the subject of the formula. 2 marks

c. Hence find w when $P = 26$ and $l = 8$. 1 mark

Question 6 (5 marks)

A market stall has costs $C = 120 + 4n$ dollars and revenue $R = 10n$ dollars, where n is the number of items sold.

a. Show that the stall breaks even when $n = 20$. 2 marks

b. Find the profit when 50 items are sold. 2 marks

c. In a spreadsheet the profit is calculated in cell D2 as `=10*A2-(120+4*A2)`. State the value displayed when cell A2 holds 35. 1 mark

End of Section B

Topic 1 Test B — Solutions

Question 1.

a. Move the decimal point 6 places: $4\,500\,000 = 4.5 \times 10^6$.

b. $\sqrt{121} + 2^3 = 11 + 8 = 19$.

c. (i) The tens digit is $2 < 5$, so to the nearest hundred $46\,820 \approx 46\,800$. (ii) The first two significant figures are 4 and 6; the next digit is $8 \geq 5$, so round up: $47\,000$.

d. The digit 7 is in the tenths column, so its place value is 7 tenths, that is 0.7.

Question 2.

a. The ratio $1:4$ has $1+4=5$ parts, so one part $= 30 \div 5 = 6$ kg. The cement is 6 kg and the sand is $4 \times 6 = 24$ kg (check: $6 + 24 = 30$ kg).

b. One part of white $= 12 \div 3 = 4$ L, so the blue paint is $1 \times 4 = 4$ L.

c. Fuel consumption is distance per litre:

$$\frac{240\,\text{km}}{20\,\text{L}} = 12\,\text{km per litre},$$

as required.

Question 3.

a. Hourly rate $= \dfrac{\$95.40}{6} = \$15.90$ per hour.

b. Pay for 9 hours $= 15.90 \times 9 = \$143.10$.

c. Time = volume ÷ rate:

$$\frac{240\,\text{L}}{8\,\text{L/min}} = 30\,\text{minutes},$$

as required.

Question 4.

a. $20\,\%$ of $\$85 = 0.20 \times 85 = \$17$.

b. A $15\,\%$ reduction leaves $85\,\%$: sale price $= 0.85 \times 120 = \$102$.

c. The increase is $46 - 40 = \$6$. As a percentage of the original,

$$\frac{6}{40} \times 100\,\% = 15\,\%.$$

Question 5.

a. $5x - 3 = 2x + 18$. Subtract $2x$: $3x - 3 = 18$. Add 3: $3x = 21$. Divide by 3: $x = 7$.

b. $P = 2(l + w)$. Divide both sides by 2: $\dfrac{P}{2} = l + w$. Subtract l:

$$w = \frac{P}{2} - l.$$

c. $w = \frac{26}{2} - 8 = 13 - 8 = 5$.

Question 6.

a. Break-even is where revenue equals cost: $10n = 120 + 4n$. Subtract $4n$: $6n = 120$. Divide by 6: $n = 20$, as required.

b. At $n = 50$: revenue $R = 10 \times 50 = \$500$ and cost $C = 120 + 4 \times 50 = \$320$, so profit $= 500 - 320 = \$180$.

c. With $A2 = 35$: $10 \times 35 - (120 + 4 \times 35) = 350 - (120 + 140) = 350 - 260 = \$90$.