Foundation Mathematics Units 1 & 2

Chapter 1 Working mathematically and investigations

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Contents

Theory

Most of the mathematics you meet in Foundation Mathematics is not a bare sum on a page — it is buried inside a real situation. A pay slip, a recipe, a fuel gauge, a quote for tiling a bathroom: each one hides a calculation. **Working mathematically** means pulling that calculation out of the situation, doing it accurately, and then checking that the answer makes sense. This section sets out the toolkit you will use in every chapter that follows.

Reading a problem and extracting the mathematics.

Before touching a calculator, read the problem twice and ask three questions:

- *What am I being asked to find?* — the quantity you need, and its units.
- *What information am I given?* — the numbers that matter (and any that don't).
- *What has to happen to those numbers?* — which operation or operations turn the given information into the answer.

Answering these turns a wordy situation into a clear calculation. This is the single most important skill in the subject.

Choosing the operation.

The words in a problem usually signal which operation is needed.

Situation	Operation	Everyday words
Combining amounts	Addition +	total, altogether, sum, combined
Taking away, comparing	Subtraction −	difference, how much more, left over, change
Repeated equal groups	Multiplication ×	each, per, groups of, times, at $… each
Sharing or grouping equally	Division ÷	share, split, per, how many fit, average

Many real problems need **more than one** operation, done in the right order — for example "multiply the hours by the rate, then add the allowance".

Units matter.

Every quantity in a real problem carries a **unit** — dollars, kilograms, litres, minutes, metres. Keep track of units all the way through, make sure the numbers you combine are in the **same** unit, and state the unit in your answer. "1500" and "1500 g" are not the same answer; the second one is complete. If a problem mixes units (grams and kilograms, minutes and hours), convert first.

Estimation and rounding.

An **estimate** is a quick, rough answer found by replacing the numbers with easy ones — usually by **rounding** to the nearest ten, hundred or dollar, or by keeping only the **leading digit** (the first significant figure). For example $58 \approx 60$ and $39 \approx 40$, so $39 \times 58 \approx 40 \times 60 = 2400$. An estimate is not the final answer; it is a *check*. You work it out **before** or **after** the exact calculation and compare the two.

Is the answer reasonable?

The last step of every problem is a sanity check. Ask:

- Is the exact answer **close to my estimate**? If they are wildly different, something went wrong.
- Is it the **right size** for the situation? A week's groceries costing $6 or $6000 is a warning sign; $63.65 is believable.
- Are the **units** sensible, and did I answer the actual question?

Calculators do exactly what they are told, so a slipped decimal point or a mis-keyed operation gives a confident but wrong number. Your estimate and your common sense are what catch it.

Using a scientific calculator well.

A scientific calculator follows the **order of operations** (brackets, then × and ÷, then + and −), not simply left to right. So `12 + 6 × 4` gives 36, not 72, because the multiplication is done first. To force a different order, use **brackets**: `(12 + 6) × 4` gives 72. Three habits prevent most errors:

- Put **brackets** around any sum that must be worked out before a multiplication or division.
- Key a whole multi-step calculation in **one entry** where you can, using the `ANS` or memory keys, rather than copying rounded values back in.
- **Don't round part-way** through — round only the final answer, and only when asked.

Worked examples

Example 1 — scaffolded

A mobile mechanic charges a call-out fee of \$65 plus \$95 for each hour of labour. A repair takes 3 hours. Find the total charge.

Working	Comment
Find: total charge (in dollars). Given: \$65 fixed fee, \$95 per hour, 3 hours.	Read the problem: identify what is asked and what is given.
The labour is a repeated amount, so **multiply**: $95 \times 3 = 285$.	"For each hour" signals multiplication.
The call-out fee is added on once, so **add**: $285 + 65 = 350$.	Two operations, in order: multiply, then add.
The total charge is \$350.	State the answer with its unit (dollars).

Example 2 — scaffolded

A shopper buys five items costing \$18.90, \$6.20, \$11.50, \$4.75 and \$22.30. Estimate the total by rounding each price to the nearest dollar, then find the exact total and check that it is reasonable.

Working	Comment
Round each price: \$19, \$6, \$12, \$5, \$22.	Round to the nearest whole dollar for a quick estimate.
Estimated total $= 19 + 6 + 12 + 5 + 22 = 64$, so about \$64.	Add the easy rounded numbers.
Exact total $= 18.90 + 6.20 + 11.50 + 4.75 + 22.30 = 63.65$.	Now the exact calculation, keeping the cents.
\$63.65 is very close to the \$64 estimate, so the answer is reasonable.	Compare exact with estimate — the check.

Example 3 — unscaffolded

A garden bed is 8 m long and 1.5 m wide, and is to be covered with mulch spread 0.1 m deep. Mulch is sold by the cubic metre. Estimate the volume needed, then calculate it, and decide whether ordering 1 cubic metre is enough.

The three measurements are multiplied to give a volume, so first estimate: $8 \times 1.5 \approx 12$ m² of surface, and $12 \times 0.1 = 1.2$ m³. Now the exact calculation:

$$V = 8 \times 1.5 \times 0.1 = 1.2\,\mathrm{m}^3.$$

The estimate and the exact value agree. Since $1.2 > 1$, one cubic metre would fall short.

∴ the bed needs 1.2 m³ of mulch, so ordering only 1 m³ is **not** enough.

Example 4 — unscaffolded

In a day a café sells 32 coffees at \$4.50 each and 18 muffins at \$3.80 each, and pays \$95 for supplies. Find the café's takings after supplies, entering the calculation in one line, and check the answer is reasonable.

Each product is a repeated amount (multiply), the two are combined (add), and the supplies are taken off (subtract). As a single calculator entry:

$$(32 \times 4.50) + (18 \times 3.80) - 95 = 144 + 68.40 - 95 = 117.40.$$

Checking by estimation: coffees $\approx 30 \times 4.50 = 135$, muffins $\approx 18 \times 4 = 72$, giving about $135 + 72 - 95 = 112$ — close to \$117.40.

∴ the café's takings after supplies are \$117.40, and the estimate confirms this is reasonable.

Practice questions

Scaffolded

Q1. A school orders 6 basketballs at \$35 each and 4 ball pumps at \$18 each. (a) State which operations are needed to find the total cost. (b) Find the cost of the basketballs and the cost of the pumps. (c) Find the total cost of the order.

Q2. A family's four weekly grocery bills were \$142.80, \$158.10, \$137.45 and \$164.90. (a) Round each bill to the nearest \$10. (b) Use your rounded values to estimate the total. (c) Find the exact total and state whether your estimate was reasonable.

