Foundation Mathematics Units 1 & 2

Chapter 2 Number and place value

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Theory

Every day you meet numbers of many different kinds — the temperature on your phone, the balance in a bank account, the price of a coffee, the fraction of a tank of fuel left. Choosing the right **kind of number** to describe a quantity, and being able to read, compare and combine those numbers, is the starting point for all of Foundation Mathematics.

Kinds of number.

- **Whole numbers** $0, 1, 2, 3, \ldots$ are used for *counting* things that cannot be split — the number of students in a class, cars in a car park, or messages in an inbox.
- **Integers** are the whole numbers *together with their negatives* $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$. A **negative integer** describes a quantity *below* an agreed zero: a temperature below 0°C, a height below sea level, an overdrawn bank balance, or a basement level below the ground floor.
- **Fractions** such as $1/2$, $3/4$, $2/3$ describe *part of a whole* — half a pizza, three-quarters of a cup, two-thirds of the class.
- **Decimals** such as 4.50, 1.75, 45.2 are used whenever we measure or deal with money — $\$4.50$, a height of 1.75 m, 45.2 litres of fuel.

Fractions and decimals are just two ways of writing the same “in-between” numbers: $1/2 = 0.5$, $3/4 = 0.75$, $1/4 = 0.25$.

Reading and writing large whole numbers.

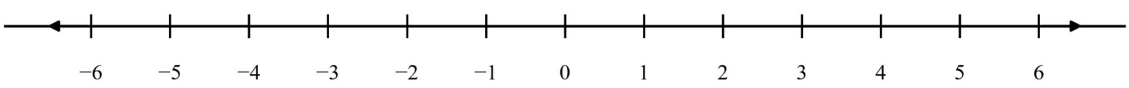
Large numbers are grouped in **threes**, counting from the right, to make them easy to read. In Australia the groups are separated by a **thin space** (not a comma): $1\,704\,630$ is read “one million, seven hundred and four thousand, six hundred and thirty”. Each digit has a **place value** set by its position:

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units
1	7	0	4	6	3	0

So in $1\,704\,630$ the digit 7 stands for $700\,000$ (seven hundred thousand), while the digit 4 stands for $4\,000$ (four thousand).

The number line.

Numbers are pictured on a **number line**. Zero sits in the middle; numbers get **larger** as you move to the **right** and **smaller** as you move to the **left**, so every negative number is to the left of zero.



To **compare** two numbers, the one further to the **right** is the larger. This needs care with negatives: -8 is *less than* -3 (written $-8 < -3$), because -8 is further left — it is colder, deeper or more in debt. A useful check: **the more negative a number, the smaller it is.**

Combining directed numbers in context.

Adding a positive number, or a rise, moves you to the **right**; subtracting, or a fall, moves you to the **left**.

- A temperature of -4°C that **rises** 9°C becomes $-4 + 9 = 5^{\circ}$C.

- A balance of $-\$35$ (overdrawn) after a $\$90$ **deposit** becomes $-35+90=\$55$.

To find the **difference** between two values — how far apart they are on the number line — subtract the smaller from the larger. From a temperature of -4°C up to 5°C is a rise of

$$5-(-4)=5+4=9^{\circ}\text{C}.$$

Notice that subtracting a negative is the same as adding: $5-(-4)=5+4$.

Worked examples

Example 1 — scaffolded

The overnight minimum temperatures at five towns were -5°C, 4°C, -2°C, 9°C and -8°C. Order them from coldest to warmest, and find the difference between the warmest and the coldest.

Working	Comment
Colder means smaller, so further **left** on the number line.	The more negative the temperature, the colder it is.
Order: $-8,-5,-2,4,9$ (all °C).	Line the values up from left (coldest) to right (warmest).
Coldest $=-8^{\circ}$C, warmest $=9^{\circ}$C.	The two ends of the ordered list.
Difference $=9-(-8)=9+8=17^{\circ}$C.	Subtracting a negative adds; this is the gap on the number line.

Example 2 — scaffolded

A student's bank account has a balance of $\$40$. The account allows an **overdraft** (the balance may go negative). In order, a $\$75$ textbook is bought on the card, then a $\$90$ part-time wage is deposited. Find the balance after each event.

Working	Comment
Start: balance $=\$40$.	A positive balance is money you have.
After the $\$75$ purchase: $40-75=-\$35$.	Buying subtracts; the negative means the account is **overdrawn by** $\$35$.
After the $\$90$ deposit: $-35+90=\$55$.	A deposit adds; the balance moves back to positive.
The account was overdrawn once (after the purchase) and finished at $\$55$.	Track the sign at each step.

Example 3 — unscaffolded

A crowdfunding campaign displays a total of $\$3\,275\,040$ raised. (i) Write this amount in words. (ii) State the place value of the digit 2. (iii) Round the amount to the nearest ten thousand dollars.

Grouping the digits, $3\,275\,040$ is three million, two hundred and seventy-five thousand and forty, so the amount is **three million, two hundred and seventy-five thousand and forty dollars**. The digit 2 sits in the hundred-thousands column, so its place value is $200\,000$ — **two hundred thousand**. To round to the nearest ten thousand, look at the thousands digit, 5: since $5\geq5$ we round the ten-thousands up, giving $3\,280\,000$.

$\therefore$ the amount is three million, two hundred and seventy-five thousand and forty dollars; the 2 means $\$200\,000$; and rounded to the nearest ten thousand it is $\$3\,280\,000$.

Example 4 — unscaffolded

Beside a salt lake, the water surface is 12 m **below** sea level, written -12 m. A hill next to the lake rises to 148 m **above** sea level. (i) How much higher is the hilltop than the lake surface? (ii) A drone climbs from the lake surface to a point 65 m above sea level. How far does it climb?

Heights above sea level are positive and heights below are negative, so the lake surface is -12 m and the hilltop is $+148$ m. The vertical distance between two heights is the larger minus the smaller:

$$148-(-12)=148+12=160\,\text{m}.$$

For the drone, it rises from -12 m to $+65$ m, a climb of

$$65-(-12)=65+12=77\,\text{m}.$$

∴ the hilltop is 160 m higher than the lake surface, and the drone climbs 77 m.

Practice questions

Scaffolded

Q1. The overnight minimum temperatures at five towns were -3°C, 7°C, -6°C, 2°C and -1°C. (a) Order the temperatures from coldest to warmest. (b) State the coldest and the warmest temperature. (c) Find the difference between the warmest and the coldest.

Q2. A savings account has a balance of $\$28$ and allows an overdraft. In order, a $\$45$ direct debit is taken out, then a $\$30$ deposit is made, then a $\$20$ purchase is made. (a) Find the balance after the $\$45$ direct debit. (b) Find the balance after the $\$30$ deposit. (c) Find the balance after the $\$20$ purchase.

Q3. A city's population is recorded as $5\,278\,064$. (a) Write this number in words. (b) State the place value of the digit 7. (c) Round the population to the nearest ten thousand.

Q4. At 6 am the temperature in a town was -6°C. (a) By 11 am it had risen 8°C. Find the temperature at 11 am. (b) By midnight it had fallen 10°C from the 11 am reading. Find the temperature at midnight. (c) Find the difference between the warmest and the coldest of the three readings.

Q5. A walking trail runs beside a salt lake whose surface is 9 m below sea level. (a) Write the lake-surface height as a directed number (in metres, relative to sea level). (b) A lookout on the trail is 214 m above sea level. How much higher is the lookout than the lake surface? (c) A bird hovers 30 m directly above the lake surface. What is its height relative to sea level?

Q6. Consider the five numbers $-2, 11/2, 0.75, -0.5, 2$. (a) Write $11/2$ as a decimal. (b) Order all five numbers from smallest to largest. (c) Which two of the numbers are closest together on the number line?

Unscaffolded

Q7. Order the integers $3, -7, 0, -2, 5, -5$ from lowest to highest, then state the difference between the highest and the lowest.

Q8. The overnight minimums at five alpine sites were: Site A -6°C, Site B -11°C, Site C -2°C, Site D 1°C, Site E -8°C. (a) Which site was coldest? (b) Find the difference between the coldest and the warmest site. (c) How much colder was Site E than Site D?