Q3. A recipe uses 250 g of sugar for one batch of biscuits. A baker makes 6 batches. (a) State the operation needed and the unit of the answer. (b) Calculate the total mass of sugar, in grams. (c) Convert your answer to kilograms.

Q4. Using the correct order of operations, evaluate: (a) $12 + 6 \times 4$ (b) $(12 + 6) \times 4$ (c) $40 - 24 \div 8$

Q5. A student wants 3 concert tickets at \$28.50 each. Her calculator display shows \$8.55. (a) Estimate the total cost by rounding \$28.50 to the nearest \$10. (b) Explain why the calculator's \$8.55 is not reasonable. (c) Find the correct total cost.

Q6. A tradesperson has a 30 m roll of cable and uses 4.5 m on each of 5 jobs. (a) How much cable is used in total? (b) How much cable is left on the roll? (c) Is there enough left for one more 4.5 m job? Explain.

Unscaffolded

Q7. A plumber charges an \$80 call-out fee plus \$110 per hour. Find the cost of a job that takes 2.5 hours.

Q8. Estimate, by rounding each number to its leading digit, the cost of 39 chairs at \$58 each. Then find the exact cost and compare it with your estimate.

Q9. A water tank holds 4500 L, and 320 L is drawn off each day. Estimate how many days the water will last, then calculate it to the nearest whole day.

Q10. Using the correct order of operations, evaluate: (a) $50 - 3 \times 8$ (b) $(15 - 7) \times (4 + 2)$ (c) $100 \div 4 + 6 \times 3$

Q11. A recipe for 8 people needs 1.2 kg of potatoes. Extract the mathematics and find how many kilograms are needed for 20 people.

Q12. A phone plan costs \$45 per month. Estimate the yearly cost, then find the exact yearly cost, stating the units of your answer.

Q13. A carpenter needs 6 shelves, each 0.85 m long, cut from a single 6 m board. Show a calculation to decide whether the board is long enough, and check the result is reasonable.

Q14. A student keys `18 + 24 ÷ 6` into a scientific calculator, expecting the answer 7, but the display shows 22. Using the order of operations, explain why the calculator is correct and write the calculation the student meant to enter.

Q15. A café sells 45 coffees at $4.20 each and 28 teas at $3.50 each in a morning. Estimate the total takings first, then find the exact total.

Q16. The distance by road from a town to the city is 138 km, and a car uses about 7.5 L of fuel for every 100 km. Estimate, then calculate, the fuel used for the trip, to the nearest litre.

Q17. A pay slip shows 38 hours worked at $27.40 per hour, plus a $65 tool allowance. Using a single calculator entry with brackets, calculate the gross pay, and use estimation to check the answer is reasonable.

Q18. A hall floor measuring 12.5 m by 8 m is to be coated with sealant. One tin of sealant covers 25 m^2. Find the floor area, work out how many tins are needed, and state clearly how many tins must be bought (you cannot buy part of a tin).

Answers

Q1. (a) multiplication, then addition. (b) basketballs \$210, pumps \$72. (c) \$282. **Q2.** (a) \$140, \$160, \$140, \$160. (b) \$600. (c) \$603.25; the estimate \$600 is close, so it is reasonable. **Q3.** (a) multiplication; grams. (b) 1500 g. (c) 1.5 kg. **Q4.** (a) 36. (b) 72. (c) 37. **Q5.** (a) about \$90. (b) \$8.55 is far below the \$90 estimate and less than the price of a single ticket, so it cannot be right (a likely keystroke or decimal error). (c) \$85.50. **Q6.** (a) 22.5 m. (b) 7.5 m. (c) Yes — 7.5 m is more than 4.5 m. **Q7.** \$355. **Q8.** estimate $40 \times 60 = \$2400$; exact $39 \times 58 = \$2262$; the estimate is close, confirming the exact value is reasonable. **Q9.** estimate $4500 \div 300 = 15$ days; exact $4500 \div 320 = 14.06\ldots$, so 14 whole days. **Q10.** (a) 26. (b) 48. (c) 43. **Q11.** 3 kg. **Q12.** estimate $50 \times 12 = \$600$; exact $45 \times 12 = \$540$ per year. **Q13.** $6 \times 0.85 = 5.1$ m needed; 5.1 m ≤ 6 m, so the board is long enough. **Q14.** The calculator does the division first ($24 \div 6 = 4$) then adds 18, giving 22; the student meant $(18 + 24) \div 6 = 7$. **Q15.** estimate about \$280; exact total \$287. **Q16.** estimate about 10.5 L; exact 10.35 L, so about 10 L. **Q17.** \$1106.20 (estimate about \$1145, so reasonable). **Q18.** area 100 m²; $100 \div 25 = 4$; buy 4 tins.

Fully-worked solutions

Q1. (a) Each item is a repeated cost (multiply), and the two amounts are combined (add), so the operations are **multiplication then addition**. (b) Basketballs: $6 \times 35 = 210$, so \$210. Pumps: $4 \times 18 = 72$, so \$72. (c) Total $= 210 + 72 = 282$, so \$282.

Q2. (a) To the nearest \$10: \$142.80 → \$140, \$158.10 → \$160, \$137.45 → \$140, \$164.90 → \$160. (b) Estimate $= 140 + 160 + 140 + 160 = 600$, about \$600. (c) Exact $= 142.80 + 158.10 + 137.45 + 164.90 = 603.25$, so \$603.25. This is very close to the \$600 estimate, so it is reasonable.

Q3. (a) Six equal batches means **multiplication**; the sugar is measured in **grams**. (b) $250 \times 6 = 1500$, so 1500 g. (c) $1500 \div 1000 = 1.5$, so 1.5 kg.

Q4. (a) Multiplication first: $6 \times 4 = 24$, then $12 + 24 = 36$. (b) Brackets first: $12 + 6 = 18$, then $18 \times 4 = 72$. (c) Division first: $24 \div 8 = 3$, then $40 - 3 = 37$.

Q5. (a) Round \$28.50 to \$30: $30 \times 3 = 90$, about \$90. (b) The display \$8.55 is far smaller than the \$90 estimate — it is even less than one ticket — so it cannot be the cost of three tickets; the numbers were likely mis-keyed. (c) $28.50 \times 3 = 85.50$, so the correct total is \$85.50.

Q6. (a) $4.5 \times 5 = 22.5$, so 22.5 m is used. (b) $30 - 22.5 = 7.5$, so 7.5 m is left. (c) One more job needs 4.5 m, and $7.5 \geq 4.5$, so **yes**, there is enough.

Q7. The hourly labour is $110 \times 2.5 = 275$, and the call-out fee is added once: $275 + 80 = 355$. The job costs \$355.