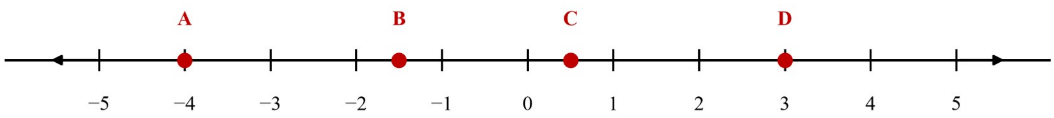
Q9. An account begins a week at $\$52$ and allows an overdraft. The transactions, in order, are: $-\$70$ (rent), $+\$40$ (refund), $-\$35$ (groceries), $+\$120$ (pay). Find the running balance after each transaction, state on how many occasions the account was overdrawn, and give the final balance.

Q10. An infrastructure project has a total cost written as $\$48\,705\,000$. (a) Write this amount in words. (b) State the place value of the digit 8. (c) Round the cost to the nearest million dollars.

Q11. A research submarine sits at a depth of 320 m below sea level. A weather balloon released from its support ship rises to $1\,450$ m above sea level. (a) Write both heights as directed numbers relative to sea level. (b) Find the vertical distance between the submarine and the balloon. (c) The submarine then ascends 185 m. What is its new depth relative to sea level?

Q12. In a golf tournament, four players finished with scores relative to par of Player P -4, Player Q $+2$, Player R -1, Player S -7 (in golf, a lower score is better). (a) Order the players from best to worst. (b) By how many strokes did the winner beat the last-placed player? (c) Player R later improves by 5 strokes. What is R's new score relative to par?

Q13. The number line below shows four points.



(a) State the value at each of A, B, C and D.
(b) Find the distance between A and D.
(c) Which point is closest to zero?

Q14. The table gives each city's time-zone offset from UTC (ignoring daylight saving): London 0, Perth $+8$, New York -5, Auckland $+12$ hours. (a) Which city's clock is furthest ahead of UTC? (b) Find the time difference between Perth and New York. (c) When it is 3 pm in London, what is the time in New York?

Q15. Consider the numbers $0.6, -11/4, 2, -0.8, 3/4, -2, 1.5$. (a) Order them from smallest to largest. (b) Which of them lie between 0 and 1? (c) Which number is closest to zero?

Q16. For each quantity, state whether a negative value is possible, and give an example value where relevant. (a) the temperature inside a freezer (in °C) (b) the number of students in a class (c) a bank account balance (d) a person's height (in cm)

Q17. At midnight the temperature in a ski village was -3°C. It then fell 4°C by 4 am, rose 2°C by 8 am, rose 9°C by 1 pm, and fell 6°C by 8 pm. (a) Find the temperature at 4 am, 8 am, 1 pm and 8 pm. (b) What was the lowest temperature during the day? (c) Find the difference between the highest and the lowest temperatures.

Q18. A cable-car station at a ski resort is $1\,620$ m above sea level. Skiers descend to a valley restaurant 240 m lower, and a service tunnel drops a further 35 m below the restaurant. A nearby lake surface is recorded at -6 m (that is, 6 m below sea level). (a) Find the height of the valley restaurant above sea level. (b) Find the height of the tunnel floor above sea level. (c) Find the vertical distance between the tunnel floor and the lake surface.

Answers

Q1. (a) $-6,-3,-1,2,7$ (°C). (b) coldest -6°C, warmest 7°C. (c) 13°C. **Q2.** (a) $-\$17$ (overdrawn $\$17$). (b) $\$13$. (c) $-\$7$. **Q3.** (a) five million, two hundred and seventy-eight thousand and sixty-four. (b) $70\,000$. (c) $5\,280\,000$. **Q4.** (a) 2°C. (b) -8°C. (c) 10°C. **Q5.** (a) -9 m. (b) 223 m. (c) 21 m above sea level. **Q6.** (a) 1.5. (b) $-2,-0.5,0.75,1.5,2$. (c) 1.5 and 2 (only 0.5 apart). **Q7.** $-7,-5,-2,0,3,5$; difference $=5-(-7)=12$. **Q8.** (a) Site B. (b) 12°C. (c) 9°C colder. **Q9.** Running balances $-\$18,\$22,-\$13,\$107$; overdrawn on 2 occasions; final balance $\$107$. **Q10.** (a) forty-eight million, seven hundred and five thousand dollars. (b) $8\,000\,000$. (c) $\$49\,000\,000$. **Q11.** (a) submarine -320 m, balloon $+1\,450$ m. (b) 1770 m. (c) -135 m (135 m below sea level). **Q12.** (a) S, P, R, Q. (b) 9 strokes. (c) -6. **Q13.** (a) A $=-4$, B $=-1.5$, C $=0.5$, D $=3$. (b) 7. (c) C. **Q14.** (a) Auckland. (b) 13 hours. (c) 10 am. **Q15.** (a) $-2,-1.25,-0.8,0.6,0.75,1.5,2$. (b) 0.6 and $3/4\,(=0.75)$. (c) 0.6. **Q16.** (a) yes, e.g. -18°C. (b) no — a count is a whole number, never negative. (c) yes, e.g. $-\$50$ when overdrawn. (d) no — a height is always positive. **Q17.** (a) 4 am -7°C, 8 am -5°C, 1 pm 4°C, 8 pm -2°C. (b) -7°C. (c) 11°C. **Q18.** (a) 1380 m. (b) 1345 m. (c) 1351 m.

Fully-worked solutions

Q1. Colder temperatures are more negative, hence further left on the number line. (a) Ordered coldest to warmest: $-6,-3,-1,2,7$ (°C). (b) The ends of the list: coldest -6°C, warmest 7°C. (c) Difference $=7-(-6)=7+6=13^{\circ}$C.

Q2. Start at $\$28$; a debit or purchase subtracts, a deposit adds. (a) $28-45=-\$17$ — the account is overdrawn by $\$17$. (b) $-17+30=\$13$. (c) $13-20=-\$7$ — overdrawn again by $\$7$.

Q3. The number is $5\,278\,064$. (a) Grouped as $5|278|064$, it reads five million, two hundred and seventy-eight thousand and sixty-four. (b) The 7 is in the ten-thousands column, so its place value is $70\,000$. (c) To round to the nearest ten thousand, look at the thousands digit, 8. Since $8\ge 5$ we round up, giving $5\,280\,000$.

Q4. Start at -6°C at 6 am. (a) A rise of 8: $-6+8=2^{\circ}$C at 11 am. (b) A fall of 10 from 2°C: $2-10=-8^{\circ}$C at midnight. (c) The three readings are $-6,2,-8$; the warmest is 2°C and the coldest is -8°C, so the difference is $2-(-8)=10^{\circ}$C.

Q5. Above sea level is positive, below is negative. (a) The lake surface is 9 m below sea level, so -9 m. (b) The lookout is $+214$ m, so it is $214-(-9)=214+9=223$ m higher. (c) The bird is 30 m above the surface: $-9+30=21$ m above sea level.

Q6. The five numbers are $-2,1\,1/2,0.75,-0.5,2$. (a) $1\,1/2=1.5$. (b) Writing each as a decimal ($-2,1.5,0.75,-0.5,2$) and ordering from left to right on the number line: $-2,-0.5,0.75,1.5,2$. (c) The gaps between neighbours are $1.5,1.25,0.75,0.5$, so the smallest gap is 0.5, between 1.5 and 2.

Q7. Ordering from most negative to most positive: $-7,-5,-2,0,3,5$. The highest is 5 and the lowest is -7, so the difference is $5-(-7)=5+7=12$.

Q8. The five values are $-6,-11,-2,1,-8$ (°C). (a) The most negative is -11, so **Site B** is coldest. (b) Warmest is Site D at 1°C; difference $=1-(-11)=12^{\circ}$C. (c) Site D is 1°C and Site E is -8°C, so Site E is $1-(-8)=9^{\circ}$C colder.

Q9. Start at $\$52$ and apply each transaction in turn:

$$52-70=-18,\ -18+40=22,\ 22-35=-13,\ -13+120=107.$$

The running balances are $-\$18,\$22,-\$13,\$107$. The balance was negative after the 1st and 3rd transactions, so the account was overdrawn on 2 **occasions**; the final balance is $\$107$.

Q10. The cost is $\$48\,705\,000$. (a) Grouped as $48|705|000$: forty-eight million, seven hundred and five thousand dollars. (b) The 8 is in the millions column, so its place value is $8\,000\,000$. (c) To round to the nearest million, look at the hundred-thousands digit, 7. Since $7\ge 5$ we round up, giving $\$49\,000\,000$.

Q11. Below sea level is negative. (a) Submarine -320 m; balloon $+1\,450$ m. (b) Vertical distance $=1\,450-(-320)=1\,450+320=1\,770$ m. (c) Ascending 185 m from -320 m: $-320+185=-135$ m, i.e. 135 m below sea level.

Q12. Scores relative to par: P -4, Q $+2$, R -1, S -7; lower is better. (a) From best (most negative) to worst: S (-7), P (-4), R (-1), Q $(+2)$. (b) Winner S is -7 and last-placed Q is $+2$, so the margin is $2-(-7)=9$ strokes. (c) Improving by 5 strokes means subtracting 5: $-1-5=-6$.

Q13. Reading the number line: (a) Reading directly from the line, A $=-4$, B $=-1.5$ (halfway between -2 and -1), C $=0.5$ (halfway between 0 and 1), D $=3$. (b) Distance between A and D $=3-(-4)=3+4=7$. (c) The distances from zero are $4, 1.5, 0.5, 3$, so **C** (0.5) is closest to zero.

Q14. Offsets from UTC: London 0, Perth $+8$, New York -5, Auckland $+12$. (a) The largest offset is $+12$, so **Auckland** is furthest ahead. (b) Time difference between Perth and New York $=8-(-5)=13$ hours. (c) New York is 5 hours behind London, so at 3 pm ($15{:}00$) London time it is $15-5=10$, i.e. 10 am in New York.

Q15. Writing each as a decimal: $0.6, -1.25, 2, -0.8, 0.75, -2, 1.5$. (a) Ordered smallest to largest: $-2, -1.25, -0.8, 0.6, 0.75, 1.5, 2$. (b) Between 0 and 1 are 0.6 and $3/4=0.75$. (c) The distances from zero are $0.6, 1.25, 2, 0.8, 0.75, 2, 1.5$; the smallest is 0.6, so 0.6 is closest to zero.

Q16. A value can be negative only when the quantity is measured against a zero that it can drop below. (a) Yes — a freezer is colder than 0°C, e.g. -18°C. (b) No — a count of students is a whole number and cannot be negative. (c) Yes — an overdrawn account has a negative balance, e.g. $-\$50$. (d) No — a height is measured up from zero and is always positive.

Q17. Start at -3°C and apply each change:

$$-3-4=-7, -7+2=-5, -5+9=4, 4-6=-2.$$

(a) 4 am -7°C, 8 am -5°C, 1 pm 4°C, 8 pm -2°C. (b) The readings are $-3, -7, -5, 4, -2$; the lowest is -7°C (at 4 am). (c) The highest is 4°C and the lowest is -7°C, so the difference is $4-(-7)=11^{\circ}$C.

Q18. Above sea level is positive. (a) Restaurant $=1\,620-240=1\,380$ m above sea level. (b) Tunnel floor $=1\,380-35=1\,345$ m above sea level. (c) The lake surface is -6 m, so the distance to the tunnel floor is $1\,345-(-6)=1\,345+6=1\,351$ m.

Theory

Our number system is a **place-value system**: the *value* of a digit depends on the **position** it occupies. The same digit 5 is worth five hundred in 538 but only five hundredths in 2.05. Reading place value correctly is the foundation of estimating, rounding and checking whether an answer is reasonable.

Place-value columns.

Moving left, each column is **ten times** the one on its right; moving right, each column is **one tenth** of the one on its left. The decimal point separates the whole-number part from the fractional part.

Ten-thousands	Thousands	Hundreds	Tens	Ones	.	Tenths	Hundredths	Thousandths
10 000	1000	100	10	1	.	0.1	0.01	0.001

For example, in 3 486.72 the digit 4 sits in the **hundreds** column, so its value is 400; the digit 7 sits in the **tenths** column, so its value is 0.7.

Expanded form.

Writing a number in **expanded form** shows the value contributed by each digit:

$$3\,486.72 = 3000 + 400 + 80 + 6 + 0.7 + 0.02\,.$$

Multiplying and dividing by powers of 10.

Because each column is ten times the next, multiplying or dividing by $10\,,100\,,1000\,,\ldots$ just shifts the digits relative to the decimal point:

- **multiplying** by 10, 100, 1000 moves each digit **one, two, three** places to the **left** (the number gets bigger);
- **dividing** by 10, 100, 1000 moves each digit the same number of places to the **right** (the number gets smaller).

So $4.62 \times 100 = 462$, while $4.62 \div 100 = 0.0462$. This is exactly how we convert between metric units such as metres and millimetres.

Rounding.

To **round** a number we keep it to a chosen accuracy and replace the rest. Look at the **next digit** after the place you are rounding to: if it is 5 or more, round **up**; if it is 4 or less, round **down** (leave the digit unchanged). We can round to a given **place value** (nearest ten, hundred, …), to a number of **decimal places**, or to a number of **significant figures**.

The **significant figures** of a number are its meaningful digits, counted from the first non-zero digit. In 0.03858 the leading zeros are *not* significant, so the first significant figure is the 3; rounded to 2 significant figures it is 0.039.

Truncation.

To **truncate** is to simply *cut off* the digits beyond the chosen place, without looking at what follows. Truncating 52.678 to two decimal places gives 52.67, whereas **rounding** to two decimal places gives 52.68. Rounding is usually more accurate; truncation is what happens when a display or a fuel pump just drops the extra digits.

Leading-digit approximation.

A quick estimate keeps only the **first significant digit** and replaces the rest with zeros — this is **leading-digit approximation** (rounding to one significant figure). It turns 6843 into 7000 and 0.0724 into 0.07, which is ideal for a fast mental check of a calculation.

Order of magnitude.

The **order of magnitude** of a positive number is the power of 10 in the column of its leading digit. Writing the number as $a \times 10^n$ with $1 \le a < 10$, the order of magnitude is 10^n:

$$6843 = 6.843 \times 10^3 \Rightarrow \text{order of magnitude } 10^3, 4\,500\,000 = 4.5 \times 10^6 \Rightarrow 10^6.$$

Two quantities that differ by k orders of magnitude differ by a factor of about 10^k. Comparing orders of magnitude is a powerful way to judge "how many times bigger" one quantity is than another.

Worked examples

Example 1 — scaffolded

A city's water-storage display reads $48\,216.35$ megalitres. State the place value of the digit 2, and write the number in expanded form.

Working	Comment
Set the digits in their columns: 4 ten-thousands, 8 thousands, 2 hundreds, 1 ten, 6 ones, 3 tenths, 5 hundredths.	Read left to right, the point after the ones.
The digit 2 is in the **hundreds** column, so its value is 200.	Place value = digit × column value.
Expanded form: $40\,000 + 8000 + 200 + 10 + 6 + 0.3 + 0.05$.	Each digit times its column value, added.

Example 2 — scaffolded

A fuel receipt records 63.847 litres. Round this amount to (a) one decimal place, (b) two decimal places, and (c) two significant figures.

Working	Comment
(a) The digit after the first decimal place is 4 ($63.8\underline{4}7$). Since $4 < 5$, round down: 63.8 L.	Look at the next digit only.
(b) The digit after the second decimal place is 7 ($63.84\underline{7}$). Since $7 \ge 5$, round up: 63.85 L.	$84 \to 85$.
(c) The first two significant figures are 6 and 3; the next digit is $8 \ge 5$, so round up: 64 L.	Significant figures start at the first non-zero digit.

Example 3 — unscaffolded

A factory makes a batch of 1000 identical bolts with a total mass of 42.5 kg. Find the mass of one bolt, in kilograms and in grams. If the factory instead makes 10 times as many bolts, what is the total mass of the larger batch?

Dividing by 1000 moves each digit three places to the right:

$$42.5 \div 1000 = 0.0425\,\text{kg}.$$

To convert to grams, multiply by 1000 (three places left): $0.0425 \times 1000 = 42.5$ g. A batch of $10 \times 1000 = 10\,000$ bolts has 10 times the mass:

$$42.5 \times 10 = 425\,\text{kg}.$$

$\therefore$ each bolt has mass 0.0425 kg $= 42.5$ g, and the larger batch has mass 425 kg.

Example 4 — unscaffolded

In a recent year Australia's population was about $26\,380\,000$ and the Pacific nation of Nauru's was about $12\,500$. Use leading-digit approximation and order of magnitude to estimate how many times larger Australia's population is.