Q8. Leading-digit estimate: $39 \approx 40$ and $58 \approx 60$, so $40 \times 60 = 2400$, about \$2400. Exact: $39 \times 58 = 2262$, so \$2262. The exact cost is close to the estimate, confirming it is reasonable.

Q9. Estimate by rounding 320 to 300: $4500 \div 300 = 15$ days. Exact: $4500 \div 320 = 14.0625$. The water runs out during the 15th day, so it lasts 14 **whole** days.

Q10. (a) $3 \times 8 = 24$ first, then $50 - 24 = 26$. (b) Each bracket first: $15 - 7 = 8$ and $4 + 2 = 6$, then $8 \times 6 = 48$. (c) × and ÷ before +: $100 \div 4 = 25$ and $6 \times 3 = 18$, then $25 + 18 = 43$.

Q11. First find the amount per person: $1.2 \div 8 = 0.15$ kg each. For 20 people, $0.15 \times 20 = 3$, so 3 kg of potatoes are needed.

Q12. There are 12 months in a year. Estimate: $50 \times 12 = 600$, about \$600. Exact: $45 \times 12 = 540$, so \$540 per year.

Q13. The six shelves need $6 \times 0.85 = 5.1$ m of board in total. Since 5.1 m is less than the 6 m board, the board is long enough (with 0.9 m to spare). This is reasonable, as each shelf is under a metre and six of them is close to but under six metres.

Q14. A scientific calculator applies the order of operations, doing division before addition: $24 \div 6 = 4$, then $18 + 4 = 22$ — so the display of 22 is correct. The student intended to add first and then divide, which requires brackets: $(18 + 24) \div 6 = 42 \div 6 = 7$.

Q15. Estimate: coffees $\approx 45 \times 4 = 180$ and teas $\approx 28 \times 3.50 = 98$, giving about \$280. Exact: $45 \times 4.20 = 189$ and $28 \times 3.50 = 98$, so the total is $189 + 98 = 287$, i.e. \$287 — close to the estimate.

Q16. The trip is $138 \div 100 = 1.38$ lots of 100 km. Estimate: about $1.4 \times 7.5 = 10.5$ L. Exact: $1.38 \times 7.5 = 10.35$ L, which rounds to about 10 L of fuel.

Q17. Gross pay $= (38 \times 27.40) + 65$. The hours give $38 \times 27.40 = 1041.20$, then $1041.20 + 65 = 1106.20$, so \$1106.20. Estimate: $40 \times 27 = 1080$, plus \$65 is about \$1145 — close to the exact value, so it is reasonable.

Q18. Floor area $= 12.5 \times 8 = 100$ m². Number of tins $= 100 \div 25 = 4$ exactly. Since this works out to a whole number, exactly 4 tins are needed, so **buy 4 tins**.

Chapter 1 Working mathematically and investigations — 1D The problem-solving cycle

Theory

Most of the mathematics you use in everyday life does not arrive as a neat sum with an equals sign already waiting. It arrives as a **situation**: *Can I afford this day out? How much paint do I need? Which option is cheaper?* **Problem solving** (also called **mathematical modelling**) is the skill of turning a real, messy situation into a mathematical question, answering that question, and then explaining what the answer means back in the real situation.

The three-stage cycle.

Every investigation in this course follows the same three stages. You **formulate**, you **explore**, and you **communicate**.

revise if the result is not reasonable

Formulate.

This is the *setting-up* stage. You read the situation carefully and decide:

- What exactly is being asked? Turn the situation into a clear mathematical question (this is called **mathematisation**).
- What information do I have, and what do I still need? You may have to **estimate** or look up a missing value.
- What **assumptions** am I making? An assumption is something you take to be true so the problem can be solved — for example “all tickets are the adult price” or “the car uses fuel at a constant rate”. Good problem solvers always **write their assumptions down**, because the answer is only trustworthy if the assumptions are reasonable.
- Which operations or method will I use — adding costs, dividing to share, multiplying a rate?

Explore.

This is the *doing* stage. You carry out the calculations, using a scientific calculator where it helps, and organise your work with **tables, diagrams or sketches** so it is easy to follow. As you go, keep asking whether each result is **reasonable** — a quick estimate (“about $300”) tells you at once if a calculator answer of $34.50 must be a mistake.

Communicate.

This is the *reporting* stage. A bare number is not an answer. You must:

- **interpret** the result in the context — what does it *mean*?
- state the answer **with its units** and to a sensible accuracy;
- decide the **practical** answer. If 8.3 buses are needed you must round **up** to 9, because you cannot hire part of a bus;
- present your findings clearly so someone else can understand them.

It is a cycle, not a straight line.

If the result is not reasonable — the paint runs out, the budget is blown, an assumption turns out to be wrong — you go back, adjust the model, and work through the stages again. That is why the three stages are drawn as a loop.

A worked model.

Maya has $50 and plans a museum visit: return train $9.20, entry $15, lunch $12. **Formulate:** the question is "what is the total cost, and is $50 enough?"; assume these are the only costs. **Explore:** total $= 9.20 + 15 + 12 = \$36.20$. **Communicate:** $36.20 is less than $50, so Maya can afford the day and has $50 - 36.20 = \$13.80$ left over.

Worked examples

Example 1 — scaffolded

Maya plans a day trip into the city. Her costs are a return train ticket $8.40, gallery entry $18, lunch $14.50 and an afternoon snack $6. She has $60 to spend. Find the total cost of the day, and how much she will have left.

Working	Comment
Question: total cost = sum of the four amounts; then compare with $60.	*Formulate* — decide what to calculate.
Assume these four items are her only spending.	State the assumption.
$8.40 + 18 + 14.50 + 6 = 46.90$	*Explore* — add the costs.
Total cost $= \$46.90$. Since $46.90 < 60$, she can afford it.	*Communicate* — interpret with units.
Left over $= 60 - 46.90 = \$13.10$.	Answer the second part.

Example 2 — scaffolded

A painter needs to give a wall 4 m long and 2.5 m high **two** coats of paint. One litre of paint covers $11\,\mathrm{m}^2$, and paint is sold only in whole litres at $18 per litre. Find how many litres must be bought and the cost.

Working	Comment
Question: area to cover, then litres, then cost. Assume no doors or windows and no wastage.	*Formulate* — plan and state assumptions.
Wall area $= 4 \times 2.5 = 10\,\mathrm{m}^2$.	*Explore* — area of a rectangle.
Two coats cover $2 \times 10 = 20\,\mathrm{m}^2$.	Both coats need paint.
Litres needed $= 20 \div 11 = 1.82\,\mathrm{L}$ (to 2 d.p.).	Divide area by the coverage rate.
Paint is sold in whole litres, so **round up** to 2 litres.	*Communicate* — the practical answer.
Cost $= 2 \times \$18 = \$36$.	State the cost with units.