Leading-digit approximation (round each to one significant figure):

$$26\,380\,000 \approx 30\,000\,000\,,\,12\,500 \approx 10\,000\,.$$

Writing each in the form $a \times 10^n$:

$$26\,380\,000 = 2.638 \times 10^7 \Rightarrow 10^7\,,\,12\,500 = 1.25 \times 10^4 \Rightarrow 10^4\,.$$

The orders of magnitude differ by $7-4=3$, so Australia's population is about $10^3=1000$ times larger.

$\therefore$ Australia's population is roughly 10^3, that is about **a thousand times**, larger than Nauru's.

Practice questions

Scaffolded

Q1. A stocktake value is displayed as $57\,903.48$. (a) State the place value of the digit 7. (b) State the place value of the digit 4. (c) Write the number in expanded form.

Q2. Calculate, using place value (no calculator needed): (a) 3.6×100. (b) $148 \div 1000$. (c) $0.052 \times 10\,000$.

Q3. A football match drew a crowd of $43\,728$ people. Round this figure to the nearest (a) ten, (b) hundred, (c) thousand.

Q4. A surveyor measures a boundary as 8.4637 m. Round this length to (a) one decimal place, (b) two decimal places, (c) three decimal places.

Q5. Round each number to the stated number of significant figures. (a) 6214 to 2 significant figures. (b) 0.05863 to 2 significant figures. (c) 0.05863 to 3 significant figures.

Q6. (a) Write the leading-digit approximation of 7361. (b) Write the leading-digit approximation of 0.0836. (c) State the order of magnitude of $5\,400\,000$ as a power of 10.

Unscaffolded

Q7. In the number 806.375, state the place value of the digit 8, the digit 6 and the digit 7.

Q8. Write $24\,059.6$ in expanded form, then state which digit has the greatest place value.

Q9. A reel holds 2500 m of fencing wire. (a) Express this length in kilometres. (b) The wire is cut into $10\,000$ equal pieces. Find the length of each piece in metres and in millimetres.

Q10. A phone bill total is calculated as $\$284.7391$. Round it to the nearest (a) cent (two decimal places), (b) dollar, (c) ten dollars.

Q11. A petrol pump shows a delivery of 47.856 litres. (a) Round this to two decimal places. (b) Truncate it to two decimal places. (c) Explain why the two answers differ.

Q12. The average distance from the Earth to the Sun is about $149\,597\,871$ km. (a) Round this to 3 significant figures. (b) Round this to 1 significant figure. (c) Write your answer to (b) in the form $a \times 10^n$.

Q13. A bacterium is about 0.000002 m long and an ant is about 0.005 m long. (a) State the order of magnitude of each length as a power of 10. (b) Use the orders of magnitude to estimate how many times longer the ant is than the bacterium.

Q14. Estimate 6812×39 using leading-digit approximation, then find the exact value and comment on whether your estimate has the right order of magnitude.

Q15. When 4.85 is multiplied by 10^n the result is 4850. Find the value of n.

Q16. A measurement, rounded to one decimal place, is recorded as 6.3. (a) State the smallest value the exact measurement could have been. (b) State the value it must be *below*.

Q17. A community fun-run raised $\$37\,500$ from exactly 1000 donors. (a) Find the mean donation. (b) If instead $10\,000$ donors had each given that mean amount, how much would have been raised in total? (c) Express the original $\$37\,500$ to one significant figure.

Q18. A grain of rice has a mass of about 0.025 g and an African elephant has a mass of about $6\,000\,000$ g. (a) State the order of magnitude of each mass as a power of 10. (b) Estimate, to the nearest power of 10, how many grains of rice have the same mass as the elephant.

Answers

Q1. (a) thousands, value 7000. (b) tenths, value 0.4. (c) $50\,000+7000+900+3+0.4+0.08$. **Q2.** (a) 360. (b) 0.148. (c) 520. **Q3.** (a) $43\,730$. (b) $43\,700$. (c) $44\,000$. **Q4.** (a) 8.5 m. (b) 8.46 m. (c) 8.464 m. **Q5.** (a) 6200. (b) 0.059. (c) 0.0586. **Q6.** (a) 7000. (b) 0.08. (c) 10^6. **Q7.** 8: hundreds, value 800; 6: ones, value 6; 7: hundredths, value 0.07. **Q8.** $20\,000+4000+50+9+0.6$; the digit 2 (ten-thousands) has the greatest place value. **Q9.** (a) 2.5 km. (b) 0.25 m $=250$ mm. **Q10.** (a) $\$284.74$. (b) $\$285$. (c) $\$280$. **Q11.** (a) 47.86 L. (b) 47.85 L. (c) see solution. **Q12.** (a) $150\,000\,000$ km. (b) $100\,000\,000$ km. (c) 1×10^8 km. **Q13.** (a) bacterium 10^{-6} m, ant 10^{-3} m. (b) about $10^3=1000$ times. **Q14.** estimate $7000\times40=280\,000$; exact $265\,668$; same order of magnitude ($\approx10^5$). **Q15.** $n=3$. **Q16.** (a) 6.25. (b) 6.35. **Q17.** (a) $\$37.50$. (b) $\$375\,000$. (c) $\$40\,000$. **Q18.** (a) rice 10^{-2} g, elephant 10^6 g. (b) about 10^8.

Fully-worked solutions

Q1. Placing $57\,903.48$ in columns gives 5 ten-thousands, 7 thousands, 9 hundreds, 0 tens, 3 ones, 4 tenths, 8 hundredths. (a) The 7 is in the thousands column, value 7000. (b) The 4 is in the tenths column, value 0.4. (c) Expanded form: $50\,000+7000+900+3+0.4+0.08$ (the tens digit is 0, so no tens term).

Q2. (a) Multiplying by 100 moves each digit two places left: $3.6\times100=360$. (b) Dividing by 1000 moves each digit three places right: $148\div1000=0.148$. (c) Multiplying by $10\,000$ moves each digit four places left: $0.052\times10\,000=520$.

Q3. For $43\,728$: (a) nearest ten — the ones digit is $8\ge5$, so round up: $43\,730$. (b) nearest hundred — the tens digit is $2<5$, so round down: $43\,700$. (c) nearest thousand — the hundreds digit is $7\ge5$, so round up: $44\,000$.

Q4. For 8.4637: (a) one decimal place — the second decimal is $6\ge5$, round up: 8.5 m. (b) two decimal places — the third decimal is $3<5$, round down: 8.46 m. (c) three decimal places — the fourth decimal is $7\ge5$, round up: 8.464 m.

Q5. (a) 6214: the first two significant figures are $6,2$; the next digit is $1<5$, so 6200. (b) 0.05863: the first significant figure is 5 (leading zeros do not count). Two significant figures $5,8$; the next digit is $6\ge5$, so round up to 0.059. (c) Three significant figures $5,8,6$; the next digit is $3<5$, so round down to 0.0586.

Q6. (a) Keep the leading digit 7 and replace the rest with zeros; the next digit $3<5$, so $7361\approx7000$. (b) The first significant digit is 8; the next digit $3<5$, so $0.0836\approx0.08$. (c) $5\,400\,000=5.4\times10^6$, so the order of magnitude is 10^6.

Q7. In 806.375: the 8 is in the hundreds column (value 800), the 6 is in the ones column (value 6), and the 7 is in the hundredths column (value 0.07).

Q8. $24\,059.6=20\,000+4000+50+9+0.6$ (the hundreds digit is 0). The digit 2 sits in the ten-thousands column, the largest column present, so it has the greatest place value.

Q9. (a) Dividing by 1000: $2500\div1000=2.5$ km. (b) Dividing by $10\,000$: $2500\div10\,000=0.25$ m. Converting to millimetres, multiply by 1000: $0.25\times1000=250$ mm.

Q10. For $\$284.7391$: (a) two decimal places — the third decimal is $9\ge5$, round up: $\$284.74$. (b) nearest dollar — the first decimal is $7\ge5$, round up: $\$285$. (c) nearest ten dollars — the ones digit is $4<5$, round down: $\$280$.

Q11. (a) Rounding 47.856 to two decimal places: the third decimal is $6\ge5$, so round up to 47.86 L. (b) Truncating to two decimal places simply drops everything after the second decimal: 47.85 L. (c) Rounding examines the next digit (6) and, because it is 5 or more, increases the second decimal from 5 to 6. Truncation ignores the following digits entirely and just cuts them off, so it leaves the 5 unchanged.