Example 3 — unscaffolded

A family of two adults and one child plans a day at the zoo. Adult entry is $32 each, child entry is $18, parking is $10, and they expect to spend about $25 on food. They budget $120 for the day. Decide whether $120 will be enough.

Formulate: the question is whether the total cost is within $120; assume the food estimate is accurate and there are no other costs. **Explore:** the total cost is

$$2 \times 32 + 18 + 10 + 25 = 64 + 18 + 10 + 25 = \$117.$$

Communicate: $117 is less than the $120 budget, with $120 - 117 = \$3$ to spare.

∴ yes, $120 is enough — but only just, with $3 left over, so any extra spending on food or souvenirs would put them over budget.

Example 4 — unscaffolded

To get to a concert Jordan compares two options. **Driving:** the venue is 46 km away, the car uses 8 litres of petrol per 100 km, petrol costs $1.80 per litre, and parking is $22. **Train:** a return ticket is $13.60 plus a $6 bus connection at each end. Assuming petrol is used at a constant rate, find the cost of each option and which is cheaper.

Formulate: for driving, cost = fuel + parking; fuel cost = distance × rate × price. For the train, cost = ticket + two bus fares. **Explore:** the return distance is $2\times 46=92$ km, so the fuel used is

$$92\times\frac{8}{100}=7.36\,\text{L}\,,\,\text{fuel cost}=7.36\times 1.80=\$13.25\,.$$

Driving total $=13.25+22=\$35.25$. Train total $=13.60+2\times 6=\$25.60$.

Communicate: the train costs $25.60 and driving costs $35.25, so the train is cheaper by $35.25-25.60=\$9.65$.

∴ Jordan should take the **train**, saving $9.65 (and avoiding the stress of parking).

Practice questions

Scaffolded

Q1. For a night out, Priya's costs are a bus fare $5.60, a cinema ticket $21, popcorn $9.50 and a drink $5. (a) Write down the calculation for the total cost. (b) Find the total cost. (c) If Priya has $45, how much will she have left?

Q2. A decorator will stain identical timber fence panels, each 3 m long and 1.8 m high, with **one** coat. The stain covers $8\,\text{m}^2$ per litre and is sold in whole litres at $24 per litre. (a) Find the area of one panel. (b) There are 6 panels. Find how many litres must be bought (rounding **up** to whole litres). (c) Find the total cost of the stain.

Q3. A cinema listing says a 90-minute film starts at $6:45$ pm. (a) A student says it will finish at $8:15$ pm. State the assumption behind this. (b) In fact there are 15 minutes of advertisements first. What time does the film actually finish? (c) If it then takes 20 minutes to travel home, what time do they arrive home?

Q4. A minibus seats 12 passengers. A school needs to transport 100 students on an excursion. (a) Write down the division that gives the number of minibuses. (b) Work it out. (c) How many minibuses are needed? Explain why you round the way you do.

Q5. A shopper buys six items priced $4.95, $3.20, $7.80, $2.15, $5.50 and $6.35. (a) Estimate the total by rounding each price to the nearest dollar. (b) Find the exact total. (c) Is the estimate close to the exact total? By how much do they differ?

Q6. Two portable chargers are compared. Brand A costs $39 and lasts 18 months; Brand B costs $55 and lasts 30 months. (a) Find the cost per month for Brand A, to the nearest cent. (b) Find the cost per month for Brand B, to the nearest cent. (c) Which brand is better value per month?

Unscaffolded

Q7. Four friends plan a beach day. The costs are a return train ticket of $11.20 for **each** friend, sunscreen $13 shared, a picnic $48 shared, and $6 each for ice cream. Find the total cost for the group and the cost per person.

Q8. A driver makes a 260 km trip. The car uses 7 litres of petrol per 100 km and petrol costs $1.85 per litre. Assuming a constant fuel-use rate, find the fuel cost for the trip, to the nearest cent.

Q9. A function is set up with tables that each seat 8 guests. There are 90 guests. Find how many tables are needed, and, if each table costs $15 to hire, the total hire cost.

Q10. A tradesperson estimates a job will cost about $600. The actual costs are materials $342, labour $210 and a $35 call-out fee. Find the actual total, and state whether the estimate was too high or too low and by how much.

Q11. To move house, option A is a truck hire costing \$89 per day plus \$0.40 per km for a 60 km trip; option B is a removalist with a flat fee of \$145. Find the cost of each option and state which is cheaper and by how much.

Q12. A caterer has 150 sausages to pack into trays of 8. (a) How many **full** trays can be filled? (b) How many sausages are left over?

Q13. A train departs at 9:50 am and the journey takes 2 hours 35 minutes. Assuming it runs on time, at what time does it arrive? If the passenger then has a 25-minute walk, at what time do they reach their destination?

Q14. A ceiling measuring 8 m by 5 m needs **two** coats of paint. Paint covers $12\,\mathrm{m}^2$ per litre and is sold only in 4-litre tins at \$52 each. Find how many tins must be bought and the total cost.

Q15. A recipe that serves 4 people uses 600 g of rice and 500 mL of stock. A cook needs to serve 10 people. Assuming the recipe scales in proportion, find how much rice and stock are needed.

Q16. A student calculates that 28 people each paying \$19 for a ticket raises \$53.20 in total. Using estimation, explain why this answer must be wrong, and find the correct total.

Q17. A class of 25 students plans an excursion. The costs are bus hire \$340, museum entry \$12 per student, and a \$150 workshop fee. (a) Find the total cost. (b) Find the cost per student if it is shared equally, to the nearest cent. (c) If each student is asked to pay \$30, is that enough to cover the excursion?

Q18. Two campsites are compared for a 3-night stay. Campsite P charges \$45 per night plus a one-off \$30 booking fee; Campsite Q charges \$54 per night with no booking fee. Find the total cost of each for 3 nights and recommend the cheaper campsite.

Answers

Q1. (a) $5.60+21+9.50+5$. (b) \$41.10. (c) \$3.90. **Q2.** (a) $5.4\ \mathrm{m}^2$. (b) 5 litres. (c) \$120. **Q3.** (a) that the film starts exactly at $6:45$ pm with no advertisements first. (b) $8:30$ pm. (c) $8:50$ pm. **Q4.** (a) $100 \div 12$. (b) 8.33 (to 2 d.p.). (c) 9 minibuses — round **up**, because 8 buses seat only 96 students so a 9th is needed for the rest. **Q5.** (a) \$30. (b) \$29.95. (c) yes; they differ by only \$0.05. **Q6.** (a) \$2.17. (b) \$1.83. (c) Brand B. **Q7.** total \$129.80; per person \$32.45. **Q8.** \$33.67. **Q9.** 12 tables; \$180. **Q10.** actual \$587; the estimate was too **high** by \$13. **Q11.** A \$113, B \$145; option A is cheaper by \$32. **Q12.** (a) 18 full trays. (b) 6 left over. **Q13.** arrives $12:25$ pm; reaches destination $12:50$ pm. **Q14.** 2 tins; \$104. **Q15.** 1500 g rice and 1250 mL stock. **Q16.** an estimate of $28 \times 20 = \$560$ shows \$53.20 is far too small; correct total \$532. **Q17.** (a) \$790. (b) \$31.60. (c) no — \$30 each raises only \$750, which is \$40 short. **Q18.** P \$165, Q \$162; recommend **Campsite Q** (cheaper by \$3).