Q12. For $149\,597\,871$: (a) three significant figures $1,4,9$; the next digit is $5\ge5$, so $149\to150$, giving $150\,000\,000$ km. (b) one significant figure — leading digit 1, next digit $4<5$, so $100\,000\,000$ km. (c) $100\,000\,000=1\times10^8$ km.

Q13. (a) $0.000002 = 2 \times 10^{-6}$, order of magnitude 10^{-6} m; $0.005 = 5 \times 10^{-3}$, order of magnitude 10^{-3} m. (b) The exponents differ by $-3-(-6)=3$, so the ant is about $10^3 = 1000$ times longer than the bacterium.

Q14. Leading-digit approximation: $6812 \approx 7000$ and $39 \approx 40$, so the estimate is $7000 \times 40 = 280\,000$. The exact product is $6812 \times 39 = 265\,668$. Both the estimate and the exact value are a little over 2×10^5, so the estimate has the right order of magnitude and confirms the exact answer is reasonable.

Q15. $4.85 \times 10^n = 4850$ means $10^n = \dfrac{4850}{4.85} = 1000$. Since $1000 = 10^3$, $n = 3$.

Q16. A value that rounds to 6.3 at one decimal place must lie in the interval that rounds to 6.3. (a) The smallest such value is 6.25 (which rounds up to 6.3). (b) Any value from 6.35 upwards would round to 6.4, so the exact measurement must be below 6.35.

Q17. (a) Mean donation $= 37\,500 \div 1000 = \$37.50$. (b) With $10\,000$ donors each giving $\$37.50$: $37.50 \times 10\,000 = \$375\,000$. (c) $\$37\,500$ to one significant figure — leading digit 3, next digit $7 \geq 5$, so round up to $\$40\,000$.

Q18. (a) $0.025 = 2.5 \times 10^{-2}$, order of magnitude 10^{-2} g; $6\,000\,000 = 6 \times 10^6$, order of magnitude 10^6 g. (b) The exponents differ by $6-(-2)=8$, so about 10^8 grains of rice have the same mass as the elephant. (Checking: $6\,000\,000 \div 0.025 = 240\,000\,000 = 2.4 \times 10^8$, which is 10^8 to the nearest power of 10.)

Theory

Percentages are one of the most common ways numbers appear in everyday life: a "30 % off" sale, a 10 % tip, a 3.5 % interest rate, "62 % of voters agreed". This section shows how to read percentages, change them into other forms, and use them to calculate and compare.

Per cent means "per hundred".

The symbol % means "out of 100". So 37 % means 37 out of every 100, which is the fraction $\frac{37}{100}$ and the decimal 0.37. A percentage is just a convenient way of writing a fraction whose denominator is 100.

Converting between fractions, decimals and percentages.

Because a percentage is a fraction out of 100, you can move freely between the three forms:

To convert	Method	Example
percentage → decimal	divide by 100	$45\% = 45 \div 100 = 0.45$
decimal → percentage	multiply by 100	$0.6 = 0.6 \times 100 = 60\%$
percentage → fraction	write over 100, then simplify	$40\% = \frac{40}{100} = \frac{2}{5}$
fraction → percentage	multiply by 100 %	$\frac{3}{8} = \frac{3}{8} \times 100\% = 37.5\%$

To turn a fraction such as $\frac{3}{8}$ into a percentage on a scientific calculator, divide $3 \div 8 = 0.375$ and then multiply by 100 to get 37.5 %.

Finding a percentage of a quantity.

"p % **of** a quantity" means multiply the quantity by $\frac{p}{100}$:

$$p\% \text{ of } A = \frac{p}{100} \times A.$$

For example, 15 % of $\$80$ is $\frac{15}{100} \times 80 = 0.15 \times 80 = \$12$. Useful "benchmark" percentages are worth knowing: 10 % is found by dividing by 10, 25 % by dividing by 4, and 50 % by dividing by 2.

Expressing one quantity as a percentage of another.

To write one amount as a percentage of a total, form the fraction $\frac{\text{part}}{\text{whole}}$ and multiply by 100 %:

$$\frac{\text{part}}{\text{whole}} \times 100\%.$$

For example, if a student scored 18 out of 24, that is $\frac{18}{24} \times 100\% = 75\%$. Make sure both amounts are in the **same units** before dividing.

Percentage increase and decrease.

To **increase** an amount by $p\%$, you add $p\%$ of it; to **decrease** by $p\%$, you subtract $p\%$ of it. The quickest way is to multiply by a single number:

$$\text{increase by } p\%: \times\left(1+\frac{p}{100}\right), \text{decrease by } p\%: \times\left(1-\frac{p}{100}\right).$$

So a $\$50$ item increased by 20% becomes $50\times1.20=\$60$, and an $\$80$ item discounted by 25% becomes $80\times0.75=\$60$. This is how discounts, mark-ups, tips and GST are calculated. To find a **percentage change** when you know the old and new values, use

$$\text{percentage change}=\frac{\text{change}}{\text{original}}\times100\%.$$

Percentage points.

When a percentage itself goes up or down, be careful with language. If a savings rate rises from 3% to 5%, it has risen by 2 **percentage points** — that is the simple difference. Its *relative* rise, however, is $\frac{2}{3}\times100\%\approx67\%$. "Percentage points" describes the gap between two percentages; a "percentage change" describes that gap **relative to** the starting value. The two are different, and mixing them up is a common way statistics mislead.

Worked examples

Example 1 — scaffolded

A market researcher records the same result in several forms. (a) Write $\frac{3}{8}$ as a decimal and as a percentage. (b) Write 0.06 as a percentage. (c) Write 45% as a fraction in simplest form.

Working	Comment
(a) $3\div8=0.375$.	Divide the numerator by the denominator.
$0.375\times100=37.5\%$.	Multiply the decimal by 100 to get a percentage.
(b) $0.06\times100=6\%$.	Decimal → percentage: multiply by 100.
(c) $45\%=\frac{45}{100}$.	Percentage → fraction: write over 100.
$\frac{45}{100}=\frac{9}{20}$.	Simplify by dividing top and bottom by 5.

Example 2 — scaffolded

A netball club has 250 members. (a) 62% of the members are juniors. How many juniors are there? (b) Of the 250 members, 30 are volunteers. What percentage of the members are volunteers?

Working	Comment
(a) $62\%\text{ of }250=\frac{62}{100}\times250$.	"$p\%$ of" means multiply by $\frac{p}{100}$.
$=0.62\times250=155$ juniors.	A whole number, as it must be for people.
(b) $\frac{30}{250}\times100\%$.	Write the part over the whole, then $\times100\%$.
$=0.12\times100\%=12\%$.	30 volunteers is 12% of the club.

Example 3 — unscaffolded

A shop buys a bike for $\$180$ and marks the price up by 35% to set the retail price. In a later sale the retail price is discounted by 20%. Find (a) the retail price and (b) the sale price.

(a) Increasing by 35% means multiplying by $1+\frac{35}{100}=1.35$:

$$\text{retail price}=180\times1.35=\$243.$$

(b) Discounting by 20% means multiplying by $1-\frac{20}{100}=0.80$:

$$\text{sale price}=243\times0.80=\$194.40.$$

∴ the retail price is $\$243$ and the sale price is $\$194.40$.

Example 4 — unscaffolded

A bank raises the interest rate on a savings account from 3.5% to 4.2%. (a) By how many percentage points has the rate risen? (b) What is the percentage increase in the rate, relative to the original 3.5%?

(a) The rise in percentage points is the simple difference between the two rates:

$$4.2\%-3.5\%=0.7\text{ percentage points}.$$

(b) As a percentage change relative to the starting rate,

$$\frac{0.7}{3.5}\times100\%=20\%.$$

∴ the rate rose by 0.7 percentage points, which is a 20% increase on the original rate — the same change described in two different, correct ways.

Practice questions

Scaffolded

Q1. Convert between forms.

(a) Write $\frac{7}{20}$ as a percentage.

(b) Write 0.8 as a percentage.

(c) Write 15% as a fraction in simplest form.

Q2. A monthly phone bill is $\$90$.

(a) Find 10% of $\$90$.

(b) Find 25% of $\$90$.

(c) Find 5% of $\$90$.