Fully-worked solutions

Q1. (a) The total is the sum of the four costs: $5.60+21+9.50+5$. (b) $5.60+21+9.50+5=\$41.10$. (c) Left over $=45-41.10=\$3.90$.

Q2. (a) Each panel is a rectangle: area $=3\times1.8=5.4\ \mathrm{m}^2$. (b) Six panels cover $6\times5.4=32.4\ \mathrm{m}^2$. Litres needed $=32.4\div8=4.05\ \mathrm{L}$. Since stain is sold in whole litres, round **up** to 5 litres. (c) Cost $=5\times\$24=\$120$.

Q3. (a) The claim $6:45+90\ \mathrm{min}=8:15$ pm assumes the film begins exactly at $6:45$ pm with nothing (no advertisements or trailers) before it. (b) With 15 minutes of ads first, the film starts at $7:00$ pm; adding 90 minutes gives a finish time of $8:30$ pm. (c) Adding a 20-minute trip to $8:30$ pm gives an arrival home of $8:50$ pm.

Q4. (a) Number of buses $=100\div12$. (b) $100\div12=8.33$ (to 2 d.p.). (c) You cannot hire part of a bus, so round **up**: 8 buses would seat only $8\times12=96$ students, leaving 4 without a seat, so 9 minibuses are needed.

Q5. (a) Rounding to the nearest dollar: $5+3+8+2+6+6=\$30$. (b) Exact total $=4.95+3.20+7.80+2.15+5.50+6.35=\$29.95$. (c) The estimate \$30 is very close to the exact \$29.95; they differ by only $30-29.95=\$0.05$, confirming the exact answer is reasonable.

Q6. (a) Brand A: $39\div18=\$2.17$ per month (to the nearest cent). (b) Brand B: $55\div30=\$1.83$ per month (to the nearest cent). (c) \$1.83 per month is less than \$2.17 per month, so **Brand B** is better value.

Q7. Train fares $=4\times11.20=\$44.80$; ice cream $=4\times6=\$24$. Total $=44.80+13+48+24=\$129.80$. Cost per person $=129.80\div4=\$32.45$.

Q8. Fuel used $=260\times\dfrac{7}{100}=18.2\ \mathrm{L}$. Fuel cost $=18.2\times1.85=\$33.67$.

Q9. Tables needed $=90\div8=11.25$, so round **up** to 12 tables (11 tables seat only 88 guests). Hire cost $=12\times\$15=\$180$.

Q10. Actual total $=342+210+35=\$587$. The estimate of \$600 is larger than the actual \$587, so it was too **high** by $600-587=\$13$.

Q11. Option A $=89+0.40\times60=89+24=\$113$. Option B $=\$145$. Since $113<145$, option A is cheaper by $145-113=\$32$.

Q12. (a) $150\div8=18$ remainder 6, so 18 **full** trays can be filled ($18\times8=144$ sausages). (b) The number left over is $150-144=6$ sausages.

Q13. Starting at $9:50$ am and adding 2 hours reaches $11:50$ am; adding a further 35 minutes gives an arrival time of $12:25$ pm. Adding the 25-minute walk gives $12:50$ pm at the destination.

Q14. Ceiling area $=8\times5=40\ \mathrm{m}^2$. Two coats cover $2\times40=80\ \mathrm{m}^2$. Litres needed $=80\div12=6.67\ \mathrm{L}$ (to 2 d.p.). Each tin holds 4 litres, and $6.67\div4=1.67$, so round **up** to 2 tins (one tin gives only 4 L). Cost $=2\times\$52=\$104$.

Q15. The scale factor is $10 \div 4 = 2.5$. Rice $= 600 \times 2.5 = 1500$ g; stock $= 500 \times 2.5 = 1250$ mL.

Q16. A quick estimate is $28 \times \$20 = \$560$, so the true total is around \$560 — an answer of \$53.20 is ten times too small (the student probably misplaced a decimal point). The correct total is $28 \times 19 = \$532$.

Q17. (a) Museum entry for 25 students $= 12 \times 25 = \$300$. Total cost $= 340 + 300 + 150 = \$790$. (b) Cost per student $= 790 \div 25 = \$31.60$. (c) At \$30 each the class raises $25 \times 30 = \$750$. Since $750 < 790$, this is **not** enough — it falls \$40 short of the \$790 needed.

Q18. Campsite P $= 3 \times 45 + 30 = 135 + 30 = \$165$. Campsite Q $= 3 \times 54 = \$162$. Since $162 < 165$, **Campsite Q** is cheaper, by $165 - 162 = \$3$, so it is the recommended choice.

Theory

Solving a real problem well is more than getting a number out of a calculator. You have to break the problem down, choose a sensible method, use technology properly, and then *check* that the answer makes sense and means something. This section looks at the critical issues that separate a reliable solution from a careless one: **computational thinking**, **following and writing algorithms**, **using technology and its limits**, **accuracy and sensible rounding**, and **checking and communicating** your result.

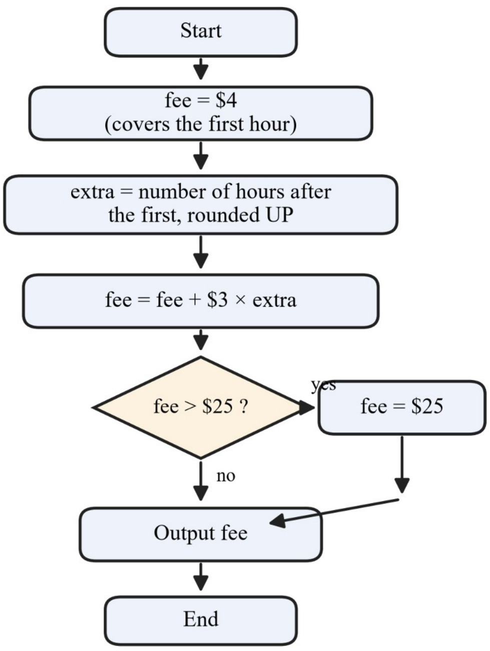
Computational thinking.