Q3. In one game a basketball player attempted 40 shots and scored 28 of them.

(a) What percentage of her shots did she score?

(b) What percentage did she miss?

(c) In the next game she scored 24 out of 30 attempts. What percentage was this?

Q4. A jacket is priced at $\$140$.

(a) Increase $\$140$ by 10%.

(b) Decrease $\$140$ by 15%.

(c) Find the price if the $\$140$ jacket is reduced by 25%.

Q5. A tablet is priced at $\$560$. In a sale it is reduced by 30%.

(a) Find the discount in dollars.

(b) Find the sale price.

(c) A different store sells the same tablet with $25\,\%$ off its $\$560$ price. Find that store's price, and state which store is cheaper.

Q6. Answer each part.

(a) The price of a loaf of bread rises from $\$4.00$ to $\$4.50$. Find the increase in dollars.

(b) Express that increase as a percentage of the original $\$4.00$ price.

(c) A café raises its public-holiday surcharge from $8\,\%$ to $10\,\%$. By how many percentage points has it risen?

Unscaffolded

Q7. In a car park, $\frac{5}{8}$ of the 240 spaces are full. Find what percentage of the spaces are full, and how many spaces that is.

Q8. A worker earns $\$960$ per week and saves $15\,\%$ of it. How much does she save each week, and how much does she have left to spend?

Q9. A student scored 63 out of 84 on an exam. Express the mark as a percentage.

Q10. A $\$75$ pair of shoes is discounted by $20\,\%$. Find the sale price.

Q11. A café marks up the wholesale price of a $\$12$ bag of coffee by $45\,\%$ to set the retail price. Find the retail price.

Q12. A restaurant bill comes to $\$86.40$ and a customer adds a $10\,\%$ tip. Find the total amount paid.

Q13. A tradesperson quotes $\$340$ for a job before GST. GST of $10\,\%$ is then added. Find the GST amount and the total price including GST.

Q14. Over five years a town's population grew from 8500 to 9860. Find the percentage increase in the population.

Q15. A car bought for $\$28\,000$ is worth $\$21\,000$ three years later. Find the percentage decrease in its value.

Q16. The unemployment rate in a region fell from $6.5\,\%$ to $5.2\,\%$ over a year.

(a) By how many percentage points did it fall?

(b) Express the fall as a percentage of the original $6.5\,\%$ rate.

Q17. Two stores sell the same $\$250$ headphones. Store A offers $30\,\%$ off; Store B takes $\$80$ off. Find each store's price and state which is cheaper, and by how much.

Q18. A $\$150$ jacket has its price increased by $20\,\%$ for the new season, then in a later clearance the new price is reduced by $20\,\%$. Find the final price, and explain why it is not back to $\$150$.

Answers

Q1. (a) 35%. (b) 80%. (c) $\frac{3}{20}$. **Q2.** (a) $\$9$. (b) $\$22.50$. (c) $\$4.50$. **Q3.** (a) 70%. (b) 30%. (c) 80%. **Q4.** (a) $\$154$. (b) $\$119$. (c) $\$105$. **Q5.** (a) $\$168$. (b) $\$392$. (c) Store B's price is $\$420$, so Store A ($\$392$) is cheaper. **Q6.** (a) $\$0.50$. (b) 12.5%. (c) 2 percentage points. **Q7.** 62.5%, which is 150 spaces. **Q8.** Saves $\$144$; has $\$816$ left to spend. **Q9.** 75%. **Q10.** $\$60$. **Q11.** $\$17.40$. **Q12.** $\$95.04$. **Q13.** GST $\$34$; total $\$374$. **Q14.** 16%. **Q15.** 25%. **Q16.** (a) 1.3 percentage points. (b) 20%. **Q17.** Store A $\$175$, Store B $\$170$; Store B is cheaper by $\$5$. **Q18.** $\$144$; the 20% reduction is taken off the larger $\$180$ price, not the original $\$150$.

Fully-worked solutions

Q1. (a) $\frac{7}{20} = 7 \div 20 = 0.35 = 35\%$. (b) $0.8 \times 100 = 80\%$. (c) $15\% = \frac{15}{100} = \frac{3}{20}$ (dividing top and bottom by 5).

Q2. (a) 10% of $\$90 = \frac{10}{100} \times 90 = 90 \div 10 = \$9$. (b) 25% of $\$90 = \frac{1}{4} \times 90 = 90 \div 4 = \$22.50$. (c) 5% of $\$90 = \frac{5}{100} \times 90 = 0.05 \times 90 = \$4.50$.

Q3. (a) $\frac{28}{40} \times 100\% = 0.7 \times 100\% = 70\%$. (b) She missed $40 - 28 = 12$ shots, so $\frac{12}{40} \times 100\% = 30\%$ (or $100\% - 70\%$). (c) $\frac{24}{30} \times 100\% = 0.8 \times 100\% = 80\%$.

Q4. (a) $140 \times \left(1 + \frac{10}{100}\right) = 140 \times 1.10 = \$154$. (b) $140 \times \left(1 - \frac{15}{100}\right) = 140 \times 0.85 = \$119$. (c) $140 \times (1 - 0.25) = 140 \times 0.75 = \$105$.

Q5. (a) Discount $= 30\%$ of $\$560 = 0.30 \times 560 = \$168$. (b) Sale price $= 560 - 168 = \$392$ (or $560 \times 0.70 = \$392$). (c) The other store's price $= 560 \times (1 - 0.25) = 560 \times 0.75 = \$420$. Since $\$392 < \$420$, Store A (with 30% off) is cheaper.

Q6. (a) Increase $= 4.50 - 4.00 = \$0.50$. (b) $\frac{0.50}{4.00} \times 100\% = 0.125 \times 100\% = 12.5\%$. (c) The surcharge rose from 8% to 10%, a rise of $10 - 8 = 2$ percentage points.

Q7. $\frac{5}{8} = 5 \div 8 = 0.625 = 62.5\%$. The number of full spaces is $\frac{5}{8} \times 240 = 150$ spaces.

Q8. Saving $= 15\%$ of $\$960 = 0.15 \times 960 = \$144$. Left to spend $= 960 - 144 = \$816$.

Q9. $\frac{63}{84} \times 100\% = 0.75 \times 100\% = 75\%$.

Q10. Sale price $= 75 \times (1 - 0.20) = 75 \times 0.80 = \$60$.

Q11. Retail price $= 12 \times \left(1 + \frac{45}{100}\right) = 12 \times 1.45 = \$17.40$.

Q12. Total $= 86.40 \times (1 + 0.10) = 86.40 \times 1.10 = \$95.04$.

Q13. GST $= 10\%$ of $\$340 = 0.10 \times 340 = \$34$. Total $= 340 + 34 = \$374$ (or $340 \times 1.10 = \$374$).

Q14. Change $= 9860 - 8500 = 1360$. Percentage increase $= \frac{1360}{8500} \times 100\% = 0.16 \times 100\% = 16\%$.

Q15. Loss in value $= 28\,000 - 21\,000 = 7000$. Percentage decrease $= \dfrac{7000}{28\,000} \times 100\,\% = 0.25 \times 100\,\% = 25\,\%$.

Q16. (a) Fall $= 6.5 - 5.2 = 1.3$ percentage points. (b) As a percentage of the original rate: $\dfrac{1.3}{6.5} \times 100\,\% = 0.2 \times 100\,\% = 20\,\%$.

Q17. Store A: $250 \times (1 - 0.30) = 250 \times 0.70 = \$175$. Store B: $250 - 80 = \$170$. Since $\$170 < \$175$, Store B is cheaper, by $175 - 170 = \$5$.

Q18. New-season price $= 150 \times 1.20 = \$180$. Clearance price $= 180 \times (1 - 0.20) = 180 \times 0.80 = \$144$. It is not back to $\$150$ because the $20\,\%$ reduction is calculated on the larger $\$180$ price ($20\,\%$ of $\$180 = \$36$), which is a bigger reduction than the $\$30$ that was added, so the final price is lower than the original.

Theory

In everyday life you rarely meet a bare sum like “3×4” written out for you. Instead you meet a **situation described in words** — a shopping trip, a pay slip, a recipe — and you have to decide *which* calculation to do before you can do it. Choosing the right operation is just as important as being able to carry it out.

Matching the situation to an operation.

Certain words are clues to the operation, but the situation itself is the real guide.