Computational thinking is a way of organising a problem so it can be solved step by step. It has four parts:

- **Abstraction** — strip away detail that does not matter and keep only the mathematics. ("How much will the trip cost?" becomes "distance × rate + fixed fees".)
- **Decomposition** — break a big problem into smaller problems you can solve one at a time.
- **Pattern recognition** — notice repetition or a rule that lets you predict the next case without starting over.
- **Algorithms** — write the solution as a clear, ordered list of steps that always gives the right answer for any valid input.

Algorithms.

An **algorithm** is a finite list of unambiguous steps. A good algorithm can be followed by anyone (or a spreadsheet) to get the same result. Algorithms are often shown as a numbered list or as a **flowchart**, in which a diamond marks a **decision** (a yes/no question) that sends you down one of two branches. The flowchart below is an algorithm for a car-park fee.



To *run* an algorithm you take an **input**, follow the steps in order, and read the **output**. For a stay of 5.5 hours: $\text{fee} = \$4$; the extra time is $5.5 - 1 = 4.5$ hours, rounded up to 5; so $\text{fee} = 4 + 3 \times 5 = \$19$; this is not more than $\$25$, so the output is $\$19$.

Using technology, and its limits.

A scientific calculator and a spreadsheet do the arithmetic, but *you* decide what to enter and whether the result is trustworthy. Technology has limits:

- It computes exactly what you type — a mis-keyed digit or a missing bracket gives a confident but wrong answer (**“garbage in, garbage out”**).
- It cannot tell you whether the *model* is sensible or whether an **assumption** is reasonable.
- Long decimals on the screen are not automatically the right level of accuracy.

In a **spreadsheet**, a cell formula begins with = and refers to other cells, for example `=A2*B2` to multiply two cells, or `=SUM(C2:C5)` to total a column. When an input cell changes, every formula that depends on it updates automatically — this is why a spreadsheet is really an algorithm you can re-run instantly.

Accuracy, estimation and sensible rounding.

Before trusting a calculator answer, make a quick **estimate** by rounding each number to one leading digit. If a till shows $48 \times \$19.99 = \$9595.20$, an estimate of $48 \times \$20 = \$960$ shows the real answer is about $\$960$, so the screen value is about ten times too big — a decimal-point or digit error. (The correct value is $\$959.52$.)

Choose a level of accuracy that suits the context:

- **Round up** when any leftover still needs a whole item — tins of paint, buses, packs.
- **Round down** when you can only use complete items you already have — lengths cut from a fixed roll.
- **Round to a sensible number of decimals** for money (2 d.p.) or a stated accuracy; keep full precision in intermediate steps and round only at the end.

Checking and communicating.

Finish every problem by asking *“Is this reasonable?”* — check the size (order of magnitude), the units, and whether the answer fits the real situation. Then communicate it clearly: state the answer **with its units**, and where a model was used, note any **assumptions or limitations** (for example, “this assumes no traffic delays”).

Worked examples

Example 1 — scaffolded

A student council is running a sausage sizzle for 120 people, allowing 2 sausages per person and one piece of bread per sausage. Sausages come in packs of 18 costing $\$7.50$, and bread comes in packs of 20 costing $\$3.20$. Use **decomposition** to find the total cost of sausages and bread.

Working	Comment
Break the job into steps: (1) total sausages, (2) packs of sausages, (3) sausage cost, (4) bread packs, (5) bread cost, (6) total.	**Decomposition** — solve one small piece at a time.
Total sausages $= 120 \times 2 = 240$.	Two per person.
Sausage packs $= 240 \div 18 = 13.3\ldots \to 14$ packs.	**Round up** — a part-pack must be bought whole.
Sausage cost $= 14 \times \$7.50 = \$105$.	Packs bought × price.
Bread: $240 \div 20 = 12$ packs, cost $12 \times \$3.20 = \$38.40$.	240 pieces needed, exactly 12 packs.
Total $= \$105 + \$38.40 = \$143.40$.	Add the two subtotals.

Example 2 — scaffolded

Use the car-park algorithm in the flowchart above to find the fee for a stay of 9 hours, and explain why the final decision step matters.

Working	Comment
Start: $\text{fee} = \$4$.	Flat charge covering the first hour.

Working	Comment
Extra time $= 9 - 1 = 8$ hours (already whole, so rounded up stays 8).	Hours after the first.
$\text{fee} = 4 + 3 \times 8 = 4 + 24 = \$28$.	Add $\$3$ for each extra hour.
Decision: is $\$28 > \$25$? **Yes**, so set $\text{fee} = \$25$.	The cap is applied on the "yes" branch.
Output: $\$25$.	Without the decision step the algorithm would over-charge at $\$28$.

Example 3 — unscaffolded

A receipt lists 32 boxes of notebooks at $\$29.99$ each and prints a total of $\$9596.80$. Without a calculator, decide whether that total is reasonable, then give the correct total.

Estimate by rounding: $32 \approx 30$ and $\$29.99 \approx \$30$, so the total should be about $30 \times \$30 = \$900$ — a few hundred to about a thousand dollars. The printed $\$9596.80$ is roughly **ten times** too large, so it is not reasonable; a digit or decimal point is in the wrong place. Computing exactly,

$$32 \times \$29.99 = \$959.68\,.$$

∴ the printed total is wrong by a factor of about ten; the correct total is $\$959.68$.

Example 4 — unscaffolded

To plan a 480 km country drive, a driver estimates the time as $480 \div 100 = 4.8$ hours "because the speed limit is 100 km/h". Identify the assumption in this model, and give a more realistic estimate that allows an average speed of 90 km/h and one 30-minute rest break.

The model assumes the car travels at a **constant** 100 **km/h for the whole trip** — it ignores town speed limits, traffic, corners and stops, and it ignores rest breaks. A more realistic average speed is 90 km/h, giving a driving time of

$$480 \div 90 = 5.33\ldots\,\text{hours} = 5\,\text{hours}\,20\,\text{minutes}\,.$$

Adding a 30-minute break gives 5 h 20 min $+ 30$ min $= 5$ h 50 min.

∴ the 4.8-hour figure is optimistic because it assumes a constant top speed; a realistic estimate is about 5 **hours** 50 **minutes**.

Practice questions

Scaffolded

Q1. A rectangular courtyard is to be paved. Tiles come in boxes, and each box covers $1.5\,\text{m}^2$. (a) List the quantities you must work out before you can decide how many boxes to buy. (b) The courtyard is $6\,\text{m}$ long and $4\,\text{m}$ wide. Find its area. (c) How many boxes of tiles are needed?