Operation	Often signalled by	Typical situation
Addition +	total, sum, altogether, combined, in all	putting amounts together
Subtraction −	difference, how much more, change, left, remaining	taking away, comparing
Multiplication ×	each, per, of, times, at (a rate), groups of	repeated equal amounts
Division ÷	shared, split equally, per (unit), how many groups, average	breaking into equal parts

Be careful: the same word can point either way. “Per” signals **multiplication** in “$\$28$ per hour for 38 hours” (a rate times a quantity) but **division** in “the price *per* kilogram” (a total shared over a quantity). Always picture what is happening.

Translating a problem into a calculation.

A reliable routine:

1. Read the whole problem and decide **what is being asked**.
2. Pick out the numbers and note **what each one means**, with its units.
3. Decide the operation(s) from the **situation**, not just a keyword.
4. Write the calculation, using **brackets** to control the order.
5. Work it out — mentally, by hand, or with a scientific calculator.
6. **Check** that the answer is reasonable.

Multi-step calculations and order of operations.

Many real problems need more than one step. Use the order of operations — **b**rackets, then × and ÷, then + and − — so the steps happen in the right order. A scientific calculator applies this order automatically, so you can type an expression such as

$$3 \times 25 + 2 \times 18$$

in one line and it will do the two multiplications before the addition. Use **brackets** whenever you need an addition or subtraction to happen first, for example $\left(45 + 12\right) \times 3$.

Estimating to check — leading-digit approximation.

Before trusting a calculator answer, estimate what it should be. A quick method is **leading-digit (front-end) approximation**: round each number to its first digit — its largest place value — and calculate with the rounded numbers. For example,

$$312 + 489 \approx 300 + 500 = 800 .$$

If the calculator shows 801 the answer is reasonable; if it shows 80.1 or 8010 you have made a keying error and should try again.

Is the answer reasonable?

A final check asks whether the answer *makes sense*:

- Is it the right **size** — roughly the size your estimate predicted?
- Is the **sign** sensible (you cannot receive negative change)?
- Are the **units** right, and is the value **possible** in real life (you cannot have 2.5 people, and money is rounded to the nearest cent)?

Working efficiently with a calculator.

- Keep a multi-step calculation on **one line** with brackets: $(45.50+12.80)\times 3$.
- Use the **memory** keys (M+, M−, MR) or the **Ans** key to carry a result into the next step without retyping it.
- **Never round part-way through** a calculation — carry full precision and round only the final answer.

Worked examples

Example 1 — scaffolded

At a hardware store Priya buys 4 tins of paint at $\$38$ each and 3 paint rollers at $\$12$ each. She pays with a $\$200$ note. How much change should she receive?

Working	Comment
Paint: $4\times\$38=\$152$.	"each" → repeated amount → multiply.
Rollers: $3\times\$12=\$36$.	Same reasoning for the rollers.
Total spent $=\$152+\$36=\$188$.	"combined" → add the two costs.
Change $=\$200-\$188=\$12$.	"change" from a note → subtract.
Estimate check: $4\times 40+3\times 10=190$, so change $\approx 200-190=\$10$.	Leading-digit estimate; $\$12$ is close, so it is reasonable.

Example 2 — scaffolded

A 2 kg bag of rice costs $\$7.60$ and a 5 kg bag costs $\$17.50$. Which bag is better value, and by how much per kilogram?

Working	Comment
2 kg bag: $\$7.60\div 2=\$3.80$ per kg.	"per kilogram" → share the price over the mass → divide.
5 kg bag: $\$17.50\div 5=\$3.50$ per kg.	Same calculation for the larger bag.
$\$3.50<\$3.80$, so the 5 kg bag is cheaper per kilogram.	Compare the two unit prices.
Difference $=\$3.80-\$3.50=\$0.30$ per kg.	"by how much" → subtract.
Both prices are about $\$3$–$\$4$/kg, which is sensible for rice.	Reasonableness: right size and units.

Example 3 — unscaffolded

A muffin recipe for 12 muffins needs 300 g of flour, 2 eggs and 180 mL of milk. Jordan wants to make 30 muffins. How much of each ingredient is needed?

The recipe must be scaled up, so first find the **scale factor**:

$$\text{scale factor}=30\div 12=2.5.$$

Every ingredient is multiplied by 2.5:

$$\text{flour}=300\times 2.5=750\,\text{g},\ \text{eggs}=2\times 2.5=5,\ \text{milk}=180\times 2.5=450\,\text{mL}.$$

As a check, 30 muffins is about two-and-a-half times 12 muffins, so each amount should be a little more than double — 750 g, 5 eggs and 450 mL all fit.

∴ Jordan needs 750 g of flour, 5 eggs and 450 mL of milk.

Example 4 — unscaffolded

Five friends share a restaurant bill. The food comes to $\$184$ and the drinks to $\$56$. They have a voucher for $\$40$ off the total, and they split what remains equally. Write the whole calculation as a single expression, then find how much each person pays.

The total is reduced by the voucher **before** it is shared, so the addition and subtraction must happen first — this needs brackets:

$$(184+56-40)\div 5.$$

Working inside the brackets first,

$$184+56-40=200, 200\div 5=40.$$

A quick estimate, $(180+60-40)\div 5=200\div 5=\$40$, agrees.

∴ each person pays $\$40$.

Practice questions

Scaffolded

Q1. For each situation, state the operation (+, −, × or ÷) you would use, then find the answer. (a) A cinema sells 8 tickets at $\$19$ each. Find the total takings. (b) A $\$50$ gift card has $\$32.75$ left after a purchase. Find how much was spent. (c) 24 chocolates are shared equally among 6 children. Find how many each child gets.

Q2. At a market Leo buys 3 kg of apples at $\$4.50$/kg and 2 kg of grapes at $\$6.00$/kg. (a) Find the cost of the apples. (b) Find the cost of the grapes. (c) Leo pays with a $\$50$ note. Find his change.

Q3. A school orders 18 boxes of markers. Each box costs $\$23$. (a) Estimate the total cost using leading-digit approximation. (b) Find the exact total cost. (c) State whether your exact answer is reasonable compared with the estimate.

Q4. A 600 mL bottle of juice costs $\$3.60$ and a 1.5 L bottle costs $\$8.40$. (a) Find the price per litre of the 600 mL bottle. (Hint: 600 mL $=0.6$ L.) (b) Find the price per litre of the 1.5 L bottle. (c) State which bottle is better value.

Q5. A punch recipe for 8 people uses 2 L of lemonade and 500 mL of orange juice. (a) Find the scale factor needed to serve 20 people. (b) How much lemonade is needed for 20 people? (c) How much orange juice is needed for 20 people?

Q6. Four friends hire a boat. The hire fee is $\$180$ and fuel costs $\$36$. They also have a $\$16$ discount coupon, taken off the total. They split the final cost equally. (a) Write a single expression, using brackets, for the amount each friend pays. (b) Evaluate the expression. (c) Use a leading-digit estimate to check your answer is reasonable.

Unscaffolded

Q7. State whether you would add, subtract, multiply or divide, then answer. (a) A worker earns $\$28$ per hour and works 38 hours. Find the week's pay. (b) A 3.5 m length of timber has 1.2 m cut off. Find the length remaining. (c) A 4.5 kg bag of dog food lasts a dog 30 days. Find the amount eaten per day, in grams.

Q8. A café buys 6 crates of milk at $\$19.50$ each and 4 boxes of coffee beans at $\$34$ each. Find the total cost, and check it with a leading-digit estimate.

Q9. A recipe for 6 servings needs 450 g of pasta. Yuki wants to make only 4 servings. How much pasta is needed?

Q10. Three families share the cost of a holiday house. The house costs $\$1560$ for the week and cleaning is $\$140$. They split the total equally among the three families. Write a single expression, then find each family's share to the nearest cent.

Q11. A phone plan costs $\$45$ per month. (a) Find the total cost over 2 years. (b) A second plan costs $\$59$ per month but includes a phone. Over 2 years, how much more does the second plan cost than the first?

Q12. A 12 kg box of laundry powder does 240 washes and costs $\$54$. (a) Find the cost per wash, in cents. (b) Find the amount of powder used per wash, in grams.

Q13. Nadia estimates that 6×48 is about 300. (a) Show how she used leading-digit approximation. (b) Find the exact value. (c) By how much does the estimate differ from the exact value?