Q2. A taxi charges a flag fall of $\$4.20$ plus $\$2.10$ per kilometre, by the algorithm $\text{fare} = 4.20 + 2.10 \times (\text{km})$. (a) Find the fare for an 8 km trip. (b) Find the fare for a 12 km trip. (c) A fare came to $\$25.20$. How many kilometres was the trip?

Q3. A student needs to work out 312×19. (a) Estimate the answer by rounding each number to one leading digit. (b) Find the exact value. (c) The student's calculator shows $59\,280$. Use your estimate to explain what has gone wrong.

Q4. A school is taking 200 students on an excursion. Each bus seats 45 students. (a) Work out $200 \div 45$. (b) How many buses are needed? (c) Explain why you round the number of buses **up** rather than to the nearest whole number.

Q5. A market stall uses the spreadsheet below, where column C should show (quantity × price).

	A (item)	B (price $)	C (value $)
2	apples	3	
3	pears	4	
4	plums	2	
5	grapes	6	
6	**Total**		

Here the quantities are stored in another column. Assume the quantity for each row is in column D. (a) Write the formula that should go in cell C2. (b) Write the formula that should go in cell C6 to total column C. (c) If the price in cell B2 is changed, does cell C6 update by itself? Explain.

Q6. A student starts a savings plan: $\$5$ in week 1, $\$8$ in week 2, $\$11$ in week 3, and so on, adding $\$3$ each week. (a) Recognise the pattern and write a rule for the amount saved in week n. (b) Use the rule to find the amount saved in week 8. (c) In which week is $\$41$ saved?

Unscaffolded

Q7. A checkout screen shows a total of $\$497.20$ for 4 identical items priced at $\$12.43$ each. Use estimation to decide whether the total is reasonable, then give the correct total.

Q8. A rectangular paddock measuring $25\,\mathrm{m}$ by $40\,\mathrm{m}$ is to be fenced with a post every $5\,\mathrm{m}$ around the closed boundary. Decompose the problem and find the number of posts required.

Q9. An algorithm classifies a whole number: *Step 1 — if the number is divisible by* 2*, output "even", otherwise output "odd". Step 2 — if the number is greater than* 50*, add the word "and large".* State the full output when the input is 84.

Q10. A student's three test marks are $17/20$, $15/20$ and $18/20$. (a) Find the total mark out of 60. (b) Express the result as a percentage. (c) The student reports "$83.33333\,\%$". State a more sensible way to report the percentage and why.

Q11. A news report estimates a festival crowd by saying, "the ground holds $40\,000$ people, so $40\,000$ people attended." Identify the assumption this makes, explain why it may be wrong, and suggest a better way to estimate the crowd.

Q12. In a payroll spreadsheet, gross pay in column E should equal (hours in column C) × (rate in column D). (a) Write the correct formula for cell E2. (b) A worker typed `=C2+D2` into E2. Explain why this is wrong and what it would compute.

Q13. A social-media post claims that "6.2 million seconds is about a week." By converting 6.2×10^6 seconds to days (there are $86\,400$ seconds in a day), decide whether 6.2 million seconds is closer to one week or to two months, and justify your answer.

Q14. A craftsperson cuts $2.4\,\mathrm{m}$ lengths of timber from a $10\,\mathrm{m}$ roll. (a) Work out $10 \div 2.4$. (b) State how many complete $2.4\,\mathrm{m}$ lengths can be cut, and explain why here you round **down**, unlike the paint-and-buses situations where you round up.

Q15. A shop assistant follows a "give change" algorithm: *subtract the price from the amount paid, then hand over the fewest notes and coins that make up the difference.* A customer pays with a $\$50$ note for goods costing $\$36.80$. (a) Find the change owed. (b) List one possible combination of Australian notes and coins that makes up the change.

Q16. A tiling border is built in stages. Stage 1 uses 4 tiles, stage 2 uses 8 tiles, stage 3 uses 12 tiles, and the pattern continues. (a) Write a rule for the number of tiles at stage n. (b) How many tiles are needed for stage 10?

Q17. A runner claims to have averaged 5 metres per second for 2 hours. (a) Convert 2 hours to seconds and find the distance covered, in metres and in kilometres. (b) A half-marathon is about 21 km and a full marathon about 42 km. Comment on whether the runner's claim is reasonable.

Q18. A room's walls have a total area of $88\,\mathrm{m}^2$ and are to be given **two** coats of paint. One litre of paint covers $11\,\mathrm{m}^2$, and paint is sold only in 4-litre tins costing $\$52$ each. (a) Find the total area to be covered and the number of litres needed. (b) Find the number of tins to buy and the total cost. (c) State one assumption in this calculation that would make the true amount of paint needed a little less.

Answers

Q1. (a) courtyard area and area covered per box (then divide). (b) $24\,\mathrm{m}^2$. (c) 16 boxes. **Q2.** (a) $\$21.00$. (b) $\$29.40$. (c) 10 km. **Q3.** (a) $300 \times 20 = 6000$. (b) 5928. (c) 59 280 is about ten times the estimate — one digit too many (a place-value error). **Q4.** (a) $4.44\ldots$ (b) 5 buses. (c) 4 buses leave 20 students without a seat, so you must round up to fit everyone. **Q5.** (a) `=D2*B2`. (b) `=SUM(C2:C5)`. (c) Yes — C2 recalculates from B2, and C6 totals column C, so both update automatically. **Q6.** (a) amount $= 3n + 2$ dollars. (b) $\$26$. (c) week 13. **Q7.** Estimate $4 \times \$12 = \$48$, so about $\$50$; $\$497.20$ is about ten times too big and is unreasonable. Correct total $= \$49.72$. **Q8.** Perimeter $= 130\,\mathrm{m}$; posts $= 130 \div 5 = 26$ posts. **Q9.** "even and large". **Q10.** (a) $50/60$. (b) $83.3\,\%$ (to 1 d.p.). (c) Report $83.3\,\%$ or $83\,\%$ — marks to five decimal places is false precision for a test score. **Q11.** It assumes the ground was exactly full (at capacity). Attendance is rarely exactly capacity; better to estimate by counting a section and multiplying, or using ticket sales. **Q12.** (a) `=C2*D2`. (b) `=C2+D2` *adds* hours and rate instead of multiplying, giving a meaningless number (e.g. $38 + 25 = 63$ instead of pay). **Q13.** $6.2 \times 10^6 \div 86\,400 \approx 71.8$ days ≈ 10 weeks — much closer to two months than to one week. **Q14.** (a) $4.16\ldots$ (b) 4 complete lengths; you can only cut whole lengths from the timber you have, so the leftover $0.4\,\mathrm{m}$ is not enough for another — round down. **Q15.** (a) $\$13.20$. (b) e.g. one $\$10$ note, one $\$2$ coin, one $\$1$ coin and one 20-cent coin. **Q16.** (a) tiles $= 4n$. (b) 40 tiles. **Q17.** (a) $2\,\mathrm{h} = 7200\,\mathrm{s}$; distance $= 5 \times 7200 = 36\,000\,\mathrm{m} = 36\,\mathrm{km}$. (b) 36 km in 2 hours averages 18 km/h — close to elite marathon pace and far beyond what an ordinary recreational runner can sustain, so the claim is not reasonable for a typical runner. **Q18.** (a) $176\,\mathrm{m}^2$ to cover; $176 \div 11 = 16$ litres. (b) $16 \div 4 = 4$ tins; $4 \times \$52 = \$208$. (c) Doors and windows are not painted, so a little less paint is really needed (the $88\,\mathrm{m}^2$ counts the whole wall).

Fully-worked solutions

Q1. (a) You need the **area of the courtyard** and the **area each box covers**; the number of boxes is the first divided by the second (rounded up). (b) Area $= \text{length} \times \text{width} = 6 \times 4 = 24\,\mathrm{m}^2$. (c) Boxes $= 24 \div 1.5 = 16$ exactly, so 16 boxes.

Q2. Using $\text{fare} = 4.20 + 2.10 \times (\text{km})$: (a) $4.20 + 2.10 \times 8 = 4.20 + 16.80 = \$21.00$. (b) $4.20 + 2.10 \times 12 = 4.20 + 25.20 = \$29.40$. (c) Reverse the steps: $25.20 - 4.20 = 21.00$, then $21.00 \div 2.10 = 10$ km.

Q3. (a) $312 \to 300$ and $19 \to 20$, so the estimate is $300 \times 20 = 6000$. (b) $312 \times 19 = 5928$. (c) The true answer is about 6000, but 59 280 is about **ten times** larger, so an extra digit has crept in — a place-value (decimal-point) error.

Q4. (a) $200 \div 45 = 4.44\ldots$ (b) Four buses seat only $4 \times 45 = 180$ students, leaving 20 without a seat, so a fifth bus is needed: 5 buses. (c) Rounding to the nearest whole number would give 4, but a part-bus of students still needs a whole bus, so you must always round **up** when every leftover needs another whole item.

Q5. (a) Value = quantity × price = `=D2*B2`. (b) The total of column C is `=SUM(C2:C5)`. (c) Yes. Cell C2's formula recalculates whenever B2 changes, and C6 sums the C column, so the total updates automatically — that is the point of using formulas rather than typing numbers.

Q6. (a) The amounts $5, 8, 11, \ldots$ increase by 3 each week and start at 5, so week n saves $5 + 3(n-1) = 3n + 2$ dollars. (b) Week 8: $3 \times 8 + 2 = 24 + 2 = \$26$. (c) Set $3n + 2 = 41$, so $3n = 39$ and $n = 13$: week 13.

Q7. Estimate: $4 \times \$12 = \$48$, so the total should be about $\$50$. The screen value $\$497.20$ is about **ten times** too large, so it is unreasonable — a decimal point is misplaced. The correct total is $4 \times \$12.43 = \$49.72$.

Q8. Decompose: (1) find the perimeter, (2) divide by the post spacing. Perimeter $= 2 \times (25 + 40) = 2 \times 65 = 130\,\mathrm{m}$. Because the fence is a closed loop, the number of posts equals the number of $5\,\mathrm{m}$ gaps: $130 \div 5 = 26$ posts.

Q9. Input 84: Step 1 — 84 is divisible by 2, so output "even". Step 2 — $84 > 50$, so append "and large". Full output: **"even and large"**.

Q10. (a) Total $= 17 + 15 + 18 = 50$ out of 60. (b) $50 \div 60 = 0.8333\ldots = 83.3\,\%$ (to 1 d.p.). (c) A test percentage to five decimal places is false precision — the marks are only whole numbers. Reporting $83.3\,\%$ (or $83\,\%$) is more sensible.

Q11. The statement assumes the ground was **exactly at capacity**. In reality attendance is almost never exactly the maximum — some seats are usually empty — so $40\,000$ is an upper limit, not a count. A better estimate counts the people in one section (or one grandstand) and multiplies by the number of similar sections, or uses the number of tickets actually sold.

Q12. (a) Gross pay = hours × rate = `=C2*D2`. (b) `=C2+D2` **adds** the hours to the rate instead of multiplying them. For example 38 hours and a $\$25$ rate would give 63, which is neither a number of hours nor an amount of pay — it is meaningless.

Q13. Convert: $6.2\times10^6\div86\,400=71.76\ldots$ days. That is about $71.8\div7\approx10.3$ weeks. Ten weeks is close to **two months**, and nowhere near one week, so the "about a week" claim is far too small — one week is only about $604\,800$ seconds ($7\times86\,400$), a little over half a million, not 6.2 million.

Q14. (a) $10\div2.4=4.16\ldots$ (b) Only 4 complete $2.4\,\mathrm{m}$ lengths can be cut, using $4\times2.4=9.6\,\mathrm{m}$ and leaving $0.4\,\mathrm{m}$ — not enough for a fifth length. Here you **round down** because you can only use whole lengths already contained in the roll, whereas buying tins or buses means any leftover forces you to buy one more (round up).

Q15. (a) Change $=\$50-\$36.80=\$13.20$. (b) The fewest pieces: one $\$10$ note, one $\$2$ coin, one $\$1$ coin and one 20-cent coin, which total $10+2+1+0.20=\$13.20$.

Q16. (a) The counts $4,8,12,\ldots$ are the multiples of 4, so stage n uses $4n$ tiles. (b) Stage 10: $4\times10=40$ tiles.

Q17. (a) 2 hours $=2\times60\times60=7200$ seconds. Distance $=\text{speed}\times\text{time}=5\times7200=36\,000\,\mathrm{m}=36\,\mathrm{km}$. (b) 36 km in 2 hours is an average of 18 km/h (5 m/s). That is around elite marathon pace — the best runners cover a full marathon (42 km) in a little over two hours — so while a world-class athlete could approach it, it is far beyond what an ordinary recreational runner could keep up for two hours. For a typical runner, then, the claim is not reasonable.

Q18. (a) Two coats over $88\,\mathrm{m}^2$ is $88\times2=176\,\mathrm{m}^2$ to cover. At $11\,\mathrm{m}^2$ per litre, that needs $176\div11=16$ litres. (b) At 4 litres per tin, $16\div4=4$ tins, costing $4\times\$52=\$208$. (c) The $88\,\mathrm{m}^2$ counts the whole wall area, but doors and windows are not painted, so the true paint needed is a little less than calculated.