Q14. A tradesperson quotes $\$65$ per hour plus a $\$90$ call-out fee. A job takes 3 hours. (a) Write a single expression for the total charge. (b) Evaluate it. (c) Another tradesperson quotes a flat $\$350$ for the same job. Which is cheaper, and by how much?

Q15. A calculator shows 8.333333 when a student divides a $\$25.00$ restaurant bill equally among some friends. (a) How many friends shared the bill? (b) Rounded to the nearest cent, how much does each pay? (c) If each person hands over the rounded amount, will the total collected cover the bill? Explain.

Q16. Petrol costs $\$1.85$ per litre. Sam buys 42 litres. (a) Estimate the cost using leading-digit approximation. (b) Find the exact cost. (c) Is the exact answer reasonable? Explain by comparing it with your estimate.

Q17. A worker's normal rate is $\$32$ per hour. Overtime is paid at 1.5 times the normal rate. In one week she works 38 normal hours and 6 overtime hours. (a) Find the overtime rate. (b) Write a single expression for the week's total pay. (c) Evaluate it.

Q18. A community group buys 150 sausages at $\$0.80$ each, 150 bread rolls at $\$0.45$ each, and 12 bottles of sauce at $\$3.20$ each for a fundraiser barbecue. (a) Find the total cost of the supplies. (b) They sell each sausage in a roll for $\$3.50$. If they sell all 150, find the total takings. (c) Find the profit (takings $-$ costs).

Answers

Q1. (a) ×, $\$152$. (b) −, $\$17.25$. (c) ÷, 4 each. **Q2.** (a) $\$13.50$. (b) $\$12.00$. (c) $\$24.50$. **Q3.** (a) $\approx 20\times 20=\$400$. (b) $\$414$. (c) reasonable — $\$414$ is close to the $\$400$ estimate. **Q4.** (a) $\$6.00$/L. (b) $\$5.60$/L. (c) the 1.5 L bottle. **Q5.** (a) 2.5. (b) 5 L. (c) 1250 mL (1.25 L). **Q6.** (a) $(180+36-16)\div 4$. (b) $\$50$. (c) estimate $(200+40-20)\div 4=\$55$, close to $\$50$ — reasonable. **Q7.** (a) ×, $\$1064$. (b) −, 2.3 m. (c) ÷, 150 g/day. **Q8.** $\$253$; estimate $6\times 20+4\times 30=\$240$, close — reasonable. **Q9.** 300 g. **Q10.** $(1560+140)\div 3=\$566.67$. **Q11.** (a) $\$1080$. (b) $\$336$ more. **Q12.** (a) 22.5 cents per wash. (b) 50 g per wash. **Q13.** (a) $48\to 50$, so $6\times 50=300$. (b) 288 . (c) by 12. **Q14.** (a) $90+65\times 3$. (b) $\$285$. (c) the tradesperson is cheaper, by $\$65$. **Q15.** (a) 3 friends. (b) $\$8.33$. (c) no — $3\times\$8.33=\$24.99$, which is 1 cent short. **Q16.** (a) $\approx 2\times 40=\$80$. (b) $\$77.70$. (c) reasonable — close to the $\$80$ estimate. **Q17.** (a) $\$48$/h. (b) $32\times 38+48\times 6$. (c) $\$1504$. **Q18.** (a) $\$225.90$. (b) $\$525$. (c) $\$299.10$ profit.

Fully-worked solutions

Q1. (a) "each" signals equal groups, so **multiply**: $8\times\$19=\$152$. (b) The amount spent is what is missing from the card, so **subtract**: $\$50-\$32.75=\$17.25$. (c) Sharing equally signals **division**: $24\div 6=4$ chocolates each.

Q2. (a) 3 kg at $\$4.50$/kg: $3\times\$4.50=\$13.50$. (b) 2 kg at $\$6.00$/kg: $2\times\$6.00=\$12.00$. (c) Total spent $=\$13.50+\$12.00=\$25.50$, so change $=\$50-\$25.50=\$24.50$.

Q3. (a) Round each number to its leading digit: $18\to 20$ and $23\to 20$, giving $20\times 20=\$400$. (b) Exactly, $18\times 23=\$414$. (c) $\$414$ is very close to the $\$400$ estimate, so the exact answer is reasonable (no keying error).

Q4. (a) 600 mL $=0.6$ L, so the unit price is $\$3.60\div 0.6=\$6.00$ per litre. (b) $\$8.40\div 1.5=\$5.60$ per litre. (c) $\$5.60<\$6.00$, so the 1.5 L bottle is better value.

Q5. (a) Scale factor $=20\div 8=2.5$. (b) Lemonade $=2\times 2.5=5$ L. (c) Orange juice $=500\times 2.5=1250$ mL, i.e. 1.25 L.

Q6. (a) The coupon comes off the total before it is split, so the addition and subtraction must be done first: $(180+36-16)\div 4$. (b) Inside the brackets, $180+36-16=200$, then $200\div 4=\$50$. (c) Estimate: $(200+40-20)\div 4=220\div 4=\$55$, which is close to $\$50$, so the answer is reasonable.

Q7. (a) A rate for many hours signals **multiply**: $\$28\times 38=\$1064$. (b) Cutting a piece off signals **subtract**: $3.5-1.2=2.3$ m. (c) Sharing the food over the days signals **divide**; working in grams, 4.5 kg $=4500$ g, so $4500\div 30=150$ g per day.

Q8. Milk: $6\times\$19.50=\$117$. Coffee: $4\times\$34=\$136$. Total $=\$117+\$136=\$253$. Leading-digit estimate: $6\times 20+4\times 30=120+120=\$240$, which is close to $\$253$, so the total is reasonable.

Q9. Per serving the recipe needs $450\div 6=75$ g of pasta, so 4 servings need $75\times 4=300$ g. (Equivalently, $450\times 4/6=300$ g.)

Q10. The cleaning is added to the house cost before splitting, so brackets are needed: $(1560+140)\div 3$. Then $1700\div 3=566.666\ldots$, which rounds to $\$566.67$ per family.

Q11. (a) 2 years $=24$ months, so $\$45\times 24=\$1080$. (b) The second plan costs $\$59\times 24=\$1416$ over 2 years, which is $\$1416-\$1080=\$336$ more than the first.

Q12. (a) $\$54\div 240=\$0.225$ per wash, which is 22.5 cents per wash. (b) 12 kg $=12\,000$ g, so $12\,000\div 240=50$ g of powder per wash.

Q13. (a) Rounding to the leading digit, $48\to 50$, so $6\times 50=300$. (b) Exactly, $6\times 48=288$. (c) The estimate exceeds the exact value by $300-288=12$.

Q14. (a) The call-out fee is added to the hourly charge, so $90+65\times 3$ (the calculator does 65×3 first). (b) $65\times 3=195$, then $90+195=\$285$. (c) $\$285<\$350$, so the first tradesperson is cheaper by $\$350-\$285=\$65$.

Q15. (a) Since $\$25 \div 3 = 8.3333\ldots$, the display of 8.333333 means the bill was split among 3 friends. (b) To the nearest cent each pays $\$8.33$. (c) $3 \times \$8.33 = \$24.99$, which is 1 cent less than $\$25.00$. So the rounded amounts do **not** quite cover the bill — one person would need to add an extra cent (or someone rounds up to $\$8.34$).

Q16. (a) Round each number to its leading digit: $\$1.85 \rightarrow \$2$ and $42 \rightarrow 40$, giving $2 \times 40 = \$80$. (b) Exactly, $\$1.85 \times 42 = \$77.70$. (c) $\$77.70$ is close to the $\$80$ estimate, so the exact answer is reasonable.

Q17. (a) Overtime rate $= \$32 \times 1.5 = \$48$ per hour. (b) Normal pay plus overtime pay: $32 \times 38 + 48 \times 6$. (c) $32 \times 38 = 1216$ and $48 \times 6 = 288$, so the total pay is $1216 + 288 = \$1504$.

Q18. (a) Sausages: $150 \times \$0.80 = \$120$. Rolls: $150 \times \$0.45 = \$67.50$. Sauce: $12 \times \$3.20 = \$38.40$. Total supplies $= 120 + 67.50 + 38.40 = \$225.90$. (b) Takings $= 150 \times \$3.50 = \$525$. (c) Profit $= \$525 - \$225.90 = \$299.10$.